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Third Law of Motion: Revisited and Generalized

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ABSTRACT
Descartes third law of motion has preceded Newton's third law of motion. Both the laws appear to explain similar observations. Third law is critically analysed in view of definition and elaboration given by Newton at page 20 of *the Principia*. It is evident that law is incompletely studied. Newton did not give any mathematical equation for the law. The definition and explanation of the law is completely analyzed theoretically and discussed in view of the experimental data. Depending upon the action, the reaction is result of interactions between various bodies having different characteristics. Thus action and reaction may not be equal in all cases. Newton's explanation is simply qualitative, not quantitative which is required for scientific analysis. The law is universally applicable to all cases and all bodies without any constraint. However Newton elaborated the same in terms of just three qualitative examples which is not scientific. The terms action and reaction neither possess units nor dimensions, even expressed in terms of equations the explanation is qualitative only. The law is understood in terms of force only. However Newton expressed law in terms of both force and velocity in three examples. If law is expressed in terms of force and velocity analogous to action and reaction, then its limitations are evident. For example it does not account for the significant factors e.g. inherent characteristics, nature, compositions, flexibility, rigidity, magnitude, size, elasticity, shape , distinctiveness of interacting bodies, mode of interactions, point of impact etc. The law is universally applicable for all bodies e.g. bodies may be of steel, wood, rubber, cloth, wool, sponge, spring, typical plastic, porous material, air / fluid filled artifact, mud or kneaded flour or chewing gum specifically fabricated material etc. For all such bodies if the action is same, then the reaction must be the same. But it is not found true in all cases, as interaction between rubber and steel are different from rubber and cotton. In the third law the law of conservation of energy is obeyed, and energy is converted in various forms. Thus law is generalized; reaction is proportional to action depending upon characteristics of system. Mathematically, Reaction = -K action, the value of K may be equal to, less than, or greater than unity depending upon experimental parameters.
Keywords: Newton's third law, coefficient, generalized form of the law

1.0 Origin of laws of motion

Aristotle (384BC-322BC) believed in doctrine of cause and effect [1,2]. Everything that is in motion must be moved by something. A table is pushed or pulled on the floor then it moves as long as external force acts on it. The table stops when external force ceases to act. Aristotle stated that rest (velocity =0) is natural state of body, and it starts moving when by external force acts on it. It appears true in daily life observations. Galileo perceived experiments in hypothetical sy stem when all resistive forces (i.e. frictional, atmospheric, gravitational etc.) are eliminated from the system, then body once set in motion always remains in state of uniform velocity (constant velocity). For example, on a long smooth glass floor a marble maintains its uniform velocity on the system where resistive forces are reduced as far as possible. Thus Galileo on the basis of such experiments perceived that a moving body maintains its constant speed in straight line unless no external force acts on it. Thus Galileo put forth that movement of body with constant velocity is natural tendency of body, and it stops due to resistive forces[2] Galileo's law of inertia is given by[3]

"A body moving on a level surface will continue to move in the same direction at a constant speed unless disturbed."

1.1 Descartes law of motion.

The first two laws resemble with Galileo's law of inertia, and third law is independent of existing laws [4-5].

Law 1

Each thing, in so far as it is simple and undivided, always remains in the same state, as far as it can, and never changes except as a result of external causes. ... Hence we must conclude that what is in motion always, so far as it can, continues to move. (Principles Part II, article. 37)

Law 2

Every piece of matter, considered in itself, always tends to continue moving, not in any oblique path but only in a straight line. (Principles Part II, article. 39)

It is independent of existing laws and all the three laws precede Newton's Third Law of motion.

Law 3

When a moving body collides with another, if its power of continuing in a straight line is less than the resistance of the other body, it is deflected so that, while the quantity of motion is retained, the direction is altered; but if its power of continuing is greater than the resistance of the other body, it carries that body along with it, and loses a quantity of motion equal to that which it imparts to the other body. (Principles Part II, article. 40)

1.2 **Descartes Third Law**

Descartes and Newton's laws can be compared, as both appear to explain the same observations. Now 'the power of continuing' may be regarded as force exerted by projectile on target (action) and the 'resistance of other body' implies force (resistive in nature) which is exerted by target on the projectile (reaction). Thus action is equivalent to 'the power of continuing' and reaction to (resistive force of body); further both the laws explain the same observations but using different terminology, nomenclature and language.

The first case in Descartes third law [4-5] implies that 'the power of continuing' of projectile (action) is less than resistance of the target (reaction), then body is deflected and direction is altered. The law is qualitative in nature and no mathematical equation has been written. For example a rubber ball retraces its original path after hitting the wall. Further a sponge ball falls near the wall and chewing gum ball sticks to wall, when both strike like rubber ball. When a nail and spring are dropped on the floor from the same height, then spring rebounds upwards to original height but nail does not. When opposite pole of magnet is pushed towards the big magnet, they stick after colliding or impinging, there is no reaction at all. Newton described directly collisions or impingements in third examples.

In second case the 'the power of continuing' of projectile (action) is more than resistance of the target (reaction), and then projectile moves along with target. For example the billiards balls of same mass collide the other ball at rest, and then both balls move ahead simultaneously. On the same billiards table if the balls are replaced by cloth balls of same mass and strike identically then they move slowly comparatively. The same experiment can be speculated with chewing gum balls of same mass which stick to each other. Thus in Descartes third law of motion the power of continuing of projectile (action) and resistance offered by target (reaction) are not always same even when masses and velocities are equal and observations are taken under identical conditions. Similarly the book tears the paper when thrown on it; the same book rests gently on table and earth. This is due to inherent characteristics of materials.

Now it is evident from above examples that in general the dominance of 'power of continuing of projectile' (action) and 'resistance of other body' (reaction) i.e. target, obviously depends upon inherent properties of bodies. These factors may include inherent characteristics, nature; compositions, flexibility, rigidity, magnitude, size, elasticity, shape, distinctiveness of interacting bodies, mode of interactions, point of impact, surface etc. play significant roles. The various bodies affect the results e.g. bodies may be of steel, wood, rubber, cloth, wool, sponge, spring, typical plastic, porous material, air / fluid filled artifact, mud or kneaded flour or chewing gum specifically fabricated material etc. Such observations or experiments are quantitatively considered in various types of collisions, the simplest cases are one dimensional elastic collisions. So all possible experiments (taking materials i.e. target and projectile of different characteristics) must be conceived and conducted as they are too useful in understanding of Descartes third law of motion. The reason being that Descartes third law and Newton's third law explain similar observations. So a problem can be assessed in different ways. Further Descartes law is more elaborative and that of Newton's law is compact.

As Descartes third law and Newton's third law is also qualitative in nature without any equation explains the similar observations, so it can be checked whether above interpretation is equally applicable for Newton's law or not. Further, Descartes has anticipated law of conservation of energy stating that projectile loses a quantity of motion equal to that it imparts to target.

2.0 The Principia's Third Law of Motion

The original form of the Third Law of Motion as in the Principia[6] is:

To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts [1,]

Action = - Reaction (1)

or Mutual action of one body = - Mutual reaction of other body (1)

There is no other factor in the law. The equation indicates the action and reaction are *universally* equal without constraints for all type of bodies etc. Action and reaction do not have dimensions and units, hence not physical quantities. Now all the laws are understood in terms of mathematical equations written in terms of physical quantities, and predictions are quantitatively justified. In *The Principia* Newton gave three examples to illustrate the law. In first two examples, Newton expressed action and reaction in terms of push or pull (force), illustrating that the finger pushes the stone and horse pulls the stone. In both cases system remains at rest and work done is zero (W =FS=0). In third example, Newton describes collisions i.e. when projectile impinges (collides) the target, then if the velocity of target changes by Δv, then velocity of projectile also changes by -Δv. In this case work is non-zero. Further the velocity is related with momentum and kinetic energy.

But Newton did not give any equation relating to collisions as in Newton's time there was no tradition (as mathematical basis was not fully developed) to write equations, and laws were expressed geometrically and qualitatively only. The various equations are developed for one dimensional or two dimensional collisions; these were formulated by other mathematicians and scientists' later on, when laws of conservation of momentum and energy were defined. Thus all collision has to be studied as consequence of the third law of motion for complete understanding. These equations and equations for other perceptions based on the law are critically discussed here.

