Measuring the Thermal Conductivities of Low Heat Conducting Disk Samples by Monitoring the Heat Flow

José A. Ibáñez-Mengual, Ramón Valerdi-Pérez y José A. García-Gamuz
Departamento de Física, Universidad de Murcia, Campus de Espinardo, 30071 Murcia

ABSTRACT
This article aims to establish an experimental procedure to measure heat transmission coefficients in low heat conducting materials. The newly developed model takes as starting point the application of Fourier’s law to a disk sample when a temperature gradient is established between its faces. The power of a heating element is determined as the heat transfer coefficient of the problem disk. Initially, a glass vessel containing water is placed in direct contact with the heating element; then, a problem plastic disk is placed between this element and the glass vessel, treating the set as a composite wall. Prior to the above the water equivalent of a calorimetric set (vessel + water + accessories) and the thermal conductivity of the vessel must be determined. The thermal conductivity of the problem plastic disk sample is obtained for temperatures ranging from 30 to 70°C. The results reveal the existence of some type of structural transition for the problem material.

Keywords: thermal conductivity; heat flow; plastic material.

I. INTRODUCTION
The problem of heat conduction in solids has been widely studied, and many solutions are available for various boundary conditions, forms and dimensions. Measurement of the thermal properties of a material is an important issue since the parameters associated with it are extremely relevant both in the laboratory and in industrial design [1] [2].

The direct measurement of heat transfer has been studied using several techniques, including the use of circular heat flow discs, thermistors, as well as comparisons and calibrations of heat flux sensors [3] [4]. The devices and techniques studied include differential scanning calorimeters, improved thin film heat transfer gauges, infrared thermography [5] and liquid crystals [6] and several other visualization techniques [7]. The techniques involved are used for direct measurement of the temperature with thin film thermocouples [8], sputtered micro thermocouples, thermo fluorescence [9], thin-foil thermography [10] and fine wire thermocouples for transient measurement.

Thermal conductivity [11] [12] has been studied for a wide variety of materials [13] [14] [15] [16] using various experimental techniques, such as transient plane source, pulsed and thermal quadruples, electrical resistance thermometry [17], transient and parallel hot wire techniques, infrared thermography, laser flash, photoacoustics, etc.

The study in this case is directed towards plastic materials. Due to the great variety of new products intended for recycling, the rapid and accurate determination of their thermal properties is of great importance [18] [19] [20] [21] [22]. The aim of the experiments described was to design a modified Lee’s disk method for measuring thermal conductivities in samples of low heat conducting materials. [23] [24] [25] [26].

II. THEORY
In the case of water (mass m) in a glass vessel placed on a heating element at temperature \( \theta_1 > \theta_0 \) (room temperature), as in Figure 1, the power transferred corresponds to the heat gained due to losses from the system, characterized by \( \lambda_1 \) and \( \lambda_2 \) coefficients respectively, resulting in the following equation.

\[
\frac{\Delta Q}{\Delta t} = (mc + k) \frac{d\theta}{dt} = C \frac{d\theta}{dt} = \left( \frac{\Delta Q}{\Delta t} \right)_1 + \left( \frac{\Delta Q}{\Delta t} \right)_2 = \lambda_1 (0-\theta_0) - \lambda_2 (0-\theta_0)
\]

where \( \left( \frac{\Delta Q}{\Delta t} \right)_1 \) and \( \left( \frac{\Delta Q}{\Delta t} \right)_2 \) are the net power entering the system from the heating element and the losses respectively, \( \theta \) is its temperature, \( C \equiv mc + k \) is the heat capacity of the system (\( c_a \) the specific heat of water) and \( k \) the water equivalent of the same, which corresponds to the heat capacity of the surrounding vessel, as well as of the measuring probe (accessories). Regrouping terms in the above equation gives

\[
-(\lambda_1 + \lambda_2) \theta + \lambda_1 \theta_0 + \lambda_2 \theta_a = C \frac{d\theta}{dt},
\]

