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Free Vibration Analysis of Curved Cantilever Sandwich Structure

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ABSTRACT
Structural sandwich construction is used in many air and space vehicles, cargo containers, boats and ships. Sandwich construction offers high strength to weight ratios, good buckling resistance, formability to complex shapes and easy reparability, which are of extremely high importance in aerospace applications. The most common sandwich structure is composed of two thin face sheets with a thicker lightweight, low-stiffness core. In this work an attempt has been made to develop the formulation to find free vibration analysis of a curved cantilever three layered sandwich beam. The formulation for curved sandwich beam is developed using linear displacement field at face layer and non-linear displacement field at core layer. The face layers are assumed to behave according to Euler-Bernoulli theory and the core deforms in shear only. The governing equation of motion is to be derived using Hamilton's principle in conjunction with dynamic finite element method.The formulation is to be developed for a number of different numerical cases by varying the thickness of core layers and face layers and studied under the fixed free cantilever boundary conditions. The analytical results of the sandwich beams are to be compared with available literature values.

Keywords: Euler-Bernoulli theory, Face and core layers, Finite element method,Hamilton's principle,Sandwich beam.

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I. INTRODUCTION

The study of free vibration is an important prerequisite for all dynamic response calculations for elastic systems. The current trend in the aerospace industry of using composites and sandwich material to lighten aircraft in an attempt to make them more fuel efficient has caused a recent resurgence into research to develop a reliable method of predicting the vibration behavior of sandwich structures.K M Ahmed [1], investigated the flexural vibration characteristics of curved sandwich beams by using the finite element displacement method.The free vibration characteristic of a curved sandwich beam is analyzed by him using the principle of minimum total potential energy.Finally, he concluded that the element with four degrees of freedom per node is relied upon yield reasonable estimates of the natural frequencies of curved sandwich beams.S HE and M D Rao [2], described an analytical model for the coupled flexural and longitudinal vibration of a curved sandwich beam system. The governing equations of motion for the forced vibration of the system are derived using the energy method and Hamilton's principle. They used Rayleigh- Ritz method for solving system resonance frequencies and loss factors. To evaluate the effects of curvature, core thickness and adhesive shear modulus a parametric study was conducted by them on the system for resonance frequencies and loss factor.Bozhevolnaya and J Q Sun [3],evaluated numerical analyses of free vibrations of simply supported and clampedcurved sandwich beams and investigated effect of the curvature on the Eigen modes and their frequencies. They derived linear equations of motion and the boundary conditions with the help of calculus of variations. A coupling coefficient is introduced to study the dynamic coupling for different types of motions in an eigen mode.J R Banerjee and A J Sobey [4], investigated a dynamic stiffness theory of a three-layered sandwich beam and subsequently used to investigate its free vibration characteristics. They imposed displacement field so that the top and bottom layers behave like Rayleigh beams, while the central layer behaves like a Timoshenko beam. Using Hamilton's principle the governing differential equations of motion of the sandwich beam are derived by them.

The boundary conditions for responses and loads at both ends of the freely vibrating sandwich beam are imposed to formulate the dynamic stiffness matrix, which relates harmonically varying loads to harmonically varying responses at the ends.J R

Banerjee et al [5], investigated the free vibration characteristics by dynamic stiffness model for a three-layered sandwich beam of unequal thicknesses.Each layer of the beam is assumed to behave as Timoshenko beam theory and the combined system is reduced to a tenth-order system using symbolic computation. By relating amplitudes of harmonically varying loads to those of the responses an exact dynamic stiffness matrix is developed by them.O Rahmani et al [6], investigated free vibration analysis of sandwich beams with syntactic foam as a functionally graded flexible core by implementing a new model based on high-order sandwich panel theory.

For formulation they used the classical beam theory for the face sheets and an elasticity theory for the FG core. A numerical analysis of free vibration of simply supported beams including higher modes that cannot be detected by other models is carried out by them. M A Salam and N E Bondok [7], developed a model presenting the sandwich beams to calculate the flexural rigidity and sandwich beams dynamic characteristics. Different cases such as sandwich beams multi-layer cores, sandwich beams multi cells, sandwich beams with holes in its cores having different shapes and different orientations were investigated.B O Baba [8] investigated natural frequencies of both flat and curved sandwich beams made of fiber-glass laminate skins wrapped over polyurethane foam core beams with and without debonded beams. Using an impulse-frequency response method the vibration tests were performed for clamped-clamped end conditions by him. Results obtained by him indicates that fundamental natural frequencies are seriously affected by curvature angle and debond, whereas the higher order natural frequencies show relatively small changes.H Callioglu et al [9], investigated free vibration behavior of a multilayered symmetric sandwich beam made of Functionally Graded Material with variable cross-section.