Here Newton did not characterize any type of body in the third law so that it is applicable to all bodies i.e. having diverse characteristics. The energy is inter- convertible from one form to other, during interactions, total energy remains constant. The numerous examples of third law of motion are explained on the basis of these three compact illustrations by Newton. Another noticeable point is that Newton did not give any equation for the third law at all. In fact Newton did not give any equation for Second Law of Motion (F =ma) and law of gravitation ($F = GmM/r^2$) in *the Principia* or elsewhere. In fact Newton started beginning of physics (by defining basic terms and laws; but beginning must not be regarded as end), described by phenomena geometrically and philosophically, but now it is age of precise robotics and computer programming.

All the laws are ought to be repeatedly confirmed by diverse and sensitive experiments and inherent characteristics of the interacting systems. But Newton's Third Law does not take any characteristic of the system in account, and is universally valid for every existing interacting system as per definition. The inherent characteristics affect the results in various laws and experiments. But Principia's third law of motion is completely independent of characteristics of bodies. The bodies can be of solid, liquid, gas or mixture of all. The law is applicable for all bodies thus can be considered for waves as well. Here those examples are discussed which were considered neither by Newton nor by following scientists quantitatively. Newton did not give any mathematical equation for the third law. Further energy is inter-convertible to various forms. Likewise Newton did not give equation for Second Law of Motion and law of gravitation.

In Newton's time it was beginning of science in systematic way. The physical quantities, units, dimensions were not defined. Thus the terms action and reaction do not possess units and dimensions, as Newton did not define action and reaction in terms of specific physical quantities. However, in the qualitative explanation given after the definition to the law, Newton expressed action and reaction in terms of push or pull (force) and motion (velocity), in *the Principia* at page 20. Newton applied third law of motion to examples when body remains at rest or moves. Thus it is applicable to numerous cases, and must be confirmed quantitatively for all i.e. comparing mathematical predictions with experimental results. Newton did not write eq. (1) in *the Principia*. Newton wrote the Principia's Second Law of Motion in proportionality form like Law of Gravitation, but The Third Law is expressed in equality form, which implies that all other factors are insignificant.

Now a day's all definitions are expressed in terms of equations. If mathematical predictions of equation is quantitatively confirmed in repeated experiments in all regions, then scientifically equations hence the definition of the law is regarded as true. Thus it is most important that definition and equation of the law is expressed in terms of physical quantity which has definite units and dimensions (may be basic or derived) i.e. Say velocity (m/s, M^0LT^{-1}). Action and reaction neither have units nor dimensions, thus it is not logical to call them as scientific terms as give no quantitative information as required in scientific measurements. Also the word motion does not have units and dimensions and Newton himself used it as velocity. In Newton's first law the motion is regarded as velocity. Newton expressed the law of gravitation in *the Principia* in terms of force, mass, distance; each physical quality has definite units and dimensions.

The most of developments took place well after Newton's Principia . The dimensional analysis was initiated by Fourier [7,8] in 1822 , and dimensions of force are based upon $F = ma$ (given by Euler in 1775), not on *Principia*'s $F =(v - u)$. Also, the unit of force (dyne) was first defined in 1866, about 184 years after publication of the first edition of the Principia[9,10] The 9th Conférence Générale des Poids et Mesures held in 1948, then adopted the name "newton" for the unit of force, in a resolution [10] . Thus Newton just initiated the beginning of physics by defining few terms , and did not end the same as science is expanding. Newton estimated the speed of sound in air as 280m/s, whereas experimental value is 330m/s. Thus Laplace corrected Newton's equation, so that exact value is obtained. Newton's law of cooling states that heat loss of a body is directly proportional to the difference in the temperatures between the body and its surroundings provided the temperature difference is small. A correction to Newton's law concerning larger temperature differentials was made in 1817 by Dulong and Petit. Thus improvements in older laws continue to accommodate as more facts as they are known.

Further, in third law of motion, the action and reaction have no units and dimensions, thus are not physical quantities. In Definition section of the Principia, Newton defined 'Quantity of mass' (mass), 'Quantity of motion' (product of mass and velocity, the momentum) at page 8 , and motion (velocity) at page 10 at Scholium after Definition section. In Second law of motion in *the Principia* , Newton defined force as proportional to alteration in motion. But the main terms in the third law of motion, the action and reaction were not defined scientifically for quantitative equations, as they do not possess any dimensions and units. Thus even if action and reaction are expressed in terms of mathematical quantities, the predictions are not checked quantitatively. The terms or words action and reaction are used as verb, noun, and adjective in English literature. Thus if properly analysed, then definition of Principia's third law of motion is more philosophical than scientific. Further action and reaction can be calibrated in terms of distance travelled and time taken to understand the meaning of law completely.

2.1 Newton's Original Explanation of the Third Law and its critical analysis.

"Whatever draws or presses another is as much drawn or pressed by that other. If you press a stone with your finger, the finger is also pressed by the stone. If a horse draws a stone tied to a rope, the horse (if I may so say) will be equally drawn back towards the stone: for the distended rope, by the same endeavour to relax or unbend itself, will draw the horse as much towards the stone as it does the stone towards the horse, and will obstruct the progress of the one as much as it advances that of the other.

If a body impinges upon another and by its force change the motion of the other, that body also (because of the quality of, the mutual pressure) will undergo an equal change, in its own motion, towards the contrary part. The changes made by these actions are equal, not in the velocities but in the motions of bodies; that is to say, if the bodies are not hindered by any other impediments. For, because the motions are equally changed, the changes of the velocities made towards contrary parts are reciprocally proportional to the bodies. This law takes place also in attractions, as will be proved in the next scholium."

From above discussion it is clear that Newton expressed action and reaction in terms of force and velocity (momentum and kinetic energy are easily derived from this, as mv and $mv^2/2$). The action and reaction are expressed as force in first two examples. In these case bodies remain at rest i.e. work done is zero (W =Fdx =0). In the third example, Newton expressed action and reaction in terms of velocity; in this case body moves and work is non-zero. In this case role of characteristics of body is significant. There are numerous examples involving action and reaction, thus these may be considered or understood in two ways i.e. in terms of force and velocity (or momentum or kinetic energy). It may be useful to understand that action and reaction in terms of kinetic energy as in the phenomena, energy is converted from one form to other and total energy remains constant i.e. energy is conserved in action and reaction.

2.2 Action and Reaction in terms of Force

Newton gave only three qualitative examples to support the law in *the Principia*. In first two examples, Newton used words 'presses' or 'draws' and 'motion' in third example. Further scientists concluded that words 'presses' or 'draws' mean force in first two examples given by Newton at page 20 of *the Principia*. In the existing literature action and reaction are expressed in terms of force [11]. These examples are as the finger pushes the stone; and horse pulls the stone. The stone is the body which is pushed or pulled but remain at rest after action in both the cases. The systems remains at rest and hence work done is zero (W=0). The other simple example in this case is a book lying on the table. We say that book exerts force on the table as F_b (may be regarded as action) and in turn table also exerts force on the book ,F_t (may be regarded as reaction) . Thus

Force exerted on table by book (F_b) = -Force exerted on book by table (F_t) (2)

However when book is placed on the sheet of paper, then paper is torn as force exerted by book (F_b) is more than force exerted by sheet of paper (F_p). Thus characteristics of bodies also play significant roles in equality of action and reaction. Thus law can be re-stated in terms of force and velocity as action and reaction in the Principia. In terms of force,

"To *every force there is always opposed an equal force; or the mutual forces of two bodies upon each other are always equal, and directed to contrary parts."*

2.3 **Action and Reaction in terms of velocity**

The third example is fairly general in sense when bodies move and work is done. Let body A (projectile) exerts force on the body B (target). The body A causes change in velocity of body B by Δv, then body A itself suffers reaction -Δv in the process. Newton stated that both change in velocity of body A (-Δv) is universally equal to change in velocity of body B (Δv) i.e.