Introducing \( \alpha \equiv \lambda_1 + \lambda_2 \), and \( \gamma \equiv \lambda_1 \theta_0 + \lambda_2 \theta_a \), it follows that

\[
C \frac{d\theta}{dt} = -\alpha \theta + \gamma \Rightarrow \frac{d\theta}{dt} = \frac{\theta - \alpha \theta}{C} \theta \beta dt
\]
system, whose resolution provides expressions for the coefficients \( \lambda_1 \) and \( \lambda_2 \):

\[
\begin{align*}
\lambda_1 &= \frac{C}{1 + \theta_f} \\
\lambda_2 &= \lambda_1 \tilde{b}_f
\end{align*}
\]  

(7)

Equations (7) correspond to the gain and loss coefficients of the system, whose values give the power supplied by the heating element, in accordance with eq. (1). In particular, if we consider that a sufficient time interval has passed to reach the steady state \( (\theta = \theta_f) \), of the net heat transfer is cancelled, so that \( (\delta Q/dt) = 0 \) and the power supplied by the heating element is

\[
(\delta Q/dt)_i = \lambda_1 (\theta_c - \theta_f)
\]  

(8)

2.1. Determination of water equivalent by the mixtures method.

To calculate the power supplied by the heating element, it is necessary to know the water equivalent value of the system \( k \). For this, it is possible to use the mixtures method; which consists of introducing a known metal body (copper disk) in the glass vessel containing water at room temperature \([27]\). The body is previously heated to reach the boiling point of water (100ºC at normal atmospheric pressure). In this way it is possible to know the temperature of the material in the water, whose temperature increases. The metal is removed from the water at the end of time interval \( \Delta t \) and consequently the water temperature decreases due to the cooling effect of its surroundings. Figure 2 outlines the thermogram corresponding to the full process, which shows three stages: pre-heating, heating and post-heating which, in our case, lasted 20, 1 and 20 minutes, respectively (Figure 3).

![Figure 1. Heating source + water + system](image1)

![Figure 2. Full process thermogram](image2)
The heat balance equation corresponding to the heating stage can be written as

$$Mc_M(\theta_t-\theta_i)=(m_c c_a+k)(\theta_t-\theta_i)+\dot{Q}_p \Delta t$$

where $M$ is the body mass, $c_M$ its specific heat, $m_c$ is the water mass, $c_a$ the water specific heat capacity, $\theta_t$ and $\theta_i$ are the initial and final temperature corresponding to the heating time interval $\Delta t$, and $\dot{Q}_p$ indicates the losses of heat per unit of time in this stage, which can be estimated as the arithmetic mean of the exchanges associated to the pre-heating and post-heating stages,

$$\dot{Q}_p=\frac{\dot{Q}_{p}^{\text{pre}}+\dot{Q}_{p}^{\text{post}}}{2}$$

where $\dot{Q}_{p}^{\text{pre}}$ corresponds to thermal stabilization of the glass vessel + water system at temperature $\theta_a$ with $\theta_{i,20}$ and $\theta_i$ are the initial and final temperature of this stage and $\dot{Q}_{p}^{\text{post}}$ is the heat lost when the metal body is removed and the water temperature approaches the surrounding temperature, $\theta_t$ and $\theta_{f,20}$ are the initial and final temperatures of the stage.

Thus, the energy balance given by equation (9) can be rewritten as

$$Mc_M(\theta_t-\theta_i)=(m_c c_a+k) \left[ \theta_t-\theta_i + \frac{[(\theta_{i,20}-\theta_t)+(\theta_{t,20}-\theta_i)]}{40} \right]$$

Equation (10) enables us to determine the water equivalent $k$ value, since we know mass and specific heat capacity of the metal body employed.

### 2.2. Thickness and thermal conductivity calculation for problem materials. Composed wall method.

When a problem material disk sample, whose heat transmission coefficient we wish to know, is placed between a heating source and a glass vessel with water, the system corresponds to a composed wall arranged as shown in Figure 4 where $k_1$ and $k_2$ are thermal conductivities and $\delta_1$ and $\delta_2$ thicknesses, respectively. $\theta_{i,*} (i=1,2,3)$ represents temperatures at the three interfaces: 1) heating source/sample material; 2) sample material/glass layer; 3) glass layer/water (see Figure 5). The superscript * will be used to indicate that the affected quantities refer to the problem material disk. Given that in the steady state the heat flow is the same through every element of the composed wall, we can write

