

On Homogeneous Ternary Quadratic Diophantine Equation

$$z^2 = 45x^2 + y^2$$

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Abstract

The ternary quadratic homogeneous equation representing homogenous cone given by $z^2 = 45x^2 + y^2$ is analyzed for its non-zero distinct integer points on it. Four different patterns of integer points satisfying the cone under consideration are obtained. A few interesting relations between the solutions and special number patterns namely Polygonal number, Pyramidal number, Octahedral number, Stella Octangular number and Oblong number are presented. Also knowing an integer solution satisfying the given cone, two triples of integers generated from the given solution are exhibited.

Keywords: Ternary homogeneous quadratic, integral solutions.

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I. INTRODUCTION

The Ternary quadratic Diophantine equations offer an unlimited field for research because of their variety [1-5]. For an extensive view of various problem one may refer [6-20]. This communication concerns with yet another interesting for ternary quadratic equation $z^2 = 45x^2 + y^2$ for determining its infinitely many non-zero integral solutions. Also a few interesting relations among the solutions have been presented.

Notations used

- $T_{m,n}$ - Polygonal number of rank n with size m.
- CP_n^m - Centered Pyramidal number of rank n with size m.
- Pr_n - Pronic number of rank n.
- SO_n - Stella Octangular number of rank n.
- Obl_n - Oblong number of rank n.
- OH_n - Octahedral number of rank n.
- Pt_n - Pentatope number of rank n.
- PP_n - Pentagonal Pyramidal number of rank n

Method of analysis

The ternary quadratic equation under consideration is

$$z^2 = 45x^2 + y^2$$

(1)

we have different pattern of solutions of (1) which are illustrated below.

Pattern-I

Consider (1) as

$$z^2 = 45x^2 + y^2 \quad (2)$$

Assume

$$z = a^2 + 45b^2 \quad (3)$$

Write 1 as

$$1 = \frac{\{(22 + 2n - 2n^2) + i\sqrt{45}(2n - 1)\} \{(22 + 2n - 2n^2) - i\sqrt{45}(2n - 1)\}}{(23 - 2n + 2n^2)^2}$$

Substituting (3) and (4) in (2) and employing the method of factorization, define

$$y + i\sqrt{45}x = \frac{\{(22 + 2n - 2n^2) + i\sqrt{45}(2n - 1)\} (a + i\sqrt{45}b)^2}{(23 - 2n + 2n^2)}$$

Equating the real and imaginary parts in the above equation, we get

$$x = \frac{\{(22 + 2n - 2n^2)2ab + (2n - 1)(a^2 - 45b^2)\}}{(23 - 2n + 2n^2)}$$

$$y = \frac{[(22 + 2n - 2n^2)(a^2 - 45b^2) + (2n - 1)90ab]}{(23 - 2n + 2n^2)}$$

$$z = (23 - 2n + 2n^2)^2 [A^2 + 45B^2]$$

For simplicity and clear understanding, taking n=1 in the above equations, the corresponding integer solutions of (1) are given by

Replacing a by (23-2n+2n²)A, b by (23-2n+2n²)B in the above equation the corresponding integer solutions of (1) are given by

$$x = 23A^2 - 1035B^2 + 1012AB$$

$$y = 506A^2 - 22720B^2 - 2070AB$$

$$z = 529A^2 + 23805B^2$$

$$x = (23 - 2n + 2n^2) [(22 + 2n - 2n^2)2AB + (2n - 1)(A^2 - 45B^2)]$$

$$y = (23 - 2n + 2n^2) [(22 + 2n - 2n^2)(A^2 - 45B^2) - 90AB(2n - 1)]$$

Properties

- 1) $23x(A, 7A^2 - 4) + z(A, 7A^2 - 4) - 69828 CP_A^{14} - 1058 Pr_A \equiv 0 \pmod{1058}$
- 2) $x(B + 1, B) + y(B + 1, B) - z(B + 1, B) + 1058 Pr_B + T_{95222, B} \equiv 0 \pmod{47609}$
- 3) $23x(A, 2A^2 - 1) + z(A, 2A^2 - 1) - 1058 obl_A - 23276 SO_A \equiv 0 \pmod{1058}$
- 4) $22x(A, A(A + 1)) - y(A, A(A + 1)) - 48668 PP_A \equiv 0$
- 5) $x(A, 1) - 1035 obl_A + T_{2062, A} \equiv -1035 \pmod{1034}$
- 6) $z(1, B) - 203805 Pr_B \equiv 0 \pmod{23805}$