Change in velocity of body B (Δv) = Change in velocity of body A (-Δv) (3)

Now various cases are possible in this regard. It must be clearly noted that Newton's third law is equally applicable for all collisions, as Newton has stated, if *a body impinges upon another.* It simply means when one body collides or strikes the other or collision takes, then if velocity of target changes by Δv, then velocity of projectile changes by -Δv. For comparison in first two cases Newton has considered that stones remain at rest when pushed or pulled i.e. W =0. Newton did not limit the definition and explanation to particular or specific type of interacting bodies, hence it is applicable for all types of bodies. It is also clear from the explanation given by Newton that the law is universally applicable for all impinging (colliding or striking) bodies. When one body exerts force as action on the other body, the other body also reacts. Thus action and reaction result in collisions. Consequently action and reaction constitute collisions, both are inseparable. It is also justified by eq.(14). There cannot be collision without action and reaction .The law of conservation of energy is obeyed in the processes, energy may be converted from one form to other. In the original Latin book i.e. *Philosophiæ Naturalis Principia Mathematica* Newton had used Latin word 'impingens'. In the translated masterpiece *Mathematical Principles of Natural Philosophy* Andrew Mott used word impinges (means collide or strike) as 'impingens'. The numerous possibilities of experiments or observations are possible. But it can be discussed with help of following examples when masses of projectile and target are equal.

(i) The simplest case when a iron bar A (projectile) of mass 0.5kg (or different) impinges or collides the exactly identical iron bar B (target) of the same mass, right at the centre placed at suitable surface. If the bar B moves with velocity 10cm/s (or different) after collision (impingement), then bar A will be retrace its path with velocity 10cm/s. The role of nature of surface is significant. If such observations are precisely observed then law will be justified.

(ii) Then bar B is stretched to a thin flat sheet and bar A impinges near its outer edge. Then bar B will be deflected sideways but the bar A may not be deflected with same velocity exactly in opposite direction. Finally such observations are required to be precisely checked.

(iii) When bar A is replaced by spring of high spring constant. When such spring is pushed towards the bar B, then after striking the bar B may be pushed forward with some velocity v. But the spring is pushed backwards with much higher velocity than v. Thus action and reaction are not equal. Such experiments can also be conducted for various other having different characteristics, masses , shapes etc.

(vi) Consider specially two spheres i.e. elastic sphere (S_e) and super elastic (S_{se}) in nature having same mass, and a specially fabricated square body. Let both sphere collide with the square with same velocity (hence momentum and kinetic energy). Then both spheres must change the velocity of sphere with same velocity i.e. v m/s . Then Newton's law implies that both spheres (S_e and S_{se}) must rebound with same velocity –v m/s. However, the sphere (S_{se}) which suffers super elastic collision rebound with greater velocity than v m/s . It is not consistent with Newton's third law of motion as both elastic sphere (S_e) and super elastic (S_{se}) should have rebounded with same velocity. In the above examples simplest case has been considered when masses of projectile and target are equal. However ball rebounds after striking the wall (which is at rest) in this case target is much heavier than projectile .In such cases elastic collisions in one dimension are considered.

Here we are especially concerned with third example of third law of motion given by Newton. As the law is applicable for all types of impinging or colliding bodies it is applicable for all types of collisions (may be elastic or inelastic). The purpose of discussion is that in existing literature, third law is not directly considered in case of collisions i.e. in such cases action and reaction are not calculated. However initial and final velocities of projectile and target are calculated under various possible cases. The initial velocity of target is action and final velocity reaction, in third law of motion. Both action and reaction (initial and final velocities of projectile) can be compared to find out whether they are equal in all cases or not.

Thus law can be re-stated in terms of force and velocity as action and reaction in the Principia. In terms of force,

"To every velocity of a body, there is always opposed an equal velocity when it impinges on other body, or mutual velocities of two bodies upon each other are always equal, and directed to contrary parts."

The momentum and kinetic energy are dependent on velocity, so eq.(3) can be expressed in terms of these.

III. Calibration of force in terms of distances travelled and time elapsed.

In general, Newton has applied or elaborated the Third Law of Motion by stating that *"If a body impinges upon another and by its force change the motion of the other, that body also (because of the quality of, the mutual pressure) will undergo an equal change, in its own motion, towards the contrary part."*

The body may impinge (strike or collide) the other if it is pushed by some force due to external influence or body may strike the other body or earth due to gravitational pull. The action and reaction can be calibrated in terms of distance travelled by body (from initial point of impact; and to then back to original point after colliding). It means relation between distance travelled and force can be established, precisely for some cases and can be extended for others. Further time (when projectile starts and strikes the target, and returns to original position) can be calibrated in the process like distance. Thus the action and reaction (forces exerted) can be calibrated in terms of time and distance in some cases. By measuring the distance of body the magnitude of force (work =force x distance) can be assessed. Such calibration can be understood for simplicity as in case of pyrometer. **[**The temperature of Sun and furnaces is measured with help of pyrometer, without touching the Sun. The reason is that Pyrometer is earlier calibrated in terms of temperature. The reason is that Pyrometer is earlier calibrated in terms of temperature.
There is a definite relationship between the temperature of a body and the radiations it emits at different wavelengths. This relationship, known as the Planck's law of radiation states that the hotter the body, the brighter it is at shorter wavelengths. The Sun emits maximum at the visible or yellow wavelength and the corresponding temperature is about 6000 degree Kelvin.**]**

Thus by calibration the understanding of Third Law of Motion is fully understood and extended in terms of time and distance, in addition to force and velocity as directly perceived by Newton. Thus applications of the Third Law of Motion are extended to its fullest extent, as the definition of the law is applicable universally. Newton expressed action and reaction in terms of force and velocity; and applied third law of motion to bodies in both cases i.e. they may be at rest or moving but quantitatively only. The comparison of original and generalized form of the third law of motion is shown in table 1.

Action and reaction can be understood in different ways for wider understanding. For example when a body is dropped from some height. When it reaches just above the surface of the earth, the maximum velocity ($v = \sqrt{2gh}$) can be regarded as action. After collision body rises, the maximum velocity when body just rises or jumps from the earth may be understood as reaction. For equality of action and reaction both the velocities (velocity just above the surface of the earth when body falls, and velocity just above the surface of the earth when body rises) must be equal. Both are unequal if initial kinetic energy of body is changed to other forms of energy.

IV Newton's Third Law of Motion is regarded as true under ideal or standard conditions only.

The law is said to be general in nature if mathematical predictions based on equation of the definition are confirmed experimentally in all possible experiments and results are quantitatively repeatable under all circumstances. These experiments may be existing at time when law is defined or discovered later on. For this first and foremost condition is that there must be concrete equation for the said law in terms of specific physical quantities (having units and dimensions). Thus predictions of mathematical equations can be experimentally checked. But Newton defined the law without mathematical equations, and explained it with three qualitative examples ignoring many others. Or Newton left that the law must be explained by wider scientific community. These were not explained quantitatively. Newton expressed law in terms of force and velocity (these are associated with time and distance); also applied to bodies when they remain at rest or move. Consequently work done can be zero or non-zero. Thus there are numerous experiments where Newton's Law is applicable but not specifically applied to draw conclusions over wide range. Thus analysis of the law should not be regarded as complete. In science qualitative explanation only shows a trend of the experiments. But the law has to be fully understood with confirmation of mathematical equations in repeated experiments where Newton applied it in *the Principia.* The complete interpretation of the law has to be understood.

The Third Law may be regarded as true under ideal or standard conditions or law cannot be regarded as confirmed under all circumstances with specific experiments in each possible case. Conceptually Newton's Third Law of Motion is applicable to numerous experiments. Newton did not give even qualitative equation to confirm the law, not to speak of quantitative one. Thus Newton's interpretation is geometrical or philosophical. The terms action and reaction have no units and dimensions, thus no equation can be based on these to draw quantitative conclusions. Now all equations are expressed in terms of physical quantities which have units and dimensions. Hence these are not perfect scientific terms for final understanding the concepts quantitatively. If some scientific deduction differs from this perception then law should be quantitatively confirmed in numerous experiments, where third law of motion is applicable. No scientific law is used under selective cases. So law is regarded as true under ideal or standard cases only, not in general cases. It is further justified below with help of logical experiments.

4.1. Newton did not consider quantitative explanation to the following experiments.

In first two examples illustrating the third law Newton considered a stone is pushed by finger and stone is pulled by horse, does not move i.e. remains at rest and work done is zero. Another example of this type is book lying on the table, the book pushes the table downward, and table pushes the book upward. Thus action and reaction are same in these cases. But the book when placed on the sheet of paper, tears the sheet of paper and falls down. The sheet of paper does not provide any reaction to book.