The natural frequencies are computed by them for conventional boundary conditions of the functionally graded sandwich beam using theoretical procedure. Finally, the obtained results are compared by them with those in literature and a finite element based commercial program ANSYS and found to be consistent with each other. R ATalookolaei et al [10], presented analytical and finite element solutions for free vibration analysis of delaminated composite curved beams. They considered the effects of shear deformation, rotary inertia, deepness term and material coupling. The governing equations has been obtained using Hamilton's principle along with the boundary, continuity and equilibrium conditions at the delamination boundaries called as constraints. The accuracy and convergence of the present formulations are validated against several numerical examples in the literature.D Bensahal and M N Amrane [11], investigated the effect of various parameters such as length, thickness, density and shear modulus of the core and Young modulus of skins for various boundary conditions such as clamped-free and simply supported. The flexural vibrations of beams were analyzed by the finite element method, using the stiffness and mass matrix of beam element with three degrees of freedom per node by them. Mat lab commercial software was used to calculate the three first natural frequencies. R Chavanand P S Talmale [12],studied the equation of motion for the viscoelastic sandwich composite beam. Different specimens have been modeled by varying the core layers and face layers and studied under the fixed-fixed and cantilever boundary conditions for modal analysis. For the three layer viscoelastic sandwich beam a finite element model has been developed. The Natural frequencies were obtained for various models using different core thickness and boundary conditions by them. From the literature it is observed that the work had been carried out on curved structures by investigating its natural frequencies and mode shapes by different methods. In this work natural frequencies and mode shapes of different materials of core and face layers by varying the core thickness are to be evaluated.

II. MATHEMATICAL MODELING

In this a three layered cantilevered sandwich beam is considered for free vibration analysis. The equation of motion governing the deflection for the transverse vibration of a beam with uniform cross section and homogeneous material is determined based on the deflected beam geometry, bending stresses, shear forces, and other relevant factors by using principle of minimum potential energy method.For free vibration analysis, the assumption of simple harmonic motion is considered and dynamic finite element method is used to determine natural frequencies and mode shapes.

The straining of an infinitesimal portion of a fiber of un-deformed length or length of fiber before deformation(ds) is

$$ds = (\mathrm{R} + z)\, \mathrm{d}\theta \qquad \text{…. (1)}$$

The deformation of the element occurs in two stages, one in axial deformation and other in radial deformation. The net angular displacement of a material point from P_1 to P_2, measured at the center of curvature of the undeformed element, is given by

III.

$$\theta' = \theta + \frac{u}{R} - \frac{1}{(R+z)}\left(\frac{z}{R} \times \frac{\partial w}{\partial \theta}\right) \qquad \text{…. (2)}$$

The deformed length of the fiber from P_1 to P_3 may be approximated as (neglecting the effect of radial deformation)

$$ds' \approx (\mathrm{R} + \mathrm{z} + \mathrm{w})\mathrm{d}\theta' \approx (R+z+w)\left[d\theta + \frac{1}{R}\frac{\partial u}{\partial \theta}d\theta - \frac{z}{R(R+z)}\frac{\partial^2 w}{\partial \theta^2}d\theta\right] \text{...... (3)}$$

The strain in the fiber, determined up to linear order in displacements, is given as

$$\epsilon = \frac{ds' - ds}{ds} = \frac{1}{R\left(1+\frac{z}{R}\right)}\left[w + \frac{\left(1+\frac{z}{R}\right)\partial u}{\partial \theta} - \frac{z}{R}\frac{\partial^2 w}{\partial \theta^2}\right] \quad \text{... (4)}$$

