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Error Estimation Using Intuitionistic Fuzzy Linear Regression Analysis

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ABSTRACT

In this paper an attempt is made to reduce the error of non-fuzzy data using intuitionistic fuzzy linear regression. One numerical example have been analyzed. It is concluded from the example, error value in the normal regression method is better compared to intuitionistic fuzzy linear regression method.

Key word: Assumption Violation Problem (AVP), Regression Analysis, Linear Programming.

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I. INTRODUCTION

Mathematical models are deterministic models are given by $Y= a + b X$, the regression models are combination of deterministic and non-deterministic components. It can be stated as $Y= a + b X + u$, where 'u' is non-deterministic components used to represent uncertainty in the real world. Since the real world is plenty of uncertainties. The mathematical models are to be converted in to regression based econometric models. The behavior of variable 'u' is restricted by imposing some assumptions, namely no autocorrelation of 'u' values and constancy (homoscedasticity) of 'u' values. Violation of these assumption is referred to as autocorrelation and heteroscedasticity, for detail kindly refer [3] and [6].

The Violation of these two stochastic assumption enlarge the error values. In Mathematics and Statistics many research works are conducted to find methods to minimize the error as minimization of error would solve the aboveAssumption Violation Problems. In this paper we attempt to use the Intuitionistic Fuzzy Linear Regression (IFLR) as a solution of Assumption Violation Problems.

Regression analysis is one of the areas in which fuzzy set theory has been used frequently, since Tanaka [8] initiated research on fuzzy linear regression (FLR) analysis, this area has been widely developed and wide variety of methods have been proposed. One approach to deal with FLR is Linear Programming (LP). This approach was first introduced by Tanaka and developed by others, and next approach is least squares method, which was first introduced by celmins [2] and developed by others [4].

The fuzzy set theory was introduced by Zadeh [9] has derived meaningful applications in many field of studies. The idea of fuzzy set is welcomed because it handles uncertainty and vagueness. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. However in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to one minus the membership degree because there may be some hesitation degree. Atanassov [1] introduced the concept of IFS as a generalization of fuzz set. Atanassov added in the definition of fuzzy set a new component which determines the degree of non-membership. Fuzzy sets give the degree of membership of an element while IFS give both a degree of membership and a degree of non-membership. The only requirement is that the sum of these two degrees is not greater than 1. This paper present linear regression analysis in an intuitionistic fuzzy environment using intuitionistic fuzzy linear models with symmetric triangular intuitionistic fuzzy number (STIFN) coefficients. The aim of this intuitionistic fuzzy regression (IFR) is to find the coefficients of a proposed model for all given data sets. Here, the basic idea is to minimize the intuitionistic fuzziness of the model by minimizing the total support of the IF coefficients, subject to including all the given data. The data values are analyzed first by running IFLR and then by running normal regression.

The rest of this paper is organized as follows: In Section 2, the basic concept and definitions are presented, In Section 3, intuitionistic fuzzy linear model is given, In Section 4, intuitionistic fuzzy regression analysis with numerical example is given. Finally the conclusions are presented in Section5.

II. PRELIMINARIES

Definition 2.1: [5]An intuitionistic fuzzy set (IFS) A in X is defined as an object of the form A = $\{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ where the functions $\mu A : X \to [0, 1]$ and $\nu A : X \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively, and for every $x \in X$ in A, $0 \leq \mu A(x) + \nu A(x) \leq 1$ holds.

Definition 2.2: [5] For every common fuzzy subset A on X, Intuitionistic Fuzzy Index of x in A is defined as $\pi_A(x)$ = 1- $\mu_A(x)$ - $\nu_A(x)$. It is also known as degree of hesitancy or degree of uncertainty of the element x in A. Obviously, for every $x \in X$, $0 \leq \pi_A(x) \leq 1$.

Definition 2.3: A triangular intuitionistic fuzzy number (TIFN) $\tilde{A}^I$ is an IFS in R with membership and non-membership function as follows:

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \frac{x-(a-\alpha)}{\alpha} & for\ x \in [a-\alpha, a] \\ \frac{\alpha+\beta-x}{\beta} & for\ x \in [a,\ \alpha+\beta] \\ 0 & otherwise \end{cases}$$

And $$\nu_{\tilde{A}^I}(x) = \begin{cases} \frac{a-x}{\alpha'} & for\ x \in [a-\alpha', a] \\ \frac{x-a}{\beta'} & for\ x \in [a,\ \alpha+\beta'] \\ 1 & otherwise \end{cases}$$

Where a∈R, α, β, α', β' ≥ 0 such that $\alpha \leq \alpha'$ and $\beta \leq \beta'$. The symbolic representation of TIFN is $\tilde{A}^I{}_{TIFN}$ = [a:α, β, α', β']. Here $\alpha$and β are called left and right spreads of membership function $\mu_{\tilde{A}^I}$(x) respectively. $\alpha'$and β'represent left and right spreads of non-membership function$\nu_{\tilde{A}^I}$(x) respectively. The diagrammatic representation of a TIFN is in following

