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A study of Availability-Centered Preventive Maintenance for Aging Multi-state K-out-of-N Systems

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ABSTRACT
This study focuses on the aging multi-state k-out-of-n systems to establish a practical preventive maintenance (PM) model from the perspective of components. The established PM model aims to maximize system availability subject to total maintenance cost. Non-homogeneous continuous time Markov models (NHCTMMs) are utilized to evaluate the system availability, and using non-homogeneous continuous time Markov reward models (NHCTMRMs), to evaluate the total maintenance cost. The system availability and total maintenance cost for PM policies are substituted into a genetic algorithm (GA) to optimize the established PM model. A simulation case with the sensitivity analysis conducted verifies the efficacy of the proposed approach.
Keywords - Aging components, GA, k-out-of-n system, Markov chain, preventive maintenance

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I. INTRODUCTION

The k-out-of-n configuration is commonly seen in system design to upgrade system reliability, and is normally regarded as a redundancy design. Such system configuration is widely programmed into equipment bearing particularly high financial loss and fatal hazard to personnel owing to abrupt shutdown. Examples include the monitoring system of cockpits and multi-engine system in aircraft, as well as multi-pump hydraulic controls pressure [1]. These system configuration designs require at least k components functioning properly to ensure the entire system performs in comply with task requirements. [2-4]. Yuchang et al. [5] proposed a multi-valued decision diagram to evaluate k-out-of-n system reliability. Their proposed method is limited to use in cases involving a binary-state system; it cannot be applied to cases of a multi-state system. Erik et al. [6] aimed at a repairable k-out-of-n system to develop a cost-minimizing model capable of determining the optimal examining period and backup components. The limitation is that a periodical maintaining policy cannot provide information on real-time monitoring system performances and therefore may suffer disastrous loss due to abrupt system shutdown. Serkan [7] addressed the issue of optimal configuration design to determine the number of components, each possessing three states in a parallel system. Their proposed approach is based on minimizing the total cost. Soro et al. [8] aimed at multi-degraded systems to propose a preventive maintenance (PM) model consisting of minimal repairs and imperfect PM. Imperfect PM can restore systems from degrading states with performances surpass minimum requirement back to previous better states. The proposed model does not account for the aging impact of systems, and therefore uses the homogeneous continuous time Markov models to evaluate system reliability. Levitin and Lisnianski [9] and Nahas et al. [10] proposed the PM model of multi-state systems in which the total maintenance cost is minimized subject to allowable minimum system reliability via determining the optimal imperfect PM actions. Both studies considered the multi-state system consisting only of binary components, and thus, their findings are not directly applicable to cases involving multistate components. Tan and Raghavan [11] established a predictive PM model from the system perspective. PM actions are conducted on all the components once the system degrades to states lower than the performance requirement. Mathematically, directly monitoring system performance can greatly lessen the computation complexity than monitoring components performances to determine system performance. The PM policy monitoring components performances can refrain from disastrous loss of abrupt system shutdown because of the sudden failure of essential components. Therefore, given monitoring components performances is available, establishing PM policy from the components perspective has great practicability. Liu and Huang [12] proposed a PM model periodically monitoring the multi-state degraded system to optimize the PM period and the corresponding PM actions at maintenance points.

This model does not consider the aging impact within each time period during which the failure rate is set as constant. The discrete-time homogeneous Markov model is therefore used to describe the dynamic behavior of the system and reduce the computation difficulty compared to non-homogeneous continuous time Markov models (NHCTMMs).

As technology advances, the functional performance of a device tends to endure outside operating impact and aging effect, and therefore enables these devices to even function properly at lower performances. As such, the k-out-of-n design configured with multi-state components each possessing distinct performance constitutes a complicated multi-state system, which is extensively used in modern industry [13-15]. The multi-state reliability theorem is more appropriate to evaluate this type of system performance than the conventional binary-state theorem. Normally, the multi-state system fails functionally as it degrades in state and its performance fails to fulfill task requirements [16-18]. Cases of multi-state k-out-of-n systems involve nuclear power plants and military facility systems, for example [1,19]. Therefore, examining the multi-state k-out-of-n systems regarding aspects such as lifetime prediction and system reliability improvement is an essential issue and a difficult challenge as system states increase with complication of system [20].