$$\dot{Q}^* = \frac{\delta Q}{dt} = \frac{\delta_1}{k_1A} + \frac{\delta_2}{k_2A} = \frac{\delta_1 + \delta_2}{k_1 + k_2}$$

where “$\dot{Q}^*$” denotes the dissipated power by the heating source towards the system, $A$ being the common area of different interfaces.
When the glass vessel with water is placed in direct contact with the heating source, the power supplied by the source \( \text{pot} \) is given by the expression

\[
\text{pot} = \left( \frac{dQ}{dt} \right) = k_2 \frac{A}{\delta_1} (\theta - \theta_1) \tag{12}
\]

\( \theta_1 \) being the temperature at the top of the glass vessel (at the interface glass/water).

The thermal conductivity of the glass \( k_2 \) is determined from equation (12), if the thickness \( \delta \) is known so that it is possible to obtain the thermal conductivity of the problem material \( k_1 \) from equation (11)

\[
k_1 = \frac{\text{pot} \cdot \delta \cdot k_2}{(\theta - \theta_1) A \cdot k_2 \cdot \delta_2 \cdot \text{pot}} \tag{13}
\]

### III. EXPERIMENTAL DEVICE

An experimental device was designed, which enables a problem disk of suitable dimension (Ø 3 cm, thickness 2 mm) to be manipulated (see Figure 6). The disk was placed on top of an aluminium cylinder of 2 cm thickness. The cylinder is insulated with armaplex, and an electrical resistance placed within is used as heating element. The heating source temperature \( \theta_s \) is determined by a type J thermocouple (iron-constantan, range -210 to 1200 °C) lodged in the aluminium cylinder. The resistance and thermocouple are connected to a control device, provided with a screen that shows instantaneous values of the heating cylinder temperature, \( \theta_c \), which is the parameter that governs and regulates the heating of the system. Another J thermocouple continuously measures the temperature of the deionized water (40 cm³), \( \theta_w \) contained in a glass vessel (wall thickness 2 mm) placed on of the heating cylinder. This temperature reaches a final value \( \theta_w \) for every \( \theta_c \) considered. The surface temperature values of the heating cylinder \( \theta_c \), for every \( \theta \) value selected, was determined by means of an infrared thermometer with configured emissivity (in our case, an Optris® LS thermometer with infrared dual focus), provided with a laser indication device that directs a beam at the top of the problem body (shaping a cross oriented to the center of the disk). This provided a guide for the optimal measuring distance (object distance (D) and focal measurement area (S), whose D:S ratio in our case is 75:1 for the disk used).

### IV. RESULTS AND DISCUSSION

Equation (10) enables the determination of the water equivalent of our calorimetric set \( k (14,623 \text{ g}) \) and consequently the heat capacity, \( C \), whose value \( (64,623 \text{ cal g}^{-1} ^\circ\text{C}^{-1}) \), together with \( b \) in the exponential fitting (see eq. (4)) for the corresponding water thermogram (glass vessel + water). Figure 7 shows the result obtained for reference temperature \( \theta_r = 50^\circ\text{C} \). When temperature \( \theta_r \) ranges between 30 and 70°C (5°C steps), the coefficients \( \lambda_1 \) and \( \lambda_2 \) can be determined (see eq. 7). According to eq. (8), in this case (glass) the power \( (\delta Q/dt) = \lambda_1 (\theta_r - \theta) \) in eq. (12) gives the glass heat conductivity \( k_2 \), knowing the thickness \( \delta \). These results are listed in Table 1.

![Figure 7. Thermogram for reference temperature \( \theta_r = 50^\circ\text{C} \) in the case of water (system: water + vessel + accessories).](image-url)
Table 1: Results for the thermal scanning accomplishes in the case of a glass vessel with water placed on the heating cylinder. \( \theta_0 \) is the real temperature measures with infrared thermometer.