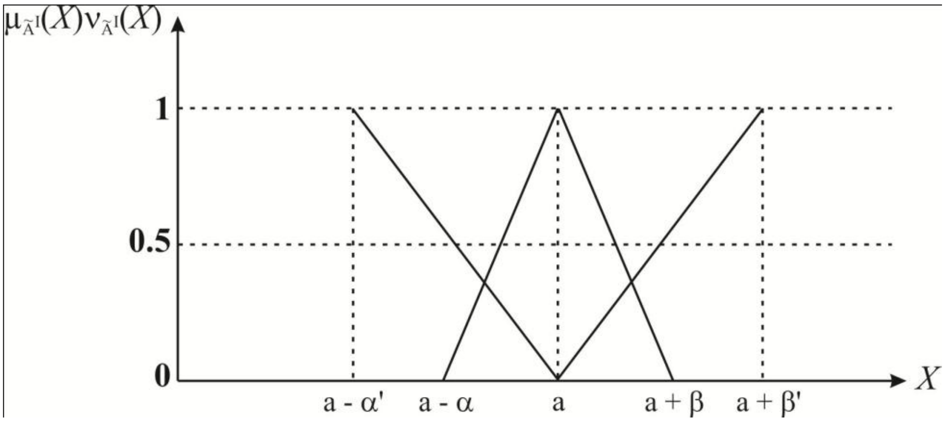

Figure 1: Triangular intuitionistic fuzzy number

Definition 2.4: The support of IFS $\tilde{A}^I$ on R is the crisp set of all x∈R such that $\mu_{\tilde{A}^I}(x) > 0$,

$\nu_{\tilde{A}^I}(x) > 0$ and $\mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \leq 1$.

Definition 2.5: A Linear Programming Problem is defined as:

Maximize z= $c\ x$

Subject to $Ax = b$

Where c = ($c_1, c_{2,} \dots, c_n$), b = $(b_1 b_2, \dots,\ b_m)^T$ and A = $[a_{ij}]_{m*n}$ where all the parameters are crisp.

III. Intuitionistic fuzzy linear regression model (IFLR)

Intuitionistic fuzzy functions and Intuitionistic fuzzy linear models [10] are presented, here for continuation.

The general form of IFR model is given by

$\tilde{y}^I$ = f(x, $\tilde{A}^I$) = $\tilde{A}^I{}_0 + \tilde{A}^I{}_1 x_1 + \tilde{A}^I{}_2 x_2 + \dots + \tilde{A}^I{}_n x_n$ …………. (1)

Where $\tilde{y}^I$ is the intuitionistic fuzzyoutput, $\tilde{A}^I{}_i$, i= $1,2,\dots,n$ is an Intuitionistic fuzzycoefficient and X = $(x_1, x_{2,} \dots, x_n)$ is an n dimensional non-fuzzy input vector. Each Triangular Intuitionistic Fuzzy Number (TIFN) coefficient $\tilde{A}^I{}_i$ can be defined by $\tilde{A}^I{}_{TIFN}$ = [a; α_i, β_i; $\alpha_i{}', \beta_i{}'$] where α_i, β_i are called the left and right spreads of membership function $\mu_{\tilde{A}^I}$(x) respectively. $\alpha_i{}'$ and $\beta_i{}'$ represent left and right spreads of non-membership function$\nu_{\tilde{A}^I}$(x) respectively. When two spreads are equal, the TIFN is known as symmetric triangular intuitionistic fuzzy number (STIFN). Hence a TIFN $\tilde{A}^I{}_{TIFN}$ = [a; α, β; α', β'] is said to be STIFN if α = β (say m) and α' = β'(say m') , this concept gives the definition of STIFN as follows:

An IFS $\tilde{A}^I$ in R is said to be a STIFN if there exist real number a, m and m' where $m \leq m'$ and $m, m' > 0$ such that the membership and non-membership functions are derived from the following diagram