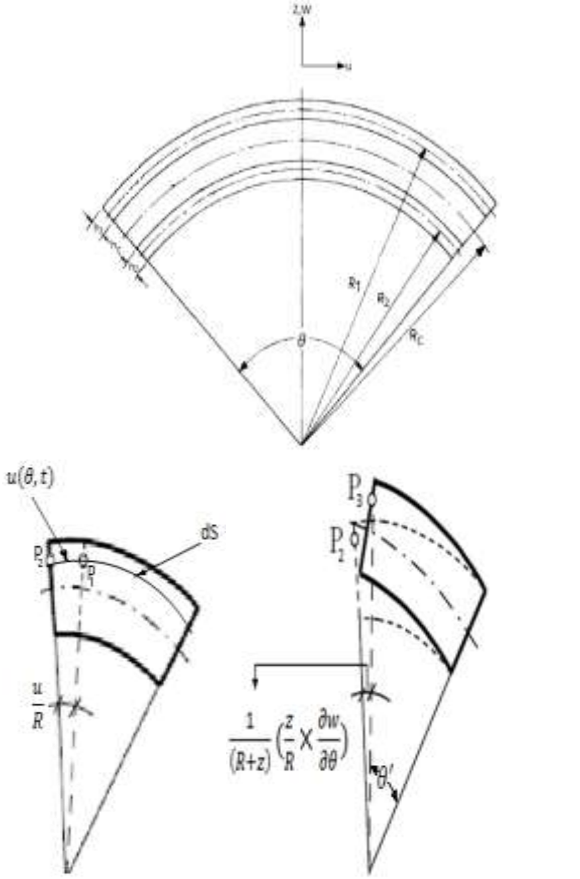

Fig. 1: Curved Beam with Face and Core Layers

Considering $\left(1+\frac{z}{R}\right) \ll 1$

$$\epsilon = \frac{1}{R}\left[w + \frac{\partial u}{\partial \theta} + \frac{z}{R}\left(\frac{\partial u}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2}\right)\right] \text{...... (5)}$$

Strain Energy Formulation for Face Layers:

$$\mathrm{U_f} = \frac{1}{2}\int \sigma\,\epsilon\,\mathrm{dv} = \frac{1}{2}\int_0^\theta \left(\int E\,\epsilon^2\,dA\right)R d\theta \quad \text{...... (6)}$$

Substituting Eq. (5) in above equation we get

$$\mathrm{U_f} = \frac{1}{2}\int_0^\theta \left[\frac{EA}{R}\left(w + \frac{\partial u}{\partial \theta}\right)^2 + \frac{EI}{R^3}\left(\frac{\partial u}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2}\right)^2\right] d\theta \text{...... (7)}$$

For Upper Layer:

$$\mathrm{U_{f1}} = \frac{1}{2}\int_0^\theta \left[\frac{EA}{R_1}\left(w_1 + \frac{\partial u_1}{\partial \theta}\right)^2 + \frac{EI}{R_1^3}\left(\frac{\partial u_1}{\partial \theta} - \right.\right.$$
$\partial 2w1\partial\theta 22d\theta$ ...... (8)

For Lower Layer:

$$\mathrm{U_{f2}} = \frac{1}{2}\int_0^\theta \left[\frac{EA}{R_2}\left(w_2 + \frac{\partial u_2}{\partial \theta}\right)^2 + \frac{EI}{R_2^3}\left(\frac{\partial u_2}{\partial \theta} - \right.\right.$$
$\partial 2w2\partial\theta 22d\theta$ ...... (9)

Kinetic Energy Formulation for Face Layers:

$$\mathrm{T_f} = \frac{1}{2}\int_0^\theta \rho A\left(\frac{\partial u}{\partial t}^2 + \frac{\partial w}{\partial t}^2\right) R d\theta \quad \text{...... (10)}$$

For Upper Layer:

$$\mathrm{T_{f1}} = \frac{1}{2}\int_0^\theta \rho A\left(\frac{\partial u_1^2}{\partial t} + \frac{\partial w_1^2}{\partial t}\right) R_1 d\theta \quad \text{...... (11)}$$

For Lower Layer:

$$\mathrm{T_{f2}} = \frac{1}{2}\int_0^\theta \rho A\left(\frac{\partial u_2^2}{\partial t} + \frac{\partial w_2^2}{\partial t}\right) R_2 d\theta \quad \text{...... (12)}$$

Formulation of Viscoelastic layer:

The face layers and the core are assumed to be under a plane stress state. The Shear strain in the core is proportional to $\frac{\mathrm{R_c}}{(\mathrm{R_c}+\mathrm{z_c})}$, where $\frac{h_c}{2} \le z_c \le \frac{h_c}{2}$ being the core thickness and R_c be the radius of curvature of the middle surface of the core layer. However, the radial deformation of the adhesive is assumed to be constant through its thickness.