However in third example Newton explained the law in terms of collisions (*if a body impinges upon another….* i.e. projectile suffers same change velocity in backward direction as targets experiences change in velocity in forward direction). In this case work is done. Depending upon the action, the reaction is result of interactions between various bodies having different characteristics. Thus action and reaction may not be equal in all cases, as interactions of bodies of sponge and super elastic are different when interact with steel. In section (5.0), equations describe elastic collisions in one direction. These equations discussed below are taken from existing literature and critically analysed.

It is revealed from the discussion that there are different examples e.g. when bodies are pushed horizontally on the surface on earth (may be floor or wooden plank) , fall vertically towards the earth in air or vacuum and move horizontally when pushed in air or vacuum strikes other surfaces (target).

(A) When body moves horizontally on some surface.

The eq.(12), as in section(5.0) , describes special case when target body (body B) remains at rest and projectile (body A) rebounds with same velocity.

$$v_1 = \frac{(M_1 - M_2)u_1}{(M_1 + M_2)} \qquad (12)$$

(i) If the target is very-2 heavy , then we get

$$v_1(\text{reaction}) = \frac{-M_2 u_1}{M_2} = -u_1(\text{action}) \qquad (14)$$

Final speed of projectile = -Initial speed of projectile.

Action(u_1)= -Reaction(v_1) (15)

Thus we start from equations of collisions and discussion leads to third law of motion.

(ii) If the target is considered very -2 small then

v_1(reaction) = $\frac{-M_2 u_1}{M_2}$ = u_1 (action) (20)

Thus in this case action and reaction are equal but directions are not different. The several cases follow from the mathematical equations which are discussed separately in section (5.0). In both cases coefficient of restitution is unity i.e. e =1, hence collisions are completely elastic.

It is added that in the equations, the characteristics of bodies and nature of surface are never considered in mathematical equations, and perception is completely ideal or standard. Practically e.g. body B can be of cloth and body A of wood, body B can be sponge ball and body A of aluminum, body B may be an air filled football and body A of gold or various types of bodies can also be considered. The surfaces on which bodies collide are equally significant.

Thus intrinsic properties of projectile, target (inherent characteristics, nature, compositions, flexibility, rigidity, magnitude, size, elasticity, shape , distinctiveness of interacting bodies, mode of interactions, point of impact etc.) and surface on which interaction takes place, are very significant . Further different bodies (may be of steel, wood, rubber, cloth, wool, sponge, spring, typical plastic, porous material, air / fluid filled artifact, mud or kneaded flour or chewing gum specifically fabricated material etc.) play significant role. It is evident from eq. (12) and eq.(20) that above factors are extremely significant , thus can never be ignored. The same conclusion follows other experiments when one body (projectile) impinges (collides) other body (target).

In all interactions energy is converted from one form to other, but the total energy is conserved in the process. It is crystal clear that in many cases action and reaction may not be always equal, thus third law needs to be extended or modified, as depending the intrinsic properties , different types of bodies and different surfaces on which interactions take place . Further we find that same conclusion holds good when bodies fall vertically downwards towards the earth in air or vacuum or pushed horizontally towards the target.

(B) When body falls vertically in air or vacuum.

(i) **Elastic collision:** Consider a rubber ball of mass 0.1 kg. Let ball is dropped from height of 1 meter, and reaches the floor in time t. Then it will have weight/force 0.98N, the action. It will be equivalent to energy (mgh), 0.98J or work, when distance travelled is 1m. It must be noted that Newton had expressed action and reaction in terms of push or pull (force), motion (velocity), just after definition of the law in *the Principia*.

After striking the floor (action) the rubber ball rebounds (reaction) identically i.e. reach the same point (1m above) in the same time t. The body strikes the surface (u=0) with velocity v^2 = 2g (KE= 0.98 J). Thus reaction is 0.98N and body again possesses energy 0.98J. Obviously action and reaction (may also be expressed in terms of force or work are same, and Newton's law holds good.

Action(0.98N)=-Reaction(0.98N) (4)

This observation is completely consistent with Newton's Third Law of Motion. It can also be understood in terms of energy i.e. ball possesses same energy before and after collision. The dissipation of other forms of energy is zero or negligible as characteristics of system are ideal or standard. It could be understood in terms of time of fall, and time of rise.

In this case energy is not dissipated in any form, and the body is rebounded with original kinetic energy to reach at original height. Or it can be understood that the floor imparts same amount of kinetic energy as ball hits it without any dissipation or absorption of energy.

(ii) Super elastic collisions [12] : Consider a specially fabricated rubber (or suitable material) ball of mass 0.1 kg. Let this ball is also dropped from height of 1 meter, on the specially fabricated floor. This ball will have action (Weight =0.98N or energy 0.98J), same as in previous case. The ball touch the surface of earth with velocity,

v^2 = 2g.(1) (5)

After striking this specially fabricated floor, the rubber ball rebounds to greater height, say 2m in time t then reaction would be double ,

Reaction=2Action (6)

or Reaction >Action

Now the action and reaction are not equal in this case i.e. reaction is twice the action. Thus potential energy (elastic) of the system is converted to additional kinetic energy to body. It is due to special characteristics of the system. Thus in this case the reaction is twice the action, which is not consistent with third law. In this case characteristics of ball and the floor are such that potential energy (elastic) is imparted to ball such that it rises to double height in the same time. Due to combined interactions of specially fabricated ball and floor, the ball rises to double height, as some additional energy is generated due to characteristics of the system and imparted to ball as kinetic energy. Thus potential energy (elastic) of the system converted to additional kinetic energy to body.

Had this specially fabricated ball (or any other typical ball) risen to height of 1m only, then reaction would have been equal to action as in previous case. Hence there would have been no scope for debate on quantitative validity of the law.

(iii) If all other parameters remain the same (rubber ball 0.1kg, height 1 meter) as in first case but now ball falls on the sponge. Again in this case ball touches the surface of sponge with velocity $v^2 = 2g$ or KE = 0.98 J. Now we find that the rubber ball does not rebound from sponge; as in previous cases (from ordinary floor, specially fabricated ball and specially floor) it rebounded to 1m and 2m.

The action (0.98N or 0.98J) is same in all the three cases but reactions are different, as characteristics are also different. In this case KE of the ball is completely converted to potential energy of the sponge . Thus ball loses all kinetic energy and no kinetic energy is imparted by the sponge to ball. Hence, it does not rebound to at all , the reaction is zero . But in literature mainly first case is discussed as for suitability of conclusion. The inherent characteristics of floor and sponge are entirely different causing different interactions between projectiles and targets. Hence

$$\text{Reaction} = 0 \quad (7)$$

or Reaction < Action

The reaction is also zero in NASA's EM Drive experiment published in peer review journal [13,14]

(C) **When body moves horizontally in air and strikes the target.**

(iv) Consider a gun or injecting /firing device is specially fabricated by throwing various bodies with known force on the target (concrete wall, say). Let the gun is placed at 5 meters away from the wall and ejects or fires rubber ball. Conceptually such experiments are similar to previous experiments when ball is dropped on the surface of the earth or one dimensional elastic collisions described by eq.(12).

The rubber ball is thrown at the wall after firing the gun and reaches the target at 5m in time t. Let the force applied on the ball be F_{action}. The ball strikes the wall and rebounds to original position travelling distance 5m in time t. Thus, the ball rebounds or retraces its path identically. The $F_{reaction}$ and F_{action} are same.

$$F_{action} = - F_{reaction} \quad (8)$$

Thus Newton's Third Law is obeyed. The ball rebounds with same kinetic energy as its original and original kinetic energy is not dissipated at all during collision of rubber ball and target wall. Thus the wall imparts the same amount of kinetic energy to ball as its original energy.

(v) Let a rubber ball is specially fabricated using different material. The rubber ball is thrown on the wall after firing the gun and reaches the target at 5m in time t. Thus, action is F_{action}. After striking, the ball rebounds to original position in time t/2 s. Thus in this case reaction will be $F_{reaction}$ or 2 F_{action}. Thus

$$F_{reaction} = 2F_{action} \quad (6)$$

or $F_{action} \neq F_{reaction}$

In this case some additional energy is imparted to ball by the wall (due to the inherent nature, characteristics, composition etc.) and it rebounds with greater energy to travel the same distance in half time. There may be transformation of potential energy of the system to additional kinetic energy to ball.