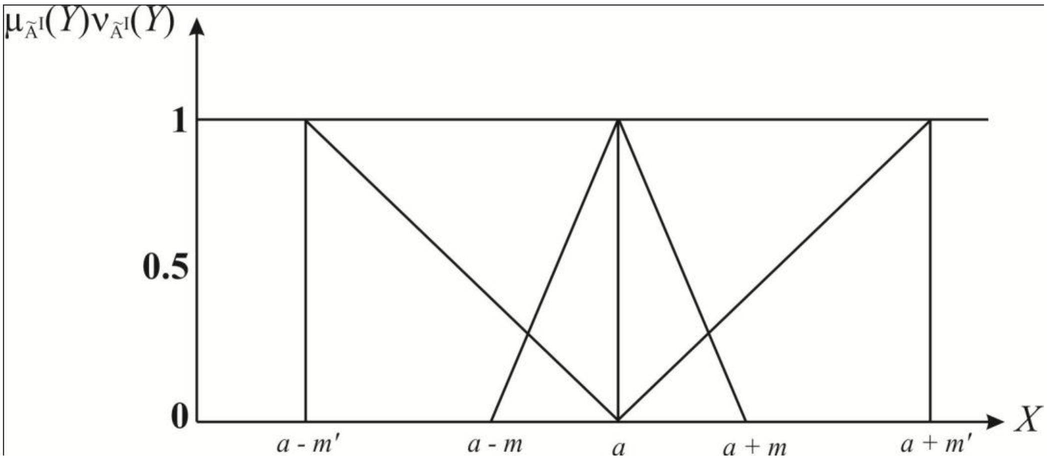

Figure 2: Symmetric triangular intuitionistic fuzzy number

$$\mu_{\tilde{A}^I}(\mathrm{x}) = \begin{cases} \frac{x-(a-m)}{m} & for\ x \in [a-m, a] \\ \frac{\alpha+m-x}{m} & for\ x \in [a,\ a+m] \\ 0 & otherwise \end{cases}$$

and $$\nu_{\tilde{A}^I}(\mathrm{x}) = \begin{cases} \frac{a-x}{m'} & for\ x \in [a-m', a] \\ \frac{x-a}{m'} & for\ x \in [a,\ a+m'] \\ 1 & otherwise \end{cases}$$

symbolically, a STIFN can be represented as $\tilde{A}^I{}_{STIFN}$ = [a: m, m, m', m'], where a is the center, m is the spread of membership function $\mu_{\tilde{A}^I}$(x) respectively and m' is spread of non-membership function$\nu_{\tilde{A}^I}$(x).

In this paper the IF components are assumed to be STIFNs having established the membership and non-membership functions for each IF coefficient $\tilde{A}^I{}_i$, the IF output from the linear model f(x, $\tilde{A}^I$) in (1) can be expressed as $\tilde{y}^I$ = f(x, $\tilde{A}^I$) = ($f^c(x)$, $f^{l_1}(x)$, $f^{l_2}(x)$) where $f^c(x)$ is the center of the of linear model f(x, $\tilde{A}^I$) and has the form $f^c(x) = a_0 + a_1x_1 + \cdots + a_nx_n$ and $f^{l_1}(x)$, $f^{l_2}(x)$ are the spreads of membership and non-membership functions of f(x, $\tilde{A}^I$).

$$f^{l_1}(x) = m_0 + m_1 \mid x_1 \mid + \cdots + m_n \mid x_n \mid \text{ and}$$
$$f^{l_2}(x) = m'_0 + m'_1 \mid x_n \mid + \ldots + m'_n \mid x_n \mid \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Then the membership and non-membership of $\tilde{y}^I$ defined in (1) can be given as

$$\mu_{\tilde{A}^I}(\mathrm{x}) = \begin{cases} \frac{y-[(a_0+\sum_i a_ix_i)-(m_0+\sum_i m_i|x_i|)]}{(m_0+\sum_i m_i|x_i|)} & for\ y \in [(a_0+\sum_i a_ix_i)-(m_0+\sum_i m_i|x_i|), (a_0+\sum_i a_ix_i) \\ \frac{[(a_0+\sum_i a_ix_i)+(m_0+\sum_i m_i|x_i|)]-y}{(m_0+\sum_i m_i|x_i|)} & for\ y \in [(a_0+\sum_i a_ix_i), (a_0+\sum_i a_ix_i)+(m_0+\sum_i m_i|x_i|), \\ 0 & otherwise \end{cases}$$

and $$\nu_{\tilde{A}^I}(\mathrm{x}) = \begin{cases} \frac{(a_0+\sum_i a_ix_i)-y}{m'_0+\sum_i m'_i|x_i|} & for\ y \in [(a_0+\sum_i a_ix_i)-(m'_0+\sum_i m'_i|x_i|),\ a_0+\sum_i a_ix_i] \\ \frac{y-(a_0+\sum_i a_ix_i)}{m'_0+\sum_i m'_i|x_i|} & for\ y \in [(a_0+\sum_i a_ix_i), (a_0+\sum_i a_ix_i)+(m'_0+\sum_i m'_i|x_i|)] \\ 1 & otherwise \end{cases}$$

The diagramatic representation of intuitionistic fuzzy output function of an IFN $\tilde{A}^I$ with h-level set is presented in the following figure.