The shear strain in the middle fiber of the core layer is approximated as

$$\gamma_{mid} = \frac{u_B + u_A}{h_c} + \frac{1}{2}\left[\frac{1}{R_c}\frac{\partial w_1}{\partial \theta} + \frac{1}{R_2}\frac{\partial w_2}{\partial \theta}\right] \quad \text{... (13)}$$

Where

$$u_A = \left(\frac{R_1+z_1}{R_1}u_1 - \frac{z_1}{R_1}\frac{\partial w_1}{\partial \theta}\right),\ u_B = \left(\frac{R_2+z_2}{R_2}u_2 - \frac{z_2}{R_2}\frac{\partial w_2}{\partial \theta}\right)$$

The radial strain of the core layer is

$$\epsilon_c = (w_2 - w_1)/h_c \quad \text{...... (14)}$$

Strain energy of the core layer is

$$U_c = \frac{1}{2}\int [G_c\gamma_c^2 + kE_c\epsilon_c^2]dv_c \text{...... (15)}$$

$$U_c = \int_0^\theta \left\{\frac{G_cR_c}{2h_c}\left(1+\frac{h_c^2}{12R_c^2}\right)\left[(a_2u_2 - a_1u_1) + \frac{h_1+h_c}{2} \times \right.\right.$$
$\partial w1\partial\theta \times 1R1 + h2 + hc2 \times \partial w2\partial\theta \times 1R22 + kEcRc2hc$
$w2 - w12d\theta$ ... (16)

Where $k = 1/[(1-\mu)(1+\mu)]$.

By substituting $u_A$and$u_B$the kinetic energy of the core layer is approximated as,

$T_c = \frac{\rho_c h_c R_c}{8} \int_0^{\theta} \left\{ \left[a_1 \frac{\partial u_1}{\partial t} + a_2 \frac{\partial u_2}{\partial t} + \frac{1}{2} \left(\frac{h_2}{R_2} w_2' - \right. \right. \right.$
h1R1w1′2+∂w1∂t+∂w2∂t2dθ...... (17)

Where ρ_c is the density of the core material and $w_1' = \partial^2 w_1/(\partial t \partial \theta)$ and $w'_2 = \partial^2 w_2/(\partial t \partial \theta)$.

Governing Equation of Motion:
The equation of motion for the mentioned element has been solved by using the Hamilton's Principle,

$\int_{t_1}^{t_2} \big(\delta(T-U)\big) dt = 0$...... (18)

Now substituting equations 2, 4, 6, 7, 16 and 17 and using principle of calculus of variations, the governing equations of motion of the system is obtained as:

$\frac{E_1 h_1}{R_1} \left(\frac{\partial^2 u_1}{\partial \theta^2} + \frac{\partial w_1}{\partial \theta} \right) + \frac{a_1 G_c R_c}{h_c} \left(1 + \frac{h_c^2}{12 R_c^2} \right) \Big[(a_2 u_2 -$
a1u1+∂∂θh1+hc2R1w1+h2+hc2R2w2−ρ1h1R1∂2u1∂t2−a1ρchcRc4∂2∂t2a1u1+a2u2+12∂3∂t2∂θh2R2w2−h1R1w1=0.

$\frac{E_2 h_2}{R_2} \left(\frac{\partial^2 u_2}{\partial \theta^2} + \frac{\partial w_2}{\partial \theta} \right) - \frac{a_2 G_c R_c}{h_c} \left(1 + \frac{h_c^2}{12 R_c^2} \right) \Big[(a_2 u_2 -$
a1u1+∂∂θh1+hc2R1w1+h2+hc2R2w2−ρ2h2R2∂2u2∂t2−a2ρchcRc4∂2∂t2a1u1+a2u2+12∂3∂t2∂θh2R2w2−h1R1− h1R1w1=0.