Pattern-II

It is worth to note that 1 in (2) may also be represented as

$$1 = \frac{\{[(45 - 4n^2) + i\sqrt{45}(4n)] \prod [(45 - 4n^2) - i\sqrt{45}(4n)]\}}{(45 + 4n^2)^2}$$

Following the analysis presented above, the corresponding integer solution to (1) are found to be

$$x = (45 + 4n^2) [(45 - 4n^2)2AB + 4n(A^2 - 45B^2)]$$

$$y = (45 + 4n^2) [(45 - 4n^2)(A^2 - 45B^2) - 360ABn]$$

$$z = (46 + 4n^2)^2 [A^2 + 45B^2]$$

For the sake of simplicity, taking n=1 in the above equations, the corresponding integer solutions of (1) are given by

$$x = 196A^2 - 8820B^2 + 4018AB$$

$$y = 2009A^2 - 90405B^2 - 17640AB$$

$$z = 2401A^2 + 108045B^2$$

Properties

- 1) $41x(A, A(A + 1)) - 4y(A, A(A + 1)) - 141120 PP_A \equiv 0$
- 2) $2009x(A, 2A^2 - 1) - 196y(A, 2A^2 - 1) - 11529602 SO_A = 0$

- 3) $2009x(A, 2A^2 + 1) - 196y(A, 2A^2 + 1) - 34588806 OH_A = 0$
- 4) $x(A, 1) - 196Pr_A \equiv -8820 \pmod{3822}$
- 5) $41x(A, (A+1)(A+2)(A+3)) - 4y(A, (A+1)(A+2)(A+3)) - 1693440 Pt_A \equiv 0$

Pattern-III

Equation (1) can be written as

$$\frac{z + y}{45x} = \frac{x}{z - y} = \frac{P}{Q} \tag{5}$$

From equation (5), we get two equations

$$45Px - Qy - Qz = 0$$

$$Qx + Py - Pz = 0$$

We get the integer solutions are

$$x = x(P, Q) = 2PQ$$

$$y = y(P, Q) = 45P^2 - Q^2$$

$$z = z(P, Q) = 45P^2 + Q^2$$

Properties

- 1) $y(a, a) + z(a, a) - 90Pr_a \equiv 0 \pmod{90}$
- 2) $x(a, -a^2) + 2CP_a^6 = 0$
- 3) $x(P, 2P^2 - 1) + y(P, 2P^2 - 1) + z(P, 2P^2 - 1) - 290Pr_P + T_{402,P} + 2SO_P \equiv 0 \pmod{489}$
- 4) $x(P, 1) + y(P, 1) - 45obl_P \equiv -1 \pmod{47}$

II. GENERATION OF INTEGER SOLUTIONS

Let (x_0, y_0, z_0) be any given integer solution of (1). Then, each of the following triples of integers satisfies (1):

Triple 1: (x_1, y_1, z_1)

$$x_1 = 3^n x_0$$

$$y_1 = \frac{1}{6} \left((8(3)^n - 2(-3)^n) y_0 + (-4(3)^n + 4(-3)^n) z_0 \right)$$

$$z_1 = \frac{1}{6} \left((4(3)^n - 4(-3)^n) y_0 + (-2(3)^n + 8(-3)^n) z_0 \right)$$

III. CONCLUSION

In this paper, we have presented four different patterns of infinitely many non-zero distinct integer solutions of the homogeneous cone given by $z^2 = 45x^2 + y^2$. To conclude, one may search for other patterns of solutions and their corresponding properties.

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