Real-time monitoring technology has sufficiently matured to enable monitoring of the sub-systems or components performance owing to the advancement of computer-related technology. Given such circumstances, from the perspective of components, establishing PM policy became available and practicable to improving or sustaining system reliability [21]. PM does indispensable work in lengthening the lifetime of a system with desired performances such as reliability and availability, given limited maintenance resources. Certainly, a well-planed PM scheme can benefit systems performances significantly and make best use of maintenance resources.

II. THE PROPOSED APPROACH

This study focuses on the multi-state k-out-of-n systems with aging multi-state components to establish a PM model from the perspective of components. The established PM model aims to maximize system availability subject to total maintenance cost, featuring limited maintenance resources. Mathematically, this study employs the NHCTMMs to evaluate system availability with the instantaneous probabilities distributions of multiple intermediate states obtained, while using non-homogeneous continuous time Markov reward models (NHCTMRMs) to evaluate the total system maintenance cost. Due to the inherent complexity of solving the NHCTMMs, this study employs the bound approximation approach [22] to solve the NHCTMMs and NHCTMRMs. The system availability and total maintenance cost corresponding to a maintenance policy is then substituted in a genetic algorithm (GA) search mechanism primarily involving crossover and mutation to optimize the established PM model for the multi-state k-out-of-n systems. A simulation case with the sensitivity analysis conducted on important parameters verifies the efficacy of the proposed approach.

2.1 MODEL ASSUMPTIONS

The established PM model has the following assumptions.

(1) The performance of components can be identified instantaneously via real-time monitoring techniques.
(2) Components in the system have multiple intermediate states with distinct performances functioning from perfect to complete failure.
(3) Components degrade randomly to deteriorated states over time without maintenance.
(4) Conducting PM can restore component states to previous better states.
(5) All components states are statistically independent.
(6) The failure rates amid intermediate states of components are increasingly time-dependent functions.

2.2 THE ESTABLISHMENT OF PM MODEL

The PM model maximizing the minimum system availability over mission duration subject to total system maintenance cost is as follows.

Objective:

$$Max \quad Min\ A(t) = \sum_{i=1}^{K} p_i(t) \times 1\,(g_i \geq w) \tag{1}$$

Constraint:

$$C_S = \sum_{l=1}^{m} C_{pm,l} < C_S^* \tag{2}$$

where C_S is the total system maintenance cost; $A(t)$ is the system availability at time t; C_S^* represents the allowable total maintenance cost; $C_{pm,l}$ is the PM cost of the l th component; $p_i(t)$ is the probability of state i at time t; K indicates the totality of system states; g_i is the system performance at state i; w is the minimum acceptable performance enabling system to function properly, and $1\,(g_i \geq w)$ is a unit function that takes a value of 1 when g_i is greater than or equal to w. The GA that efficiently solves the complicated nonlinear combinatorial optimization problem is utilized to optimize the established PM model. Accordingly, the decision variables with maintenance activities conducting in degradation states of individual components are obtained to form a PM policy from the perspective of component for the k-out-of-n system. The Matlab program is coded to execute the proposed approach.

III. OPTIMIZE THE ESTABLISHED PM MODEL WITH GA

A simulation case comprising a 2-out-of-4 subsystem and a component connected in series elucidates the proposed approach. Each component possesses three states with distinct performances. Fig. 1 shows the system configuration. Notations $\lambda_{k,k-1}(\mathrm{t})$ and $\mu_{k-1,k}(\mathrm{t})$, $\{k=1,2,3\}$ represent failure rate functions and repairing rates among three states of each component, respectively. Notably, the failure rate function mainly accounts for the impact of aging on components.

According to the system configuration of this simulation case, the established PM policy optimized by GA is detailed in the following steps.