Suppose $u$be the circumferential displacements of the faces, and w be the radial displacement which is assumed to remain constant for the three layers then, The equations of motion can be written as,

$\frac{\partial^2 u}{\partial y^2} + \frac{1}{\alpha^2} (w^2 Q_1 - 4\beta^2) u - \frac{2h\beta^2}{\alpha^2} \frac{\partial w}{\partial y} = 0$ ...... (19)

$\frac{\partial^4 w}{\partial y^4} - \frac{h^2 \beta^2}{\gamma^2} \times \frac{\partial^2 w}{\partial y^2} + \frac{1}{\gamma^2} \left(\frac{\alpha^2}{R^2} - w^2 Q_2 \right) w - \frac{2h\beta^2}{\gamma^2} \times \frac{\partial u}{\partial y} = 0$ ...... (20)

Dynamic finite element formulation:
For harmonic oscillation, the weak form of the governing equations (19) and (20) are obtained by applying a Galerkin-type integral formulation, based on the weighted-residual method. The method involves the use of integration by parts on different elements of the governing differential equations and then the discretization of the beam length into a number of two node beam elements. Applying the appropriate number of integration by parts to the governing equations and discretization lead to the following form:

$W_u^k = -\int_0^l (\delta u'' \alpha^2 + \delta u \omega^2 Q_1) u dy + \int_0^l \delta u (4\beta^2) u dy +$
δu′α2u0l+0lδu2hβ2....... (21)

$$W_W^k = \int_0^l (\delta w'''' \gamma^2 - \delta w'' h^2 \beta^2) w dy + \delta w \left(\alpha^2/_{\beta^2} - \omega^2 Q_1 \right) \Big) w dy + [\delta w' h^2 \beta^2 w]_0^1 + [\delta w' y^2 w]_0^1 + \int_0^1 \delta w (2h\beta^2)\, u dy$$ .... (22)

The non-nodal approximations are:

$\delta u = (P(y))_U [\delta a]$where, $u = (P(y))_U [a]$.... (23)

$\delta w = (P(y))_W [\delta b]$where, $w = (P(y))_W [b]$ ..... (24)

Where $\{a\}$ and $\{b\}$ are the generalized coordinates for u and w, respectively with the basic functions of approximation space expressed as:

$(P(y))_U = \frac{\cos(\varepsilon y) \sin(\varepsilon y)}{\epsilon}$ ..... (25)

$$\big(P(y)\big)_w = (\cos(\sigma y)) \frac{\sin(\sigma y)(\cosh(\gamma y) - \cos(\sigma y))\, (\sinh(\gamma y) - \sin(\sigma y))}{\sigma\, (\sigma^2 + \tau^2)(\sigma^2 + \tau^2)}$$ ...... (26)

The non-nodal approximations (23) and (24) are made for $\delta u, u, \delta w$and$w$so that the integral terms in equations (21) and (22) become zero. The first part of equation (21) has a second order differential characteristic equation of the form $A_1 D^2 + B_1 \omega^2 = 0$, where$A_1$ and $B_1$are constants and $D^2$is the second partial derivative with respect to *x*. The second part of the equation (21) has a fourth order differential characteristic equation of the form$A_2 D^4 - B_2 D^2 + C_2 \omega^2 = 0$, where$A_2, B_2andC_2$ are constants and $D^4$and$D^2$are the fourth and second partial derivatives with respect to *x*, respectively. Solving these two parts of equation (3.19) yields the solution to the uncoupled parts of equation (21) and (22).

In equations (25) and (26), ε, σ and $\tau$are calculated based on the characteristic equations i.e., first part of (21) and (22) being reduced to zero.

$\epsilon = \sqrt{\omega^2 Q_1/_{\alpha^2}}$...... (27)

IV. ANALYSIS OF CURVED CANTILEVER BEAM

The Dynamic finite element is used to compute the natural frequencies of curved symmetrical sandwich beams. Sandwich beams are made with the aluminum and mild steel as the face layers and the core layers as rubber, neoprene and Polyethane rigid foam.The details of physical and geometrical properties of specimen are given as: The thickness of top and bottom layers t=0.4572 mm, core layer thickness as t_c =12.7 mm, span S=0.7112 m and a radius of curvature of R= 4.225 m.

The material properties of sandwich beam are given in Table 1.

Table 1: Material properties of curved sandwich beam for face and core layers.