<table>
<thead>
<tr>
<th>( \theta_0 )</th>
<th>( \theta_0^* )</th>
<th>( \theta_1 )</th>
<th>( \theta_1^* )</th>
<th>( \Delta \theta = (\theta_0 - \theta_1) )</th>
<th>( \frac{C}{(\lambda_1 + \lambda_2)} )</th>
<th>( \frac{\theta_1}{(\lambda_1 + \lambda_2)} )</th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \text{pot} ) (w)</th>
<th>( k_2 ) (w/mK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>29.5</td>
<td>18.9</td>
<td>23.0</td>
<td>6.5</td>
<td>0.0296</td>
<td>1.6185</td>
<td>0.0113</td>
<td>0.0183</td>
<td>0.3075</td>
<td>0.1028</td>
</tr>
<tr>
<td>35</td>
<td>34.1</td>
<td>21.3</td>
<td>25.2</td>
<td>8.9</td>
<td>0.0444</td>
<td>2.3058</td>
<td>0.0134</td>
<td>0.0310</td>
<td>0.5000</td>
<td>0.1222</td>
</tr>
<tr>
<td>40</td>
<td>39.4</td>
<td>21.9</td>
<td>27.2</td>
<td>12.2</td>
<td>0.0467</td>
<td>2.3153</td>
<td>0.0141</td>
<td>0.0326</td>
<td>0.7200</td>
<td>0.1283</td>
</tr>
<tr>
<td>45</td>
<td>44.4</td>
<td>22.0</td>
<td>28.8</td>
<td>15.6</td>
<td>0.0467</td>
<td>2.3080</td>
<td>0.0141</td>
<td>0.0326</td>
<td>0.9228</td>
<td>0.1286</td>
</tr>
<tr>
<td>50</td>
<td>49.5</td>
<td>21.9</td>
<td>30.7</td>
<td>18.8</td>
<td>0.0468</td>
<td>2.1542</td>
<td>0.0148</td>
<td>0.0319</td>
<td>1.1651</td>
<td>0.1350</td>
</tr>
<tr>
<td>55</td>
<td>54.3</td>
<td>23.1</td>
<td>32.8</td>
<td>21.5</td>
<td>0.0475</td>
<td>2.2179</td>
<td>0.0148</td>
<td>0.0327</td>
<td>1.3258</td>
<td>0.1344</td>
</tr>
<tr>
<td>60</td>
<td>59.1</td>
<td>23.1</td>
<td>34.1</td>
<td>24.9</td>
<td>0.0476</td>
<td>2.2570</td>
<td>0.0146</td>
<td>0.0330</td>
<td>1.5225</td>
<td>0.1330</td>
</tr>
<tr>
<td>65</td>
<td>63.7</td>
<td>23.5</td>
<td>35.4</td>
<td>28.3</td>
<td>0.0527</td>
<td>2.3776</td>
<td>0.0156</td>
<td>0.0371</td>
<td>1.8442</td>
<td>0.1420</td>
</tr>
<tr>
<td>70</td>
<td>68.4</td>
<td>23.5</td>
<td>36.6</td>
<td>31.7</td>
<td>0.0556</td>
<td>2.4113</td>
<td>0.0163</td>
<td>0.0393</td>
<td>2.1618</td>
<td>0.1483</td>
</tr>
</tbody>
</table>

From the thermograms for the system incorporating the problem material disk (PMD) between the heating element and the glass vessel containing water (Figure 5). The values of \( \lambda_1^* \) and \( \lambda_2^* \) were obtained for every temperature \( \theta_0 \) between 30 and 70°C (5°C steps), as well as the heating power \( (\delta Q/dt)^* \), whose values permit the determination of the disk thermal conductivity in accordance with (13). Figure 8 shows the corresponding thermogram at \( \theta_0 = 50°C \) and Table 2 lists the results obtained when \( \theta_0 \) change over the range 30-70°C using a PMD placed between the heating cylinder and the glass vessel with water.

Table 2: Problem plastic material disk: gain and loss parameters \( (\lambda_1 \text{ and } \lambda_2) \), heating power \( \text{pot} \) and heat transmission coefficient \( k_1 \) as a function of the measured temperature \( \theta_0 \).