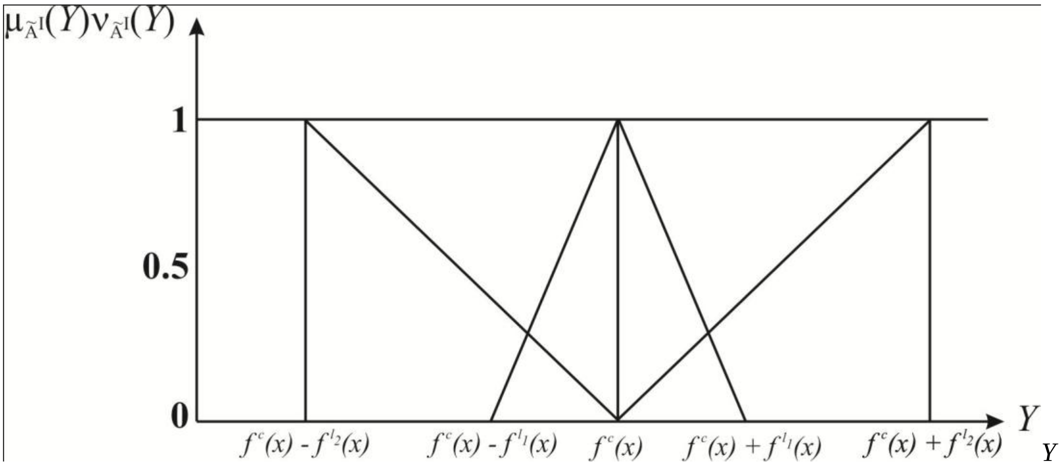

Figure 3: Triangular intuitionistic fuzzy output function

Where $f^c(x) = (a_0 + \sum_i a_i x_i)$
$f^c(x) - f^{l_1}(x) = (a_0 + \sum_i a_i x_i) - (m_0 + \sum_i m_i |x_i|)$
$f^c(x) + f^{l_1}(x) = (a_0 + \sum_i a_i x_i) + (m_0 + \sum_i m_i |x_i|)$
$f^c(x) - f^{l_2}(x) = (a_0 + \sum_i a_i x_i) - (m_0^{'} + \sum_i m_i^{'} |x_i|)$
$f^c(x) + f^{l_2}(x) = (a_0 + \sum_i a_i x_i) + (m_0^{'} + \sum_i m_i^{'} |x_i|)$

IV. ANALYSIS OF IF LINEAR REGRESSION

The objective of the IFLR method with non-fuzzy data is to determine the parameters $\tilde{A}^I$ such that the intuitionistic fuzzy output set $\{y_j\}$ is associated with $\mu_{\tilde{A}^I}(y_j) \geq h$ and $\nu_{\tilde{A}^I}(y_j) \leq 1 - b - h$ (j= 1, . . . , m) where $b,h \in [0,1]$ and 'b' is the intuitionistic index and the values of 'b' and 'h' are chosen for the purpose of generating the best-fitting model.

Now, the problem is to minimize the intuitionistic fuzziness of the output. Since, the values of the membership and non-membership functions of the intuitionistic fuzzy output are the functions of the spreads of the membership and non-membership functions, minimizing the spread of the membership function and the spread of the non-membership function leads to the minimization of the intuitionistic fuzziness of the output.

Minimize z = Min $\{f^{l_1}(x) + f^{l_2}(x)\}$ where $f^{l_1}(x)$ and $f^{l_2}(x)$ are defined in (2) it can be also written as Minimize z = $z_1 + z_2$ ………. (3)
where z_1= Min $\{f^{l_1}(x)\}$ = Min $\{(m_0 + \sum_{i=1}^{n} m_i \sum_{j=1}^{m} |x_j|\}$ and
z_2= Min $\{f^{l_2}(x)\}$ = Min $\{m'_0 + \sum_{i=1}^{n} m'_i \sum_{j=1}^{m} |x_j|\}$
Subject to the set of constraints $y_j \in [f(x_j)]_h$,
Where $[f(x_j)]_h = [\tilde{A}^I{}_0]_h + [\tilde{A}^I{}_1]_h x_j + [\tilde{A}^I{}_2]_h x_j + \ldots + [\tilde{A}^I{}_n]_h x_j$ such that $[..]_h$ denotes the h-level set of an IFN. By using the intuitionistic fuzzymembership function for the output, the two constraints of the IFR model are given by

$$\frac{y - [(a_0 + \sum_i a_i x_i) - (m_0 + \sum_i m_i |x_i|)]}{(m_0 + \sum_i m_i |x_i|)} \geq h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

And

$$\frac{[(a_0 + \sum_i a_i x_i) + (m_0 + \sum_i m_i |x_i|)] - y}{(m_0 + \sum_i m_i |x_i|)} \geq h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Simplification (4) and (5), then we have

$$y - [(a_0 + \sum_i a_i x_i) - (m_0 + \sum_i m_i |x_i|)] \geq h(m_0 + \sum_i m_i |x_i|) \ldots\ldots\ldots\ldots\ldots (6)$$

And

$$[(a_0 + \sum_i a_i x_i) + (m_0 + \sum_i m_i |x_i|)] - y \geq h(m_0 + \sum_i m_i |x_i|) \ldots\ldots\ldots\ldots\ldots (7)$$

Again simplification of (6) and (7), we get

$$a_0 + \sum_i a_i x_i) - (1-h)(m_0 + \sum_i m_i |x_i|)] \leq y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

And $$[(a_0 + \sum_i a_i x_i) + (1-h)(m_0 + \sum_i m_i |x_i|)] \geq y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Similarly, by using the IF non-membership function for the output, the last two constraints of the IFR model are given by

$$\frac{(a_0 + \sum_i a_i x_i) - y}{m'_0 + \sum_i m'_i |x_i|} \leq 1 - b - h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