Type of material	**Young's Modulus [E] (GPa)**	**Density $[\rho]\left(kg/m^3\right)$**
Aluminium	68.9	2680
Neoprene	0.0008154	950
Natural Rubber	0.00154	960
Polyethane Rigid Foam	0.00952	32.8
Mild Steel	210	7700

Fivedifferent types of sandwich beams are considered for investigation, which consists of Aluminum - Polyethane Rigid Foam – Aluminum, Aluminum – Rubber- Aluminum, Aluminum-Neoprene- Aluminum, Mild Steel- Rubber- Mild Steel, Mild Steel-Neoprene- Mild Steel.The natural frequencies for the sandwich structure of Aluminum - Polyethane Rigid Foam - Aluminum are determined by dynamic finite element formulation and are shown in Table 2.The natural frequencies obtained here are in good agreement with results of Ahmed (1971).Here small deviations between the previous and current results are noticed.

Table 2: Validation of present theory with Ahmed (1971)

Mode No.	**Natural Frequencies $\left(rad/sec\right)$**	
	Ahmed(1971)	**Present Theory**
1	1124.69	1102.19
2	1671.33	1604.47
3	3430.62	3224.78
4	5868.5	5575.07
5	8664.51	8317.92

For Aluminum-Natural Rubber-Aluminum curved sandwich beam the frequencies are calculated by dynamic finite element formulation with varying the core thickness are shown in Fig. 1. From this it is observed that as the core thickness increases, the frequency increases. With Neoprene as core material keeping aluminum as face material the frequencies are calculated by dynamic finite element formulation with varying the core thickness are shown in Fig. 2. The maximum percentage variation in frequencies by comparing Fig. 1 and Fig.2 is 7.51 and the minimum variation is 3.11. When neoprene is used as the core material frequencies obtained are very less when compared to natural rubber. By this it means that the damping effect of the sandwich containing neoprene as core material is more when compared with rubber as core material. Fig. 3 shows the frequency values obtained for curved sandwich beam from dynamic finite element formulation with mild steel as face material and rubber as core material. It is observed from this figure that as the core thickness increases, the frequency increases. With Neoprene as core materials keeping mild steel as face material the frequencies are calculated by dynamic finite element formulation with varying the core thickness is shown in Fig. 4. The maximum percentage variation in frequencies by comparing Fig. 3 and Fig. 4 is 4.62 and the minimum variation is 0.38. When neoprene is used as the core material frequencies are obtained as very less when compared to natural rubber as core material. By this it means that the damping effect of the sandwich beam containing neoprene as core material is more when compared with rubber as core material. And comparing all materials the damping effect of aluminum-Neoprene-aluminum is more.

By keeping the total thickness of a curved sandwich beam as same and varying the thickness of the core material the frequencies obtained are shown in Fig. 5 through Fig. 8. From Fig. 5 it is observed that as core thickness increases the frequencies increases. With Neoprene as core material keeping aluminum as face material the frequencies are calculated by dynamic finite element formulation with varying the core thickness are shown in Fig. 6. The maximum percentage variation in frequencies by comparing Fig.5 and Fig. 6 is 6.94 and the minimum variation is 3.03. Fig.7 shows the frequency values obtained for curved sandwich beam from dynamic finite element formulation with mild steel as face material and rubber as core material. It is observed from this figure that as the core thickness increases, the frequency increases and with Neoprene as core material keeping mild steel as face material the frequencies are calculated by dynamic finite element formulation with varying the core thickness are shown in Fig. 8. The maximum percentage variation in frequencies by comparing Fig. 7 and Fig. 8 is 4.57 and the minimum percentage variation is 0.61.

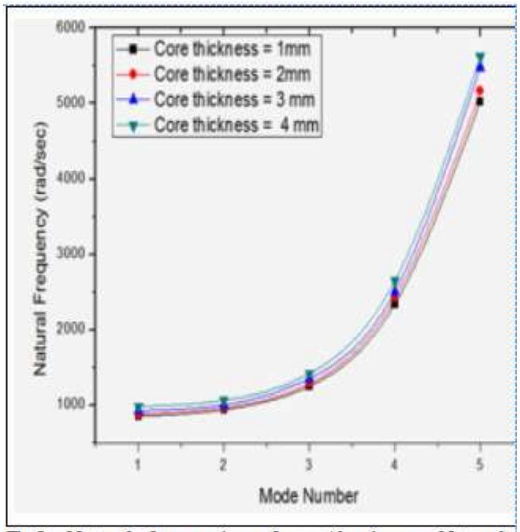

Fig.2: Natural frequencies of an Aluminum –Natural Rubber- Aluminum sandwich beam.

