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Gaussian Integer Solutions of Homogeneous Ternary Quadratic Equation $(x-y)[73(x-y)+54z]+z^2=16x(y-z)$

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ABSTRACT
The homogeneous ternary quadratic equation $(x-y)[73(x-y)+54z]+z^2=16x(y-z)$ has been analysed for its Gaussian integer solutions. Three different choices of Gaussian integer solutions are presented. Also, a few interesting relations among the solutions are given.

Keywords: Homogeneous quadratic, Ternary quadratic, Gaussian integer solutions.

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I. INTRODUCTION

The theory of Diophantine equations offers a rich variety of fascinating problems since antiquity [1, 2]. In particular, the quadratic equations with multiple variables are of interest to many Mathematicians and the solutions are represented by non-zero distinct real integers. It is worth mentioning here that an extension on ordinary integers into complex numbers is the Gaussian integers. In this context, one may refer [3, 4]. In particular, Gaussian integer solutions have been analysed for special ternary quadratic Diophantine equations in [5-11]. In [13, 14], Gaussian integer solutions of homogeneous quadratic equation with four unknowns have been presented.

In this communication, the homogeneous ternary quadratic Diophantine equation given by $(x-y)[73(x-y)+54z]+z^2=16x(y-z)$ is considered for different patterns of Gaussian integer solutions. It is worth mentioning that the Gaussian integer solutions are different from those presented in [12].

II. NOTATIONS

- Polygonal number of rank n with size m

$$t_{m,n}=n\left[1+\frac{(n-1)(m-2)}{2}\right]$$

- Pronic number of rank n

$$\mathrm{Pr}_n=n(n+1)$$

- Star number of rank n

$$S_n=6n(n-1)+1$$

III. METHOD OF ANALYSIS

The homogeneous ternary quadratic equation to be solved is

$$(x-y)[73(x-y)+54z]+z^2=16x(y-z) \tag{1}$$

The substitution

$$x=6a+2b+i3c,\quad y=6a+i6c,\quad z=2b+i9c \tag{2}$$

in (1) leads to

$$b^2+c^2=a^2=a^2*1 \tag{3}$$

Assume $a=p^2+q^2,\quad p,q\succ 0$ (4)

Choice :1
Write 1 as

$$1=\frac{(3+4i)(3-4i)}{25} \tag{5}$$

Using (4) and (5) in (3) and employing the method of factorization and equating the positive factors, we get

$$b+ic=\frac{1}{5}(p+iq)^2(3+4i) \tag{6}$$

Equating the real and imaginary parts of (6), we get

$$\left.\begin{aligned} b(p,q)&=\frac{1}{5}\left(3p^2-3q^2-8pq\right)\\ c(p,q)&=\frac{1}{5}\left(4p^2-4q^2+6pq\right)\end{aligned}\right\} \tag{7}$$

The choices $p=5P\ and\ q=5Q$ in (4) and (7) lead to

$$b(P,Q)=15P^2-15Q^2-40PQ$$
$$c(P,Q)=20P^2-20Q^2+30PQ$$
$$a(P,Q)=25P^2+25Q^2$$

In view of (2), the corresponding non-zero distinct Gaussian integer solutions of (1) are

$$x(P,Q)=180P^2+120Q^2-80PQ+i\left(60P^2-60Q^2+90PQ\right)$$

$$y(P,Q)=150\left(P^2+Q^2\right)+i\left(120P^2-120Q^2+180PQ\right)$$
$$z(P,Q)=30P^2-30Q^2-80PQ+i\left(180P^2-180Q^2+270PQ\right)$$

Properties:

- $2x(1,Q)-y(1,Q)-15S_Q\equiv 0(\mathrm{mod}5)$
- $3x(P,1)-z(P,1)-510\mathrm{Pr}_P\equiv 0(\mathrm{mod}2)$
- $3y(P,1)-2z(P,1)-63S_P-t_{26,P}\equiv 0(\mathrm{mod}3)$

Note:

In general, in (5) one may write 1 as

$$1=\frac{\left(p^2-q^2+i2pq\right)\left(p^2-q^2-i2pq\right)}{\left(p^2+q^2\right)^2}$$

$$(or)\quad 1=\frac{\left(2pq+i\left(p^2-q^2\right)\right)\left(2pq-i\left(p^2-q^2\right)\right)}{\left(p^2+q^2\right)^2},\ p\succ q\succ 0$$

Choice : 2

Instead of (5), write 1 as

$$1=i^{2n}(-i)^{2n} \tag{8}$$

For this choice, we have

$$b(p,q,n)=(-1)^n\left(p^2-q^2\right)$$
$$c(p,q,n)=(-1)^n\,2pq$$

Employing (2), the corresponding non-zero distinct Gaussian integer solutions of (1) are

$$x(p,q,n)=6\left(p^2+q^2\right)+2(-1)^n\left(p^2-q^2\right)+i\,6(-1)^n pq$$
$$y(p,q,n)=6\left(p^2+q^2\right)+i12(-1)^n\ pq$$
$$z(p,q,n)=2(-1)^n\left(p^2-q^2\right)+i18(-1)^n pq$$

Properties:

- $3x(p,1,2k)-z(p,1,2k)-t_{46,p}\equiv 2(\mathrm{mod}3)$
- $3x(1,q,2k+1)-z(1,q,2k+1)-22\mathrm{Pr}_q\equiv 0(\mathrm{mod}2)$
- $x(p,p+1,n)+y(p,p+1,n)-z(p,p+1,n)-12=24\mathrm{Pr}_p$
- $2\{x(1,q,n)+y(1,q,n)-z(1,q,n)-12\}$ is a nasty number

Choice : 3

Also, observe that, one may write 1 as

$$1=\frac{(1+i)^{2n}(1-i)^{2n}}{2^{2n}} \tag{9}$$

Following the procedure similar to the above, the corresponding another set of Gaussian integral solutions to (1) are given by

$$x(p,q,n)=6\left(p^2+q^2\right)+2\left[\left(p^2-q^2\right)\cos\frac{n\pi}{2}-2pq\sin\frac{n\pi}{2}\right]$$
$$+i\,3\left[\left(p^2-q^2\right)\sin\frac{n\pi}{2}+2pq\cos\frac{n\pi}{2}\right]$$
$$y(p,q,n)=6\left(p^2+q^2\right)$$
$$+i\,6\left[\left(p^2-q^2\right)\sin\frac{n\pi}{2}+2pq\cos\frac{n\pi}{2}\right]$$
$$z(p,q,n)=2\left[\left(p^2-q^2\right)\cos\frac{n\pi}{2}-2pq\sin\frac{n\pi}{2}\right]$$
$$+i\,9\left[\left(p^2-q^2\right)\sin\frac{n\pi}{2}+2pq\cos\frac{n\pi}{2}\right]$$

Properties:

- $3y(p,p+1,4k-3)-2z(p,p+1,4k-3)-18=44\mathrm{Pr}_p$
- $x(q,q,n)+y(q,q,n)-z(q,q,n)$ is a nasty number

IV. CONCLUSION

In this paper, we have presented three different choices of Gaussian integer solutions for the ternary quadratic equation $(x-y)[73(x-y)+54z]+z^2=16x(y-z)$ representing an elliptic cone. As the ternary quadratic equations are rich in variety, one may attempt for finding Gaussian integer solutions to the other choices of homogeneous (or) non-homogeneous ternary quadratic equations.

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