And

$$\frac{y-(a_0+\sum_i a_i x_i)}{m'_0+\sum_i m'_i|x_i|} \le 1-b-h \text{......................(11)}$$

Simplification (10) and (11), then we have

$$(a_0+\sum_i a_i x_i) - y \le (1-b-h)(m'_0+\sum_i m'_i|x_i|) \text{.........(12)}$$

And

$$y-(a_0+\sum_i a_i x_i) \le (1-b-h)(m'_0+\sum_i m'_i|x_i|) \text{......... (13)}$$

Again simplification of (12) and (13), we get

$$(a_0+\sum_i a_i x_i) - (1-b-h)(m'_0+\sum_i m'_i|x_i|) \le y \text{ .... (14)}$$

And

$$(a_0+\sum_i a_i x_i) + (1-b-h)(m'_0+\sum_i m'_i|x_i|) \ge y \text{ ....(15)}$$

Minimize z = z_1+ z_2

Wherez_1 = Min $\{(m_0+\sum_{i=1}^{n} m_i \sum_{j=1}^{m}|x_{ij}|\}$ and

z_2 = Min $\{m'_0 + \sum_{i=1}^{n} m'_i \sum_{j=1}^{m}|x_{ij}|\}$ subject to the constraints (8),(9),(14) and (15) and m_0,m'_0 ≥0 and a_i, m_i,m'_i ≥0 *for i=1,2,…,n*

By using the software TORA, the values of the center and spreads of membership and non-membership functions can be estimated. This gives the IFN coefficients as follows:

$\tilde{A}^I{}_0$ = [a_0, $m_0, m_0;m_0', m_0'$]
$\tilde{A}^I{}_1$ = [a_1, $m_1, m_1;m_1', m_1'$]
⋮
$\tilde{A}^I{}_n$ = [a_n, $m_n, m_n;m_n', m_n'$]

By using the values of $\tilde{A}^I{}_0$, $\tilde{A}^I{}_1$…… $\tilde{A}^I{}_n$ the IFR model can be given by

$$\tilde{y}^I = \tilde{A}^I{}_0 + \tilde{A}^I{}_1 x_1 + \tilde{A}^I{}_2 x_2 + \dots + \tilde{A}^I{}_n x_n \text{ .... ….. (16)}$$

The value of $\tilde{y}^I$ can be estimated by substituting the values of x_1, x_2 …. x_n in equation (16), the estimated values of $\tilde{y}^I$ are actually STIFNs which are to be defuzzified now.

STIFN $\tilde{A}^I{}_{STIFN}$ = [a: m, m, m', m'] can be transformed into a crisp number by using the below equation [5]

$$A = a + \frac{1}{6}[m' - m] \text{.............................. (17)}$$

Numerical example:

Consider the values for *X* and *Y* in the following table to calculate the coefficients of the IFLR model for *i=1*

j	1	2	3	4	5	6
x_j	1	2	4	6	7	8
y_j	3010	4500	4400	5400	7295	8195

Case (i):

Let us consider the value of *h* and *b* be *0.1*.

Minimize z = z_1+ z_2

Where z_1 = Min $\{(m_0 + m_1\sum_{j=1}^{6}|x_j|)\}$

Subject to the constraints

$a_0 + x_j a_1 + 0.9\ m_0 + 0.9\ m_1 x_j \ge y$ [from (9)]

$a_0 + x_j a_1 - 0.9\ m_0 - 0.9\ m_1 x_j \le y$ [from (8)]

and

z_2 = Min $\{(m'_0 + m'_1\sum_{j=1}^{6}|x_j|)\}$

Subject to the constraints

$a_0 + x_j a_1$ + (1-0.1-0.1) $(m'_0 + m'_1 x_j) \ge y$ [from (15)]

$a_0 + x_j a_1$ - (1-0.1-0.1) $(m'_0 + m'_1 x_j) \le y$ [from (14)]

Where x_j represents the component of *x* in the jthentry value. The number of functional constraints depend on the number of data sets available. From the example six different sets of values (x, y) will generate 12 functional constraints for z_1 and z_2 respectively. Substituting the given values, the LPP becomes,

Minimize z = z_1+ z_2

Where z_1 = Minimize $\{m_0 + 28\ m_1\}$

Subject to the constraints

$a_0 + 1a_1 + 0.9\ m_0 + 0.9\ m_1 \ge 3010$
$a_0 + 1a_1 - 0.9\ m_0 - 0.9\ m_1 \le 3010$
$a_0 + 2a_1 + 0.9\ m_0 + 1.8\ m_1 \ge 4500$
$a_0 + 2a_1 - 0.9\ m_0 - 1.8\ m_1 \le 4500$
$a_0 + 4a_1 + 0.9\ m_0 + 3.6\ m_1 \ge 4400$
$a_0 + 4a_1 - 0.9\ m_0 - 3.6\ m_1 \le 4400$
$a_0 + 6a_1 + 0.9\ m_0 + 5.4\ m_1 \ge 5400$
$a_0 + 6a_1 - 0.9\ m_0 - 5.4\ m_1 \le 5400$
$a_0 + 7a_1 + 0.9\ m_0 + 6.3\ m_1 \ge 7295$
$a_0 + 7a_1 - 0.9\ m_0 - 6.3\ m_1 \le 7295$
$a_0 + 8a_1 + 0.9\ m_0 + 7.2\ m_1 \ge 8195$
$a_0 + 8a_1 - 0.9\ m_0 - 7.2\ m_1 \le 8195$