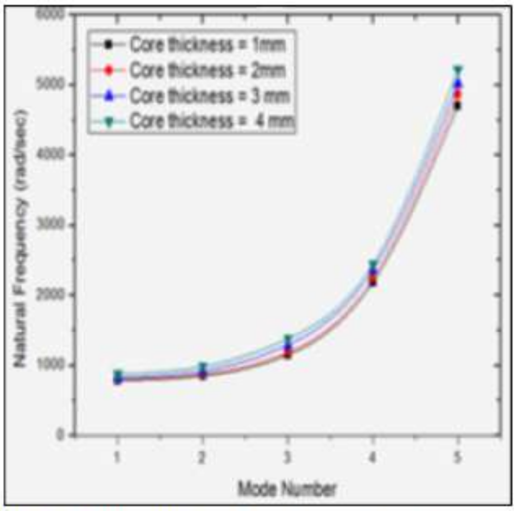

Fig 3: Natural frequencies of an Aluminum - Neoprene – Aluminum sandwich beam.

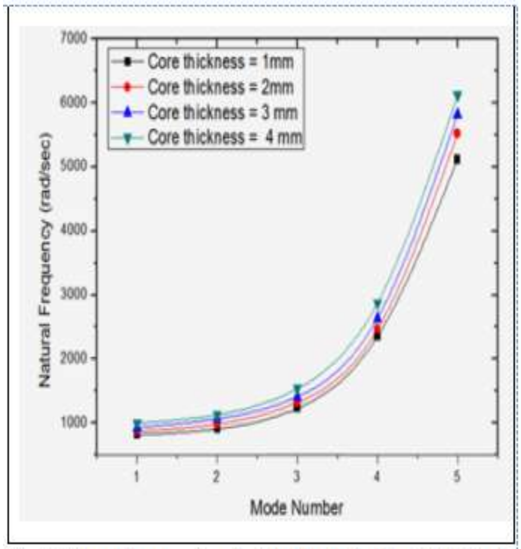

Fig 4: Natural frequencies of a Mild Steel- Rubber- Mild Steel Sandwich beam

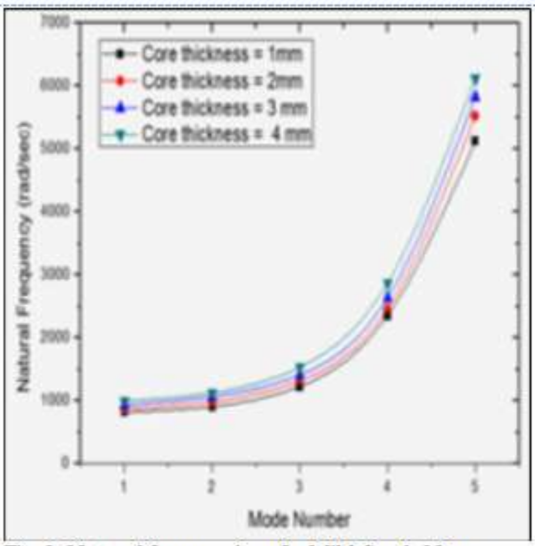

Fig 5: Natural frequencies of a Mild Steel- Neoprene - Mild Steel Sandwich beam

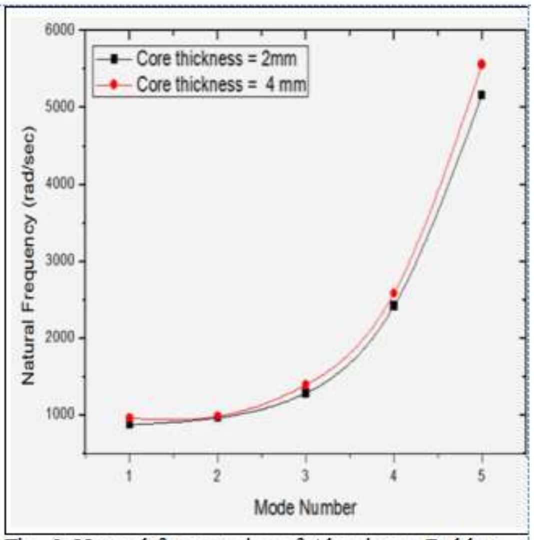

Fig. 6: Natural frequencies of Aluminum-Rubber-Aluminum sandwich beam.