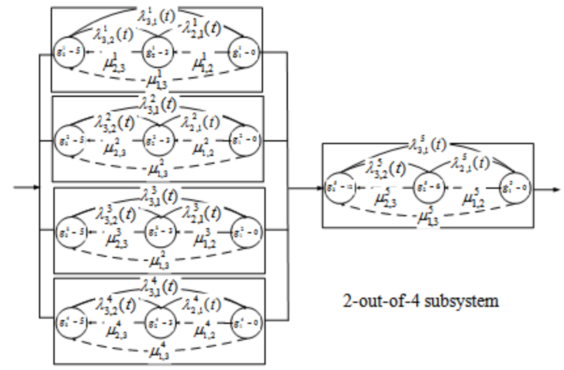

Fig. 1 System configuration

Step 1. Encode chromosome

Each chromosome comprises 15 genes corresponding to PM actions of states of 5 components each having 3 states. The genes were encoded using integers to represent varied PM actions. For the simulation case, only two alternative PM actions involving major and minor repairs are considered. The minor repair enables components to be restored to a one-step better state; while, the major repair enables components to be restored to a two-step better state.

Step 2. Construct initial chromosome population

We randomly produce 15 genes combinations featuring varied chromosome structures that each corresponds to a specific PM policy. Accordingly, 10 chromosomes are obtained to form initial population and are substituted in the subsequent GA search mechanism.

Step 3. Determine the fitness vale of chromosome

The GA fitness function for optimizing the established PM model corresponds to the objective function that maximizes minimum system availability over mission duration. Therefore, in total, 10 system availabilities of distinct PM policies in forms of 10 chromosomes representing different gene combinations are calculated based on the established NHCTMM of systems using the bound approximate approach [22]. The predominance of 10 chromosomes constituting GA population can be determined with the system availabilities calculated. Furthermore, the total maintenance cost consumed at each PM can also be determined as results of resolving the NHCTMMs and NHCTMRMs for the PM policy of each component in form of chromosome structure. The NHCTMMs and NHCTMRMs are established based on PM actions of each component in terms of transition diagrams of the five components. The established PM policy ensures that allowable total maintenance cost is unsurpassed. To elucidate the calculation process of system availability and total maintenance cost, the following content details the calculation.

(1) Calculation of system availability

Totally, 243 system states can be obtained as a result of 5 components each having 3 states, 3x3x3x3x3=243. Given the scenario that each component stays at state 3 with the highest performance, the system performance is determined to be 12. This is due to the subsystem performance amounts to $\mathrm{g}_3^1+\mathrm{g}_3^2+\mathrm{g}_3^3+\mathrm{g}_3^4=5+5+5+5=20$, and is connected to a component with performance 12 in series. Therefore, the system performance is calculated as $\min=\{20,12\}=12$. Noticeably, the 2-out-of 4 subsystem fails as the number of components that function properly are less than two, zeroing the performance. The system availability is accordingly calculated as $\mathrm{A}(t)=\sum_{i=1}^{243} p_i(t)\times 1(g_i\geq 6)$ with minimum requirement of performance at w equals 6, which includes 144 states for this scenario. The instantaneous probability distributions for all states $p_i(t)$ are determined using the NHCTMM of the system in the form of the Chapman-Kolmogorov simultaneous differential equation established as follows:

$$\frac{dp_j(t)}{dt}=\left[\sum_{\substack{i=1\\i\neq j}}^{243} p_i(t)\,\alpha_{ij}(t)\right]-p_j(t)\sum_{\substack{i=1\\i\neq j}}^{243}\alpha_{ji}(t),\ j=1,2,\ldots,243$$

(3)

where $\alpha_{i,j}(\mathrm{t})$ represents the transition intensity function between state *i* and state *j*, which is a compound of failure rates functions and repairing rates for each related component. Fig. 2 shows the transition diagram of the system with the highest performance of 12 reducing to the lowest performance of 0. Table 1 lists the failure rate functions among three states for the five components; table 2 presents the corresponding repairing rates. Table 3 lists the cost parameters related to the maintenance actions.