Wherea_0, a_1, m_0, m_1 ≥*0.*

and z_2 = Minimize $\{m_0' + 28m_1'\}$
Subject to the constraints

$a_0 + 1a_1 + 0.8\,m'_0 + 0.8\,m'_1 \geq 3010$
$a_0 + 1a_1 - 0.8\,m'_0 - 0.8\,m'_1 \leq 3010$
$a_0 + 2a_1 + 0.8\,m'_0 + 1.6\,m'_1 \geq 4500$
$a_0 + 2a_1 - 0.8\,m'_0 - 1.6\,m'_1 \leq 4500$
$a_0 + 4a_1 + 0.8\,m'_0 + 3.2\,m'_1 \geq 4400$
$a_0 + 4a_1 - 0.8\,m'_0 - 3.2\,m'_1 \leq 4400$
$a_0 + 6a_1 + 0.8\,m'_0 + 4.8\,m'_1 \geq 5400$
$a_0 + 6a_1 - 0.8\,m'_0 - 4.8\,m'_1 \leq 5400$
$a_0 + 7a_1 + 0.8\,m'_0 + 5.6\,m'_1 \geq 7295$
$a_0 + 7a_1 - 0.8\,m'_0 - 5.6\,m'_1 \leq 7295$
$a_0 + 8a_1 + 0.8\,m'_0 + 6.4\,m'_1 \geq 8195$
$a_0 + 8a_1 - 0.8\,m'_0 - 6.4\,m'_1 \leq 8195$

where $a_0, a_1, m'_0, m'_1 \geq 0$.
By using the software Tora, the estimated values of the center and spreads of membership and non-membership functions are given by
a_0= 2486.67, a_1= 615.83, m_0= 868.5185, m_1= 0, m'_0= 977.0833, m'_1= 0
where the minimized value of the objective function on the spread is
z = $z_1 + z_2$ = 868.52 + 977.08 = 1845.6
Now the IFN coefficients are as follows:
$\tilde{A}^I{}_0$ = [2486.67; 868.5185, 868.5185; 977.08, 977.0833]
$\tilde{A}^I{}_1$ = [615.83; 0, 0; 0, 0]

The IFR model is given by
$\tilde{y}^I = \tilde{A}^I{}_0 + \tilde{A}^I{}_1 x_1$
$\tilde{y}^I$ = [2486.67; 868.52, 868.52; 977.08, 977.08] + [615.83; 0, 0; 0, 0] (1)
$\tilde{y}^I$ = [3102.50; 868.52, 868.52; 977.08, 977.08]
The transformed crisp value of $\tilde{y}^I$ is given by
$Y = a + \frac{1}{6}[m' - m]$
= 3102.50+ $\frac{1}{6}$ [977.08- 868.52] = 3102.50 + 18.0941
Y = 3120.59
Therefore, the predicted value of Y = 3120.5941.
The next value of Y can be calculated here,
$\tilde{y}^I = \tilde{A}^I{}_0 + \tilde{A}^I{}_1 x_2$
$\tilde{y}^I$ = [2486.67; 868.52, 868.52; 977.08, 977.08] + [615.83; 0, 0; 0, 0] (2)
$\tilde{y}^I$ = [3718.3333; 868.52, 868.52; 977.08, 977.08]

The transformed crisp value of $\tilde{y}^I$ is given by
$Y = a + \frac{1}{6}[m' - m]$
Y = 3718.3333+ $\frac{1}{6}$ [977.08- 868.52] = 3718.3333+ 18.0941
Therefore, the predicted value of Y = 3736.4274
Similarly, the other values of Y are estimated and are presented in the following table

x_j	Observed y_j	$\hat{y}_j$	$e_j = \hat{y}_j - y_j$	e_j^2
1	3010	3120.59	110.59	12230.1481
2	4500	3736.4274	-763.573	583043.1155
4	4400	4968.094	568.094	322730.7928
6	5400	6199.7606	799.7606	639617.0173
7	7295	6815.5939	-479.406	229830.2087
8	8195	7431.4272	-763.573	583043.4209
				2370494.703

Table 1: Error for using IFR model with $h = 0.1$ and $b = 0.1$

Case (ii):