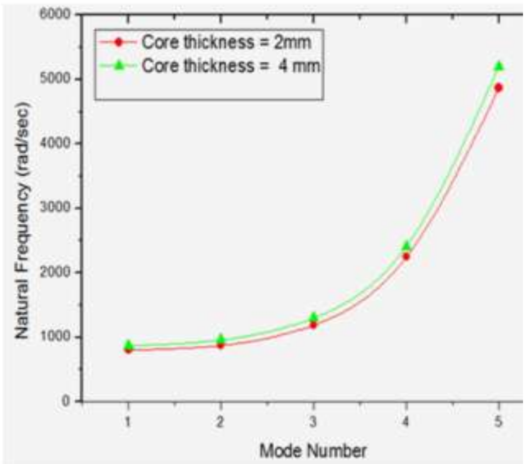

Fig. 7: Natural frequencies of a Aluminum-Neoprene- Aluminum sandwich beam.

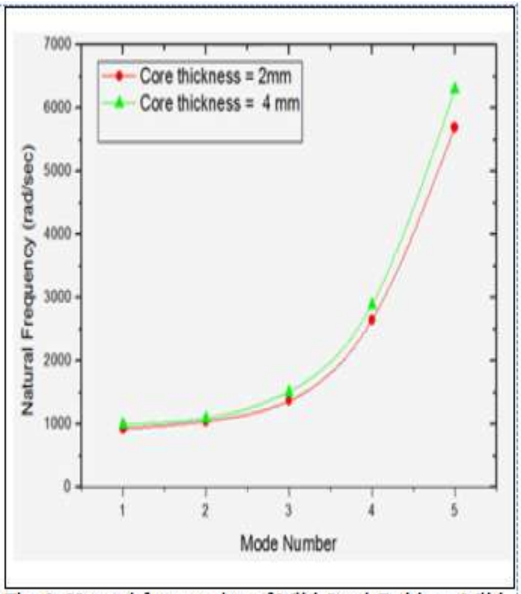

Fig. 8: Natural frequencies of Mild Steel-Rubber- Mild Steel sandwich beam.

V. CONCLUSIONS

The viscoelastic curved sandwich beam has been analyzed analytically and validated with literature results and has been found good in agreement. The developed theory has been carried out for modal analysis using dynamic element method by varying the core thickness to study the damping effect on the beams for the cantilever boundary conditions. The results obtained from the modal analysis showed that with increase in the thickness of the core layer there is an increase in the natural frequency for the same mode.It is found that the viscoelastic constrained layer damping treatment has a great significance in controlling the vibration of structures like beams, plates, etc. From the results it is concluded that damping characteristics for neoprene viscoelastic material has significant effect when compared with the rubber viscoelastic material.

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NOMENCLATURE

A	:Cross sectional area
ds, ds’	;Length of fiber before and afterdeformation
E, E_c	:Modulus of Elasticity of face and core layers
G, G_c	:Shear Modulus of face and core layers
h_1, h_2	:Thickness of the top and bottom face layers
h_c	:Thickness of the core Layer
s	:Length of Span
t	:Time

T	:Total Kinetic Energy of the beam
T_{f1}, T_{f2}	:Total Kinetic Energy of top and bottom face layers
T_c	:Total Kinetic Energy of core layer
U	:Total Strain Energy
U_{f1}, U_{f2}	:Total Strain Energy of top and bottom face Layers
u_1, u_2	:Circumferential displacement component of top and bottom face layers
u_c	: Circumferential displacement component of core layer
w_1, w_2	:Radial displacement component of top and bottom face layers
w_c	:Radial displacement component of core Layer
y	:Circumferential coordinate
z	:Normal coordinate measured from the middle surface of each layer
ρ, ρ_c	:Mass per unit volume of two Faces and core layers
ϵ	:Strain in The Fiber
$k(\omega)$	:Global Stiffness Matrix
ω	:Circular Frequency

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