(vi) Now consider that the concrete wall is replaced by cardboard wall. The rubber ball is thrown at the wall after firing the gun and reaches the target at 5m in time t. Like previous cases, in this case also third law is applicable. Let the force applied on the wall be F_{action}. Now as the rubber ball impinges (strikes) at the cardboard wall, it breaks . The rubber ball crosses across the broken cardboard wall. The cardboard does not offer any reaction (as wall does to ball in previous cases) or offers insufficient reaction or loses own identity during collision.

Thus the characteristics of projectile and target are very important in such cases; Newton's law does not take them in account. The law implies that to every action , there is equal and opposite reaction. But from where reaction comes? The target must be capable of imparting reaction after sustainable collision. It has to be mentioned. The sufficient reaction cannot be generated by every target.

(vii) Consider a chewing gum ball of the same mass (as in above two cases). The chewing gum is thrown on the wall after firing the gun and reaches the target at 5m in time t. After striking the wall the chewing gum sticks to wall and does not rebound.

This is again due to different inherent characteristics of projectile and target. Thus whole the kinetic energy is converted to potential energy (chemical) which is adhesive in nature. Or the chewing ball develops the strong binding of adhesion on the wall due to chemical energy. Thus

$$F_{reaction} = 0 \quad (9)$$

Hence in this case Newton's third law is not obeyed.

$$F_{action} \neq F_{reaction}$$

These experiments are different from Newton's perception that finger presses stone and stone presses the finger. Both remain at rest, hence work done is zero.

4.2 Various other Typical Examples Regarding Third Law Of Motion.

(i) Firstly, an example from the existing textbooks is considered for understanding. "Imagine a massive spring of high spring constant [12] ' is being compressed with an extremely delicate device (i.e. once touched will release the spring). Now we can envision that a collision between a slowly moving particle and this device (action) , would release the spring and the final kinetic energies of the massive spring and the particle will be larger than their initial one (reaction) , because the elastic potential energy was converted into kinetic energy."

In an analogous way, instead of 'massive spring of high spring constant' as 'source of chemical energy', say an explosive can be placed , in that case also huge amount of potential energy (chemical) would be released to kinetic energy when a small body touches this. Thus it is explained on the basis of inter-conversion of potential energy to kinetic energy and the other forms of energies may also be involved. The potential energy is of various forms e.g. chemical, gravitational, elastic, thermal, electrical, electrostatic, magnetic, nuclear etc. The various forms of energy are inter-convertible, and total energy remains constant. Similarly a spring and nail of same mass are dropped on the floor, and then spring rises upward abruptly whereas a nail of same mass and material does not rise upward. As in above case the potential energy of spring changes to kinetic energy, thus spring moves, but it so in case of nail. These observations are due to reason that inherent characteristics of the projectile and target which are never taken in account in the Principia's Third Law of Motion, are correctly taken in account in generalized form of third law of motion.

(ii) A golden body (square) of mass 0.1kg , rests on the table. The gold body pushes table as action. The table also pushes the body as reaction. Both action and reaction are equal in this case. Thus body rests on the table.

If the golden ball of mass 0.1kg (W=0.98N or Energy =0.98J) is dropped on the stretched paper, then it pierces through it. The reason is that the sheet of paper is unable to offer even fraction of reaction compared to action due to characteristics of target. Newton gave three examples to illustrate the third law of motion in *the Principia*, but such issues are not discussed. Newton's Principia does not address the issue specifically. Its discussion is important for completeness.

It justifies that action and reaction are not universally equal. How and why reaction is offered to action? It equally depends upon reacting body. Thus characteristics of acting and reacting bodies are equally significant. However the law implies universal equality of action and reaction, but there must some constraints on the law depending upon practical situation.

Thus due to this reason the sheet of paper cannot stop or hold golden body and later is torn and ball falls down. When the same body falls on the floor, gets sufficient reaction. Thus it rests on table. However the paper may offer sufficient reaction to an insect falling on it. In this action (weight of insect) is far less than that of ball. The similar is the reason a bullet pierces through the wall, tree or human body.

While swimming person pushes water backward with his arms, it may be regarded as action. In turn as reaction, water pushes a person forward. Thus the third law is understood to be obeyed. However, it is difficult to measure the backward momentum of water and forward momentum of swimmer. Similarly for swimming a person needs to have tank filled with water having minimum dimensions i.e. depth and breadth. If the depth of tank is less than optimum depth, then person is unable to swim. This optimum depth of tank of water is less for a child than for adult. It univocally implies that action and reaction depends upon characteristics of the system which are not taken in account definition of the third law of motion. Thus the Third Law of Motion is true under special or ideal or most suitable conditions only. Had Newton thought about these and similar other experiments he would have given the Third Law of Motion in different way i.e. in generalized form. The generalized form means the action and reaction may not be always equal and opposite depending upon characteristics of the system. When a bird flies in air, it presses the air with wings as action, and as reaction the bird moves forward. It can be further assessed if birds are made to fly in partial vacuum. Thus it is a qualitative example of application of third law of motion. Thus it is confirmed that characteristics of body and medium play significant role in this regard.

Further a person walks on the ground as he pushes the ground as action. Then as reaction the ground pushes him forward. Thus,

Pushing the ground backward with feet (Action) = Pushing the feet by ground forward (reaction) As both action and reaction are equal, hence person is able to walk on the ground. On the slippery path or on ice, the reaction offered by earth is insufficient, hence person is not able to walk. Thus it is clear that the characteristics of body and target play significant role in the process. The action and reaction are unequal here; it is clear from existing literature.

(iii) NASA's Reaction-less drive. It is drastic improvement of perception of Roger Shawyer English Satellite Engineer [8-9] , that reaction less space crafts are possible. This EM (Electromagnetic) Drive engine appeared to violate the third Law of Motion which vindicated previously published but un-reviewed results of similar devices. Harold White lead team concluded in the peer review paper; "Thrust data from forward, reverse, and null suggests that the system is consistently performing with a thrust to power ratio of 1.2 ± 0.1 millinewtons per kilowatt."

That's not an insignificant amount - to put it into perspective, the super-powerful Hall thruster generates force of 60 millinewtons per kilowatt, an order of magnitude more than the EM Drive. Hall Thruster is based on exhaust of material. Thus the results from NASA's reactionless drive are the significant, but must be repeated for final confirmation.

The Cannae Drive (plans to eventually launch a cubesat satellite) and other drive developed by Chinese scientist Yang work on the same idea. More experiments are being conducted by different scientists in different regions.

(iv) Let a rocket weighing equal to 3,000kg (just after 5s of take off) is moving with speed 2,000m/s i.e. momentum (forward) $6x10^6$ kg m/s. Let 10 kg of fuel is burnt in these 5 s. The third law implies that the backward velocity of exhaust (gases, flame, smoke etc.) of mass 10 kg (as assumed) must be equal to $6x10^5$ kg m/s i.e. $v = 6x10^6$ kg ms^{-1}/ 10kg = $6x10^5$ ms^{-1} But such measurements have never been reported, as it is impossible to measure mass of exhaust at any instant accurately. Like this speed of exhaust (gases, smokes or flames) of fireworks must be measured. As the parameters in such methods appear to be approximate, so some other precise methods be perceive and used purposely. In NASA's recent EM drive experiment published after peer review, there is no exhaust but engine moves. The toys aero planes fly with electronic batteries do not emit any exhaust required for validity of third law. Such experiments can be tried in vacuum. As exhaust is necessary for rocket, then it would have also been requirement for motion of passenger planes, helicopters, bus, truck, train, ship, moving electronic equipments etc. These machines or devices work under entirely different conditions, but the basic laws describing their motion must be the same. The repetitions can be done in vacuum to eliminate resistance due to air. The trials can be made by reducing the resistive forces to bare minimum of the system as well. NASA's EM drive experiments too have been conducted in vacuum purposely. The scientific conclusions cannot be drawn from unilateral analysis.