Let us consider the value of $h = 0.4$ and $b = 0.4$
Minimize z = $z_1 + z_2$
Where z_1 = Min $\{(m_0 + m_1 \sum_{j=1}^{6} |x_j|)\}$
Subject to the constraints
$a_0 + x_j a_1 + 0.6\,m_0 + 0.6\,m_1 x_j \geq y$ [from (9)]
$a_0 + x_j a_1 - 0.6\,m_0 - 0.6\,m_1 x_j \leq y$ [from (8)]
and
z_2 = Min $\{(m'_0 + m'_1 \sum_{j=1}^{6} |x_j|)\}$
Subject to the constraints
$a_0 + x_j a_1 + (1\text{-}0.4\text{-}0.4)\,(m'_0 + m'_1 x_j) \geq y$ [from (15)]
$a_0 + x_j a_1 - (1\text{-}0.4\text{-}0.4)\,(m'_0 + m'_1 x_j) \leq y$ [from (14)]
Where z_1 = Minimize $\{m_0 + 28\,m_1\}$
Subject to the constraints

$a_0 + 1a_1 + 0.6\,m_0 + 0.6\,m_1 \geq 3010$
$a_0 + 1a_1 - 0.6\,m_0 - 0.6\,m_1 \leq 3010$
$a_0 + 2a_1 + 0.6\,m_0 + 1.2\,m_1 \geq 4500$
$a_0 + 2a_1 - 0.6\,m_0 - 1.2\,m_1 \leq 4500$
$a_0 + 4a_1 + 0.6\,m_0 + 2.4\,m_1 \geq 4400$
$a_0 + 4a_1 - 0.6\,m_0 - 2.4m_1 \leq 4400$
$a_0 + 6a_1 + 0.6\,m_0 + 3.6\,m_1 \geq 5400$
$a_0 + 6a_1 - 0.6\,m_0 - 3.6\,m_1 \leq 5400$
$a_0 + 7a_1 + 0.6\,m_0 + 4.2\,m_1 \geq 7295$
$a_0 + 7a_1 - 0.6\,m_0 - 4.2\,m_1 \leq 7295$
$a_0 + 8a_1 + 0.6\,m_0 + 4.8\,m_1 \geq 8195$
$a_0 + 8a_1 - 0.6\,m_0 - 4.8m_1 \geq 8195$

Where $a_0, a_1, m_0, m_1 \geq 0$.
and z_2 = Minimize $\{m_0' + 28m_1'\}$
Subject to the constraints

$a_0 + 1a_1 + 0.2\,m'_0 + 0.2\,m'_1 \geq 3010$
$a_0 + 1a_1 - 0.2\,m'_0 - 0.2\,m'_1 \leq 3010$
$a_0 + 2a_1 + 0.2\,m'_0 + 0.4\,m'_1 \geq 4500$

$a_0 + 2a_1$ -0.2 m'_0- 0.4 $m'_1 \leq 4500$
$a_0 + 4a_1$ + 0.2 m'_0 + 0.8 $m'_1 \geq 4400$
$a_0 + 4a_1$ - 0.2 m'_0- 0.8 $m'_1 \leq 4400$
$a_0 + 6a_1$+ 0.2 m'_0+ 1.2 $m'_1 \geq 5400$
$a_0 + 6a_1$ - 0.2 m'_0- 1.2 $m'_1 \leq 5400$
$a_0 + 7a_1$ + 0.2 m'_0 + 1.4 $m'_1 \geq 7295$
$a_0 + 7a_1$ - 0.2 m'_0 - 1.4 $m'_1 \leq 7295$
$a_0 + 8a_1$ + 0.2 m'_0+ 1.6 $m'_1 \geq 8195$
$a_0 + 8a_1$ - 0.2 m'_0 - 1.6 $m'_1 \leq 8195$

where $a_0, a_1, m'_0, m'_1 \geq 0$.

By using the software Tora, the estimated values of the center and spreads of membership and non-membership functions are given by
a_0= 2486.6667, a_1= 615.8333, m_0= 1302.7778, m_1= 0, m'_0= 3908.3333, m'_1= 0
where the minimized value of the objective function on the spread is

z = 1302.7778+ 3908.3333= 5211.1111

Now the IFN coefficients are as follows:

$\tilde{A}^I{}_0$ = [2486.6667; 1302.7778, 1302.7778; 3908.3333, 3908.3333]

$\tilde{A}^I{}_1$ = [615.8333; 0, 0; 0, 0]

The IFR model is given by
$\tilde{y}^I = \tilde{A}^I{}_0 + \tilde{A}^I{}_1 x_1$
$\tilde{y}^I$ = [2486.6667; 1302.7778, 1302.7778; 3908.3333, 3908.3333] + [615.8333; 0, 0; 0, 0] (1)
$\tilde{y}^I$ = [3102.5; 1302.7778, 1302.7778; 3908.3333, 3908.3333]

The transformed crisp value of $\tilde{y}^I$ is given by
$Y = a + \frac{1}{6}[m' - m]$
$= 3102.5 + \frac{1}{6}[3908.3333 - 1302.7778] = 3102.5 + 434.2592$

Therefore, the predicted value of Y = 3536.7592.