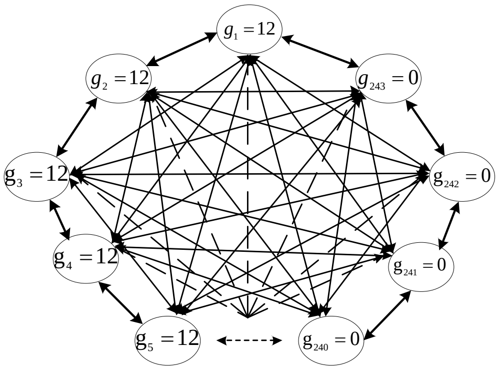

Fig. 2 Transition diagram of system

Table 1 Failure Rate Functions of Five Components

Failure Rates functions	Components				
	1	2	3	4	5
$\lambda_{3,2}(t)$	(0.20+0.09t)/24	(0.20+0.09t)/24	(0.20+0.09t)/24	(0.20+0.09t)/24	(0.22+0.09t)/24
$\lambda_{3,1}(t)$	(0.28+0.10t)/24	(0.28+0.10t)/24	(0.28+0.10t)/24	(0.28+0.10t)/24	(0.30+0.10t)/24
$\lambda_{2,1}(t)$	(0.12+0.06t)/24	(0.12+0.06t)/24	(0.12+0.06t)/24	(0.12+0.06t)/24	(0.14+0.06t)/24

Note: $\lambda_{i,j}(t)$ represents the failure rate function from state i to state j at time t

Table 2 Repairing Rates for Five Components

Repairing Rates	Components				
	1	2	3	4	5
$\mu_{1,3}$	0.125/24	0.125/24	0.125/24	0.125/24	0.125/24
$\mu_{1,2}$	0.320/24	0.320/24	0.320/24	0.320/24	0.350/24
$\mu_{2,3}$	0.500/24	0.500/24	0.500/24	0.500/24	0.525/24

Note: $\mu_{j,i}$ represents the repairing rate enabling components to restore from state j to previous better state i

Table 3 Cost Parameters of Varied Maintenance Actions

Maintenance actions	Maintenance cost				
	Component 1	Component 2	Component 3	Component 4	Component 5
Minor repair	State 1→2 : 50 State 2→3 : 30	State 1→2 : 50 State 2→3 : 30	State 1→2 : 50 State 2→3 : 30	State 1→2 : 50 State 2→3 : 30	State 1→2 : 60 State 2→3 : 40
Major repair	State 1→3 : 100	State 1→3 : 100	State 1→3 : 100	State 1→3 : 100	State 1→3 : 120

(2) Calculation of total maintenance cost

The NHCTMMs for all five components can be established according to intensity transition diagrams of the PM action related to the five components, while the corresponding NHCTMRMs can also be established given cost parameters in table 3.

NHCTMMs:

$$\frac{dp_j(t)}{dt} = \left[\sum_{\substack{i=1\\ i\neq j}}^{3} p_i(t)\,\alpha_{i\,j}(t)\right] - p_j(t)\sum_{\substack{i=1\\ i\neq j}}^{3}\alpha_{j\,i}(t),\ j=1,2,3 \quad (4)$$

NHCTMRMs:

$$\frac{dV_i(t)}{dt} = r_{i\,i} + \sum_{\substack{j=1\\ j\neq i}}^{3}\alpha_{ij}(t)\, r_{i\,j} - \sum_{j=1}^{3}\alpha_{j\,i}(t)V_j(t),\ j=1,2,3 \quad (5)$$

where $\gamma_{i,j}$ represents the reward accounted for as a state transits from state i to state j; $V_i(t)$ is the total expected reward accumulated until time t while in states i. The reward in the established PM model directly associates with maintenance costs of varied PM actions. These above NHCTMMs and NHCTMRMs are efficiently resolved using the bound approximate approach [22]. Accordingly, the expected total maintenance cost is determined based on the values of $P_i(t)$ and $V_i(t)$ resolved.