<table>
<thead>
<tr>
<th>( \theta_0 )</th>
<th>( \theta_0^* )</th>
<th>( \theta_1 )</th>
<th>( \theta_1^* )</th>
<th>( \Delta \theta = (\theta_0 - \theta_1) )</th>
<th>( \frac{C}{\lambda_1 + \lambda_2} )</th>
<th>( \frac{\theta_1}{\lambda_1 + \lambda_2} )</th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \text{pot} ) (w)</th>
<th>( k_1 ) (w/mK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>29.5</td>
<td>21.5</td>
<td>22.8</td>
<td>6.6</td>
<td>0.0399</td>
<td>3.1926</td>
<td>0.0064</td>
<td>0.0335</td>
<td>0.1794</td>
<td>0.1369</td>
</tr>
<tr>
<td>35</td>
<td>34.1</td>
<td>21.5</td>
<td>24.5</td>
<td>9.6</td>
<td>0.0384</td>
<td>3.1583</td>
<td>0.0092</td>
<td>0.0292</td>
<td>0.3705</td>
<td>0.2702</td>
</tr>
<tr>
<td>40</td>
<td>39.4</td>
<td>21.6</td>
<td>26.2</td>
<td>13.2</td>
<td>0.0412</td>
<td>2.8867</td>
<td>0.0107</td>
<td>0.0305</td>
<td>0.5898</td>
<td>0.4057</td>
</tr>
<tr>
<td>45</td>
<td>44.4</td>
<td>22.4</td>
<td>27.8</td>
<td>16.6</td>
<td>0.0438</td>
<td>3.1223</td>
<td>0.0106</td>
<td>0.0332</td>
<td>0.7376</td>
<td>0.3892</td>
</tr>
<tr>
<td>50</td>
<td>49.5</td>
<td>21.4</td>
<td>30.0</td>
<td>19.5</td>
<td>0.0465</td>
<td>2.2575</td>
<td>0.0143</td>
<td>0.0322</td>
<td>1.1618</td>
<td>3.5232</td>
</tr>
<tr>
<td>55</td>
<td>54.3</td>
<td>21.8</td>
<td>31.7</td>
<td>22.6</td>
<td>0.0467</td>
<td>2.2813</td>
<td>0.0142</td>
<td>0.0324</td>
<td>1.3441</td>
<td>3.5283</td>
</tr>
<tr>
<td>60</td>
<td>59.1</td>
<td>22.0</td>
<td>33.0</td>
<td>26.1</td>
<td>0.0467</td>
<td>2.3859</td>
<td>0.0138</td>
<td>0.0329</td>
<td>1.5068</td>
<td>2.2718</td>
</tr>
<tr>
<td>65</td>
<td>63.7</td>
<td>21.9</td>
<td>34.2</td>
<td>29.5</td>
<td>0.0467</td>
<td>2.4015</td>
<td>0.0137</td>
<td>0.0330</td>
<td>1.6955</td>
<td>1.0466</td>
</tr>
<tr>
<td>70</td>
<td>68.4</td>
<td>22.3</td>
<td>35.5</td>
<td>32.9</td>
<td>0.0469</td>
<td>2.5060</td>
<td>0.0134</td>
<td>0.0335</td>
<td>1.8419</td>
<td>0.6835</td>
</tr>
</tbody>
</table>

Figures 9 and 10 depict the meshes corresponding to the thermal behavior of each of the systems (water and PMD). Heating raises the temperature from environmental temperature \( \theta_0 \) to the final temperature \( \theta_f \) corresponding to the steady state.
Defect assessment on...

Figures 11 and 12 show the values of the heating power and heat transmission coefficient corresponding to the glass of the vessel used (system without PMD), while Figures 13 and 14 correspond to the same quantities when PMD is used. In the last case the values obtained for the heat transmission coefficient show a clear peak suggesting out some type of internal structure transition for the PMD between 28 and 34°C.

![Figure 10. Mesh for thermal behavior for the system (PMD + glass + water + accessories)](image)

![Figure 11. Heating power for the system glass +water+accessories vs. temperature.](image)

![Figure 12. Heat transmission coefficients for water vs. temperature.](image)

![Figure 13. Heating power for the complete system PMD+glass+water+accessories vs. temperature.](image)

![Figure 14. Heat transmission coefficients for PMD vs. temperature. Note the peak zone at 30-32°C.](image)

**REFERENCES**


[5]. D.A. Gonzalez et al., Defect assessment on...
radiant heaters using infrared thermography, NDT and E International 38 (6), 428 (2005).