Similarly, the other values of Y are estimated and presented in the following table

x_j	Observed y_j	$\hat{y}_j$	$e_j = \hat{y}_j - y_j$	e_j^2
1	3010	3536.7592	526.7592	277475.2548
2	4500	4152.5925	-347.408	120691.9711
4	4400	5384.2591	984.2591	968765.9759
6	5400	6615.9257	1215.926	1478475.308
7	7295	7231.759	-63.241	3999.424081
8	8195	7847.5923	-347.408	120692.11
				2970100.044

Table 2: Error for using IFR model with *h = 0.4* and *b = 0.4*

From the following graph of observed and estimated data, it is clear that the Intuitionistic fuzziness of the model is minimized by minimizing the spread of Symmetrical Triangular Intuitionistic fuzzy numbers. Since the estimated data lie on a straight line, it is concluded that the Intuitionistic fuzziness of the model is completely minimized by minimizing the total support of the intuitionistic fuzzy coefficients.

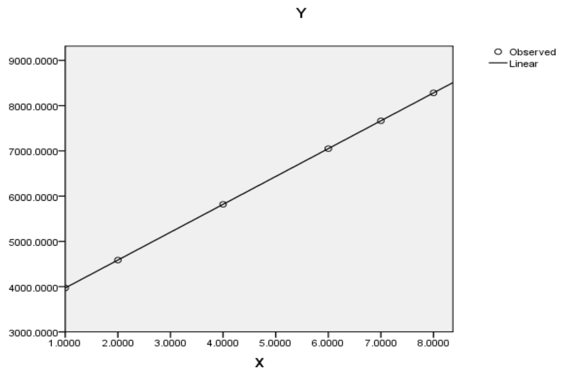

Figure 4: Graph of intuitionistic fuzzy regression model

4.1 Determination of error using ordinary regression method:

The linear regression equation is $Y = a + b X$ ……………………. (A)

The value of a and b can be determined by using the following normal equation.

$\sum X + b\sum X^2 = \sum XY$ ……………………….. (**B**)

$n\,a + b\sum X = \sum Y$ ……………………….... (C)

The values of X and Y becomes,

X	***Y***	X^2	***XY***
1	3010	1	3010
2	4500	4	9000
4	4400	16	17600
6	5400	36	32400
7	7295	49	51065
8	8195	64	65560
28	**32800**	**170**	**178635**

Table 3: Calculate Ordinary regression model

From (C) ⇒ 6a + 28b = 32800
From (B) ⇒ 28a + 170b = 178635

Solving (B) and (C) we have, a = 2433.1356 and b = 650.0423
Therefore the regression model (A) becomes, $\hat{Y}$ = 2433.1356 + 650.0423X

The other values of Y are estimated and presented in the following table

x_j	Observed y_j	$\hat{y_j}$	$e_j = \hat{y_j} - y_j$	e_j^2
1	3010	3083.1779	73.1779	5355.005048
2	4500	3733.2202	-766.78	587951.2617
4	4400	5033.3048	633.3048	401074.9697
6	5400	6333.3894	933.3894	871215.772
7	7295	6983.4317	-311.568	97074.80556
8	8195	7633.474	-561.526	315311.4487
				2277983.263

Table 4: Error for using ordinary regression model

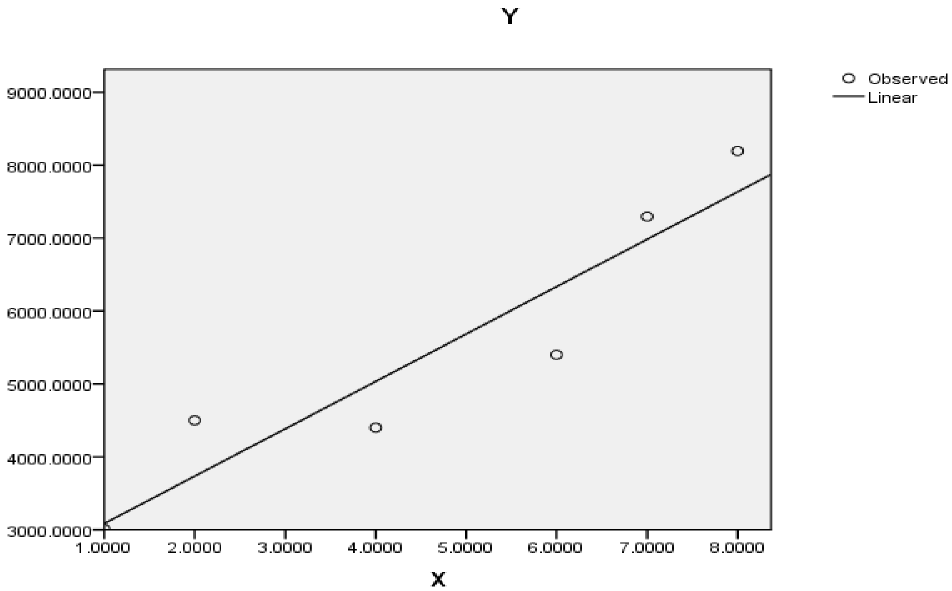

Figure 5: Graph of the regression model

V. CONCLUSION

In this paper, Intuitionistic Fuzzy Linear Regression is used for prediction of values instead of normal linear regression. Error values is also found out for both intuitionistic fuzzy linear regression and normal linear regression.

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