Step 4. Conduct GA search mechanism

The predominance of the 10 chromosomes obtained from step 3 constitutes a ground to conduct the searching mechanism that mainly includes crossover and mutation procedure. The next GA offspring is thereby evolved.

Step 5. Test of GA terminal condition

This study terminates the GA search mechanism after 50 consecutive iterations without improvement of system availability. The optimal PM policy is therefore obtained under such circumstance. Repeatedly conducting steps 3-5 until the terminal condition is satisfied obtains the optimized solution. Figures 3-7 show the optimal PM policy from the components' perspective in terms of transition diagrams of the five components. The PM policy of component 1 is conducting minor repairs at degraded state 2 and state 1, respectively; while, conducting minor repair at degraded state 2 and major repair at state 1 for both components 2, 3, and 5. However, for component 4, only minor repair is conducted at state 2. Under such PM policy, the minimum system availability over mission duration of one year is 0.987 with total maintenance cost of 2.229 unit incurred. Figure 8 displays the system availability tendency over mission duration**.** Figure 9 displays the GA convergence tendency.

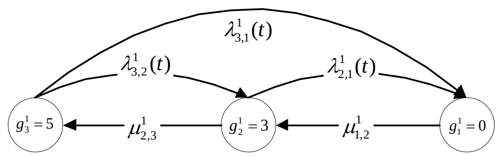

Fig. 3 Transition diagram of component 1 for optimal PM policy

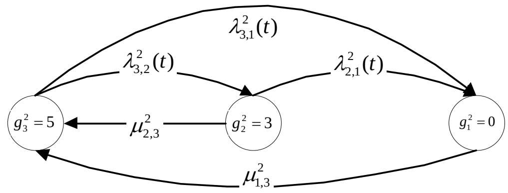

Fig. 4 Transition diagram of component 2 for optimal PM policy

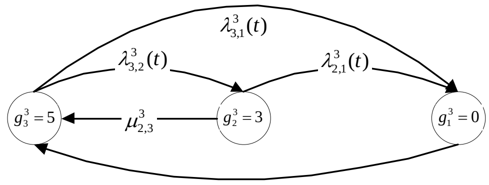

Fig 5 Transition diagram of component 3 for optimal PM policy

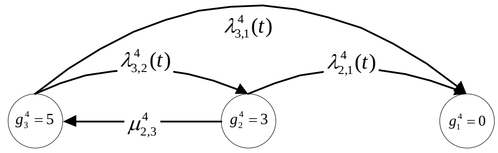

Fig. 6 Transition diagram of component 4 for optimal PM policy

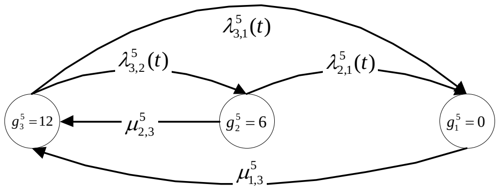

Fig. 7 Transition diagram of component 5 for optimal PM policy

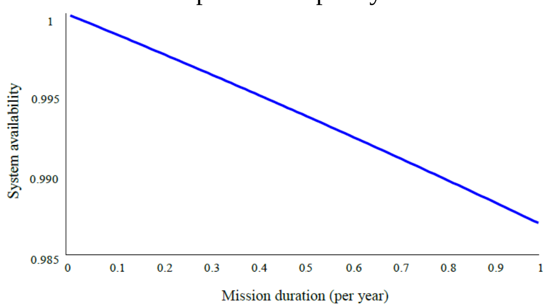

Fig. 8 System availability tendency over mission duration

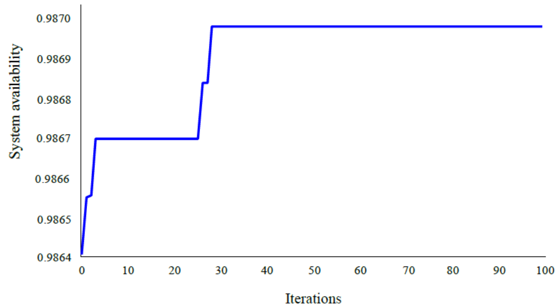

Fig. 9 GA convergence diagram

To further verify the correctness of the proposed approach, this study conducts sensitivity analysis on the minimum requirement performance w with values set at 12, 10, 8, and 6, respectively. Table 4 present the system availability and total maintenance cost varied with the decrease of w value. Figure 10 shows the tendency of system availability increased with the decrease of minimum requirement performance. Furthermore, given w equals 6 and a specific PM policy shown at figures 3-7, increasing repairing rate would heighten the total maintenance cost as demonstrated by increasing the original setting of repairing rate 5-fold and 10-fold, while conducted in combination with a 5-fold and 10-fold increase of the original setting of failure rate function, respectively. As expected, given fixed repairing rate, system availability lessens with the increase of failure rate; while, given fixed failure rate function, system availability increases with the increase of repairing rate. Table 5 lists the results associated with system availability for this sensitivity analysis.