(v) In statistical mechanics there is a variety of situations in which Newton's third law (action = reaction) is violated [15] . Generally, the action-reaction symmetry can be broken (violation of third law of motion) for mesoscopic particles, when their effective interactions are mediated by a non-equilibrium environment. This violation and the resulting breaking of the force symmetry can occur when there is a relative motion of interacting particles versus an interface. The issue is discussed in details the paper Statistical Mechanics where Newton's Third Law is broken.[11] Thus such experimental discussions support generalized form of Newton's Third Law of Motion, which implies that action and reaction may or may not be equal. Thus as investigations continue, many new facts about the law are emerging and final confirmations can be drawn with repeated experiments. The action and reaction are shown for various bodies having different characteristics on the variety of surfaces are shown in table 2.

(vi) Further Newton's law is completely silent about magnetic and similar other interactions, these may be regarded as exception in some cases. Consider two iron bars are hit on each other, it is action. As reaction the bars are also move outward as reaction. So there is reaction corresponding to action.

Consider two magnets A and B (having same mass two bars as cited above) of significant pole strengths. Let both the magnets (Newton's third law of motion is applicable for all bodies) are placed on the smooth table at distance of ½ m (say).

(a) Let the north pole of magnet B is pushed with force F_{action} towards South Pole of magnet A (stationary). As the opposite poles attract each other, so both the magnets are stuck to each other. Thus magnet B does not re-trace its path, as according to Newton to every action there must be reaction (motion in opposite direction). But in this case reaction is zero i.e.

$$F_{reaction} = 0 \quad \text{and} \quad F_{action} \neq F_{reaction}$$

This aspect is not taken in account in third law of motion.

(b) Let the north pole of magnet B is pushed with force F_{action} towards North Pole of magnet A. The similar poles of magnets repel each other, and magnet may be repelled with more velocity (normally represented by Newton in *the Principia*) than it is pushed. It depends upon the strength of magnets. Even electromagnets can be used in such observations. When magnets come in the magnetic field of each other, they repel each other and move away. There is definite reaction without physical contact or for smaller reaction. The other such experiments can be explained. For comprehensive understanding all such experiments can be conducted in space under zero or reduced gravity conditions. All these experiments relating to third law of motion can also conducted on the earth by reducing the resistive forces. The comparison of actions and reactions are shown for various bodies in the table 3.

4.3 The reasons for inconsistent results

Newton did not give any equation to measure or calculate the magnitudes of action and reaction. The action and reaction do not possess units and dimensions. The mathematical equations of the law must be expressed in terms of physical quantities which have units and dimensions. Only then law is checked quantitatively in sensitive experiments. Newton's explanation is only qualitative. Thus Newton provided conceptual, thematic and philosophical explanation of the law, not quantitative which is understandable for scientific developments in 17th century. With dawn or progress of industrial revolution between 1760 to 1840, the quantitative measurements were absolute requirements of the involved quantities.

Newton's law is unconditionally applicable to all interacting bodies, action and reaction are always equal and opposite for all interacting bodies universally. In the table 2, table 3 and table 4, the first example in each case is consistent with Principia's Third Law of Motion. But it is not prudent and scientific that only highlighting few examples, the law is regarded as universally true, like its definition. One may argue that ball bounces to different heights due to reason they have different inherent properties. It is true but the Principia's third law of motion does not take in account the inherent characteristics, nature, composition, flexibility etc. in account for projectile and target.

The Principia's law does not take in account the inherent characteristics, nature, composition, flexibility, rigidity, magnitude, size, distinctiveness of interacting bodies etc. There is no term, which accounts for the above significant factors. The bodies may be composed of steel, wood, rubber, cloth, wool, sponge, spring, elastic , plastic, typical plastic, porous material, mud or kneaded flour or chewing gum, especially fabricated etc. The bodies may be solids, liquids, gases, or mixture of all. These factors play important role in action and reaction phenomena. In the Principia's third law Newton blatantly neglected these factors as they do not exist. Depending upon the action, the reaction is result of interactions between various bodies having different characteristics. Thus action and reaction may not be equal in all cases, as interactions of bodies of sponge and super elastic are different when interact with steel. These are very significant factors affecting the results and are taken in account via a coefficient of proportionality (K) in the generalized law i.e. eq.(31). The value of K is determined experimentally, like numerous other coefficients in physics. Due to these factors different bodies experience different reaction for same action. So Newton did not mention these factors at all and summed up the basic law just giving three qualitative examples. Just possible that Edmund Halley (sponsor) wanted the manuscript as soon as possible. Also other contemporary scientists like Robert Hook and Leibniz were raising priority issues, which were diplomatically dealt by sponsor Halley. But there should be no diplomacy in science. Thus Newton might have completed the manuscript quickly, and other scientists learnt about the contents when it was published. In the preface of *the Principia,* Newton did not mention name of any scientist who had read the manuscript. In Newton's time phenomena were expressed geometrically and philosophically, so Newton did. Thus in *the Principia,* no linear equation or analytical equation were used to interpret the phenomena quantitatively. It should be noted that in the earliest days of dawn of human civilization, human might have expressed his perceptions geometrically by drawing lines and improving the understanding. But now a day it is age of accurate robotics and computer programming. Further action and reaction are used in English language as verb, noun, and adjective. In science units and dimensions are absolutely important to interpret the phenomena. These are not physical quantities hence cannot be used to represent the equation to draw conclusions quantitatively. In Newton's time units and dimensions were not discovered, it is hard reality. Due to this reason above elusive factors may have not been taken in account in third law of motion. Hence the law is generalized.

V. The various types of collisions and the Third Law of Motion.

Newton expressed action and reaction in third law of motion in terms of both 'force 'and 'velocity' at page 20 of *the Principia*. Newton's explanation implies that law is applicable for both situations when body is at rest (in this case work, W=FS is zero) and moves. Thus scientifically it is prudent that law must be studied critically and quantitatively for the fullest understanding of the law. In Newton's time laws were explained philosophically and geometrically not mathematically (algebraically and analytically) . In third example Newton quoted in *the Principia* . "*If a body impinges upon another and by its force change the motion of the other, that body also will undergo an equal change, in its own motion, towards the contrary part.*"

This Newton himself applied his third law of motion in the impinging or colliding bodies. But Newton did not give any mathematical equation, as in those days the laws were expressed by geometric methods not by mathematical equations. In *the Principia* Newton did not write even single equation. Newton stated second law of motion or law of gravitation without mathematical equation. Newton described collisions in third example of third law of motion, but the equations were written by other scientists when laws of conservation of momentum and kinetic energy were stated.

Newton's law is equally applicable even if target is at rest or moving, there is no constraint. This application of the third law of motion (impinging or colliding of one body on the other) finds many applications in various branches of physics. The target may be at rest or moving without any constraint. The projectile can cause change in motion. Newton did not characterize projectile and target in third example, whereas in first two examples Newton mentioned finger presses stone; and horse pulls stone. In the third example Newton did not mention any body specifically. However law is applicable to all bodies universally.

In highly technological physics in nuclear reactor heavy water is used as moderator (to slow down fast moving neutrons to thermal neutrons), is also based upon such mathematical calculations. However in theoretical discussion and experimental arena situation is entirely different.

Consider one-dimensional elastic collisions [16-17] when one body impinges or collides other. Furthermore, in collisions comparative size and point of impact, of target and projectile, roughness of surfaces and resistive forces play significant roles. But such factors are not mentioned except velocities and masses (M_1, M_2 in general sense) taken in equations. Also Newton did not mention about these when stated that one body impinges (collides) on the other (there are numerous bodies with entirely different characteristics). There is no factor in the equations which accounts for characteristics of bodies. Practically situation is entirely different.

The definition of the law is universally applicable irrespective of characteristics of the projectile and target. Collisions in which both momentum and kinetic energy of the system are conserved are called elastic collisions. These eqs.(10-30) are based upon conditions of applicability of elastic collisions (conservation of momentum , conservation of kinetic energy) and coefficient of restitution or coefficient of resilience is unity (e = 1). Then in view of Third Law of Motion action (initial velocity of projectile) and reaction (final velocity of projectile) can be compared to draw conclusions.

Consider projectile and target of masses M_1 and M_2 moving along the same straight line with speeds u_1 and u_2 respectively. The bodies will collide only if $u_1 > u_2$. The final speeds of projectile and target are v_1 and v_2,

$$v_1 = \frac{(M_1 - M_2)u_1 + 2M_2u_2}{(M_1 + M_2)} \tag{10}$$

$$v_2 = \frac{(M_2 - M_1)u_2 + 2M_1u_1}{M_1 + M_2} \tag{11}$$

If the target is at rest (u_2 = 0), then Eqs. (10-11) become

$$v_1 = \frac{(M_1 - M_2)u_1}{(M_1 + M_2)} \quad (12)$$

$$v_2 = \frac{2M_1u_1}{M_1 + M_2} \quad (13)$$

The initial velocity u_1 of the projectile is regarded as action, and its final velocity v_1 as reaction. The various sub-cases are discussed below.

(i) **When $M_2 >> M_1$ i.e. target is very massive compared to the projectile.** The target M_2 remains at rest v_2=0. For example, when projectile (ball) collide with huge target (wall). Thus

$M_1 - M_2 = -M_2$, $M_2 + M_1 = M_2$

In this case the final speeds of the projectile and target can be calculated from eqs.(12-13).

$$v_1 \text{ (reaction)} = \frac{-M_2u_1}{M_2} = -u_1 \text{ (action)} \quad (14)$$

Final speed of projectile = -Initial speed of projectile.

Action (u_1) = -Reaction (v_1) (15)

The velocity of target , v_2=0

Thus projectile (ball) rebounds with original velocity and target (wall) remains at rest, the third law is obeyed. The negative sign in v_1 indicates that direction of projectile reverses after collision.

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{u_1}{u_1} = 1 \quad (16)$$

As the coefficient of restitution is unity, the collision is elastic.

Ideal calculations only: However these are ideal mathematical calculations as we have not considered actual experimental characteristics at all i.e. magnitude, shapes, sizes of projectile and target, point of impact, resistive force, compositions etc. The rest of calculations are based on similar perceptions. The composition of various bodies may be different [18] e.g. body B can be of cloth and body A of wood, body B can be water filled ball and body A of aluminum, body B may be an air filled football and body A of gold or various types of bodies can also be considered. Further the characteristics of the surface on which bodies interact are significant. The definition of the third law of motion is applicable for all bodies universally. The various experiments must be conducted before drawing final conclusions. Experimentally situation is different than ideal mathematical considerations.

(ii) Mass of target is 1000 times larger than that of projectile. If we consider that the target is 1000 times more massive than the projectile i.e. M_2 = 1000M_1, M_1+M_2 = 1001M_2, M_1-M_2 = 999M_2 then eq.(9) becomes

v_1 = -0.998001998u_1 (17)

u_1 = -1.002002v_1

Initial velocity of projectile or action = 1.002002 final velocity of target or reaction

Action(u_1)=-1.002002Reaction(v_1) (18)

Thus Newton's third law of motion is not justified in this case. The collision is elastic as

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \left[\frac{2M_1u_1}{1001\ M_1} - \frac{-999\ M_1u_1}{1001\ M_1}\right]/[u_1 - 0]$$

$$= 1 \quad (19)$$

(iii) **Let us consider the case when the target is very-2 small compared to projectile.** (M_2<<M_1 or even mass of target may be negligible compared to projectile). $M_2+M_1 = M_2$. The various velocities can be calculated. Now from eqs.(12-13)

$$v_1 = \frac{M_1u_1}{M} = u_1 \quad (20)$$

Reaction = Action

Thus the projectile does not rebound at all and keep on moving with original velocity , which is contrary to original form of third law of motion i.e. *'to every action there is* ***opposite*** *and equal reaction.'* The projectile keeps on moving with same velocity.

Experimentally inherent characteristics, nature, magnitude, shape, size, point of impact of projectile and target play significant roles in experiments. The body is moving with velocity u_1 as in earlier case. These are not taken in account by original form of law; hence Principia's law is generalized. Thus for every collision reaction is not possible.

$$v_2 = \frac{2M_1u_1}{M_1} = 2u_1 \quad (21)$$

The final velocity of target = 2 initial velocity of projectile. (22)

This situation can be theoretically understood by visualizing a ball hanging in air in train track . The moving train (projectile, M_1) hits the ball (target, M_2), the ball moves with double velocity of train and train keep on moving with original velocity. But it is not experimentally confirmed under which conditions when (M_2<<M_1 or even mass of target may be negligible compared to projectile), the target moves with double velocity of projectile.

Does everybody (M_2<<M_1) move with double velocity ? Anyhow this prediction needs to be confirmed under controlled and measurable conditions in the laboratory, only then final conclusions can be drawn. Obviously it will highlight the importance of characteristics of bodies regarding experimental confirmation the law. As already mentioned Newton's law does not take the characteristics of the bodies in account, so such experiments are very significant. Further,

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{2u_1 - u_1}{u_1} = 1 \qquad (22)$$

(iv) **Let us consider the target is slightly heavier than projectile i.e. $M_2 = 1.01M_1$**
Thus, $M_1 - M_2 = M_1 - 1.01M_1 = -0.01M_1$, $M_1 + M_2 = M_1 + 1.01M_1 = 1.01M_1$
In this case the final velocities of projectile and target are can be calculated from eqs.(12-13)

$$v_1 = \frac{(M_1 - 1.01M_1)u_1}{(M_1 + 1.01M_1)} = \frac{-0.01M_1u_1}{2.01M_1} = -0.004975u_1 \qquad (23)$$

Final velocity of projectile = - 0.004975 Initial velocity of projectile.

Action= -201.005 Reaction (24)

The negative sign in v_1 indicates that direction of projectile after collision reverses. But magnitudes of action and reaction are different, thus third law is not obeyed.
This case is precisely used in the nuclear reactor to reduces the speed of fast moving neutrons (2MeV, 1,95x10^4 km/s) to thermal energy (0.025 MeV, 2.2 km/s) but under ideal conditions . The fast moving neutron (m_n = 1.00866 u) are made to hit the proton (1.007276u) in heavy water. Thus velocity of neutron can be calculated when it strikes with proton, say with individual proton (assumed) like two marbles, which is only ideal conception, can be calculated. It turns out to be nearly 13 km/s i.e. about 5.9 times the initial speed. But practically other factors are also there which are neglected, so the speed of neutron may be practically reduced to thermal neutrons which cause fission. Purposely other moderators are also used.

$$v_2 = \frac{2M_1u_1}{(M_1 + 1.01M_1)} = \frac{2M_1u_1}{2.01M_1} = 0.995024u_1 \qquad (25)$$

The coefficient of restitution in this case is also unity i.e.

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \left[\frac{2M_1u_1}{2.01M_1} + \frac{-0.01M_1u_1}{2.01M_1}\right]/[u_1 - 0] = 1 \qquad (26)$$

(v) Let us consider the case when masses of projectile and target are equal i.e. $M_1 = M_2 = M$.
Now consider the case when target is at rest (u_2 =0), then we get,

$$v_1 = \frac{0}{M + M} = 0 \qquad (27)$$

The projectile comes to rest after collision i.e. reaction =0

$$v_2 = \frac{2M\ u_1}{2M} = u_1 \qquad (28)$$

Final velocity of target = Initial velocity of projectile
Experimentally it can be easily observed if one marble properly hits other in typical collision ; then one marble (projectile) comes to rest and target starts moving with velocity of projectile. The children practically observe this situation while playing with marbles in streets in few cases. In this case direction of the projectile not reversed which is condition for validity of Third Law of Motion.

(vi) **Let us consider the case when masses of projectile and target are equal i.e. $M_1 = M_2 = M$.**
The various velocities can be calculated (u_2>0). Now from eqs.(10-11) we get

$$v_1 = \frac{(M_1 - M_2)u_1 + 2M_2u_2}{(M_1 + M_2)} = \frac{0 + 2Mu_2}{M + M} \qquad (29)$$

v_1 (reaction) = u_2

$$v_2 = \frac{(M_2 - M_1)u_2 + 2M_1u_1}{M_1 + M_2} = \frac{0 + 2Mu_1}{M + M} \qquad (30)$$

v_2 = u_1 (initial velocity of projectile)
Thus projectile and target exchange their velocities, and continue to move in the same direction. Let projectile is moving with velocity 10m/s and target 5m/s , only then they will collide ($u_1 > u_2$) . After collision projectile *does not rebound,* thus suffers no reaction and there is no motion in reverse direction, and moves with reduced velocity 5m/s. The third law of motion is not obeyed as there is no reaction (the projectile does not retrace the path). It is again added that equations are written for ideal elastic collisions, the Reaction (final velocity of Projectile) is different from Action (initial velocity of projectile) in these cases, and thus the Third Law of Motion is inconsistent in such cases. The action and reaction for projectiles and targets of different masses in one dimension are shown in Table 4 for comparison.