Table 4 Sensitivity Analysis of w Values

Performance requirements	System availability	Total maintenance cost
w=12	0.964	2.375
w=10	0.965	2.229
w=8	0.977	2.229
w=6	0.987	2.229

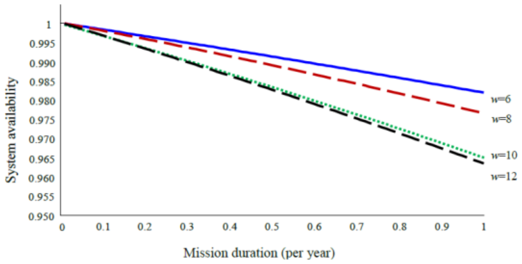

Fig. 10 System availabilities tendency with varied minimum performances requirements

Table 5 Sensitivity Analysis Of Failure Rates And Repairing Rates on System Availability

Failure rate function	Repairing rates	System availability
$\lambda(t)$	10μ	0.9877
	5μ	0.9873
	μ	0.9870
$5\lambda(t)$	10μ	0.9407
	5μ	0.9388
	μ	0.9372
$10\lambda(t)$	10μ	0.8851
	5μ	0.8815
	μ	0.8785

IV. CONCLUSION

The redundancy design of k-out-of-n configuration has been widely seen in equipment that causes devastating loss or is perilous to personnel once abrupt shutdown occurs. Typically, this type of equipment is found in nuclear plants, military facilities, and infrastructure, among others. For such equipment, establishing a PM from the perspective of components is particularly essential in preventing abrupt shutdown, due to sudden breakdown of essential components. Therefore, this study aims at equipment with k-out-of-n configuration design to establish a PM policy from the perspective of the components, given real-time surveillance of components is available. This study has the following merits.

(1)The optimal PM policy obtained is based on model maximizing system availability subject to total maintenance cost, which considers the restriction of maintenance resources practically. Engineering can easily apply the proposed approach, based on the practical requirement of system function, while simultaneously considering the allowable maintenance resources.

(2)Conversely, given that reducing maintenance resources is the primary concern, the established PM model can be revised to minimize total maintenance cost subject to minimum allowable system availability. With the solution approach proposed, the optimal PM policy in terms of maintenance actions in degrading states of each component can be determined simply.

(3)From the PM policy obtained, we found that the maintenance actions are conducted partially at some degraded states of components capable of fulfilling maintenance resources restriction, while maximizing system availability over mission duration rather than at all degraded states.

(4)The proposed approach relying heavily on the construction of NHCTMMs to determine the system performance faces resolving difficulty. Particularly, with the increase of solving complexity from model augmentation, the proposed approach will face the challenge of solving difficulty in terms of calculation precision. As a result, when solving large-scale modes, a more efficient approach developed to solving the system performances of the proposed PM model is indispensable.

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