VI. Generalized form of the Third Law of Motion

Newton stated third law of motion in terms of action and reaction, which do not possess units and dimensions. These are not physical quantities. Newton did not give any equation to quantitatively justify the law experimentally, which is the first and foremost condition for any law now days. The law is universally applicable for all bodies under all conditions. Depending upon the action, the reaction is result of interactions between various bodies having different characteristics. Thus action and reaction may not be equal in all cases, as interactions of bodies of sponge and super elastic are different when interact with steel.

In Newton's time it was beginning of physics, due to lack of systematic scientific quantities, third law was expressed philosophically. The major reason for this is that in Newton's time units and dimensions were not discovered, and also mathematical equations (algebraic or analytical) were not used to explain the phenomena, so Newton used philosophical and geometrical methods to interpret the phenomena. But now specifically scientific terms (possessing units and dimensions) can be used to understand the law, which lead to noble ideas i.e. generalization of third law of motion.

In first two examples Newton explained the third law of motion in terms of force. Newton explained that when a stone is pressed with finger or a stone is pulled by horse. In both the cases the stone remain at rest or work done is zero.

In the third example, Newton expressed action and reaction with velocity, validating or extending application of law collisions. *If a body impinges upon another and by its force change the motion of the other, that body also will undergo an equal change, in its own motion, towards the contrary part.*

Newton's this statement extends applications of the law in realm of collisions. As in this case the projectile and target move thus work is done. Here the role of characteristics of bodies (projectile and target) become self evident, as energy is inter converted from one form to other and law of inter-conversion of energy. If the Newton's third law of motion is explained to the fullest extent, the role of characteristics of system becomes significantly clear.

It is a basic principle of science that no conclusions can be drawn on the basis of a single or few qualitative observations. If the results are repeated under all conditions for different parameters, only then the law is accepted experimentally. The laws should be formulated or generalized in view of numerous experimental observations. On the basis of the above observations and illustrations given in the discussion *The Principia's* Third Law of Motion is generalized:

'*To every action there may be reaction, but may or may not be always equal and opposite; depending upon the inherent characteristics of the interacting bodies.*'

or

"*The mutual actions of two bodies may not always be equal and opposite, depending upon the inherent characteristics of the interacting system.".*

Newton did not any mathematical equation for the third law of motion in *the Principia*, in fact he did not give any equation for the second law of motion or the law of gravitation .The generalized form of the third law can be expressed as

Action $\propto$ Reaction

or Reaction = - K Action (31)

K is the coefficient of proportionality. There are numerous coefficient of proportionalities in the existing physics and experimentally determined and depends upon the inherent characteristics of interacting bodies or mode of interactions. The value of K may be equal to, less than, or greater than unity depending upon experimental parameters.

The coefficient of proportionality takes into account the inherent characteristics, nature, composition, flexibility, elasticity, rigidity, magnitude, plasticity, distinctiveness of interacting bodies or mode of interactions, and other relevant factors. The bodies may be of steel, wood, rubber, cloth, wool, clay, kneaded flour, chewing gum, sponge, spring, etc. The bodies can be of solid, liquid, gas or mixture of all. The law is conceptually applicable for all bodies thus can be considered for waves as well.

There is no factor in Newton's Third Law of Motion which could affect equality of Action and Reaction. In calculation of time period of pendulum, in verification of Ohm's law, in determination of force in Coulomb's law, in many other laws etc. the external or internal conditions play a significant role. Nuclear fission is only possible for uranium not for steel. So there are constraints on every experimental and theoretical phenomenon. These have to be taken in account in all cases. But it is not so in case of Newton's Third Law of Motion, which implies irrespective of any external or internal effect, other characteristics of bodies etc., the action and reaction must be precisely equal in all cases universally.

Some clear examples are theoretically illustrated in the discussion reading the generalized form of third law of motion. However these can be experimentally authenticated in wide range of experiments. In the generalized form of Newton's third law of motion, coefficient of proportionality takes all elusive factors in account.

The critical analysis or refinement in existing theories is continuous process in science when new experimental evidences emerge.. In the current discussion, there is generalization of the third law within realm of classical mechanics. Thus Physics is not static body of doctrine but a developing science. The speculative EM drive is being put into practical prospective in various laboratories. However the logical theoretical deductions and experimental evidences are the main criteria which decide the acceptance or rejection of a theory.

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Table 1. The comparison of original form of Newton's law (accounts for action and reaction only) and generalized law (takes all factors in account, including action and reaction) .

Sr. No	Newton's Third Law	Generalized form of Newton's Third Law of Motion
1	Action = - Reaction	Action = -K Reaction K is coefficient of proportionality
2	No factors other than Action and Reaction are involved.	"the inherent characteristics, nature, compositions, flexibility, rigidity, magnitude, size, elasticity, shape, distinctiveness of interacting bodies, mode of interactions, point of impact etc." These factors are effective and change results.
3	Neglects all other factors except action and reaction.	Takes all possible factors in account.
4	Newton gave three examples, but these are qualitative.	All examples can be explained
5	Newton expressed Action and Reaction in terms of force and velocity, which has to be mathematically studied.	In addition to force and velocity action and reaction can also be calibrated in terms of time and distance.

Table 2. Comparison of action and reaction when body of mass (0.1kg or 0.98N, Action) of different characteristics striking at different surfaces when dropped from the same height. The action is same for all bodies (0.1kg is 0.98N) but reactions may be different.

Sr. No.	Projectile Ball (0.1kg)	Striking surface	Action (body falls)	Reaction (body rebounds)	Deduction
1	Rubber	Floor	1m	1m	Action =-Reaction
2	Rubber	soil/sand	1m	0.01 m	Reaction = -0.01 Action
3	Specially fabricated rubber ball	Specially fabricated surface	1m	2m	Reaction = -2Action
4	Sponge	Soft surface	1m	0	Reaction =0
6	NASA's EM drive	Experimental set up	Microwaves Action >0	0	Reaction less device
5	Golden ball	Paper	1m	0	Reaction? (paper torn)

Table 3. Comparison of action and reaction, for various projectiles colliding various targets.

Sr. No.	Projectile	Target	Action	Reaction	Third Law of Motion
1	Rubber ball	Concrete wall	F_{action}	F_{action}	Action = Reaction
2	Rubber ball (special)	Concrete wall (specially fabricated)	F_{action}	2 F_{action}	$F_{action} \neq F_{reaction}$
3	Rubber wall	Cardboard wall	F_{action}	0 (wall breaks)	$F_{action} \neq F_{reaction}$
4	Chewing gum ball	Concrete wall	F_{action}	0 (ball sticks)	$F_{action} \neq F_{reaction}$
5	Magnet B (pushed)	Magnet A, opposite pole (stationary)	Push (F_{action})	Attraction ($F_{reaction}$)	$F_{reaction} > F_{action}$

6	Magnet A, north pole (stationary)	Magnet B, north pole (stationary)	Push (F_{action}) zero	Repulsion ($F_{reaction}$) non zero	F_{action} =0 $F_{reaction}$ is Non-zero

.

Table 4 . The theoretical comparison of action and reaction (initial and final velocities of projectile) under ideal conditions, in elastic collisions (coefficient of restitution, e=1). The Principia's Third Law is obeyed in first case only.

Sr. No	Projectile (mass)	Target (mass)	Initial velocity (action)	Final velocity (reaction)	Action = -Reaction	Third Law Obeyed
1	M_1	$M_2>>M_1$	u_1	$-u_1$	Action =-Reaction	Yes
2	M_1	$M_2=1000M_1$	u_1	$-0.998002\ u_1$	Action =-1.002 Reaction	No
3	M_1	$M_2<<M_1$ ($u_2=0$)	u_1	$+u_1$	Reaction=0, No rebound	No
4	M_1	$M_1 =M_2$	u_1	$v_1 =0$	Reaction =0, projectile at rest	No
5	M_1	$M_2=1.01M_1$	u_1	$-0.004975u_1$	Action = 201 Reaction	No
6	M_1	$M_1=M_2$	u_1	u_2	Reaction=0, No rebound	No