RESEARCH ARTICLE OPEN ACCESS

Bianchi type III Bulk viscous string cosmological model with shear free and time dependent cosmological constant in modified general relativity

R. K. Dubey[1], Shishir Kr. Srivastava[2] and Vipendra Kr. Dwivedi[3]
[1]*Govt Model Science College, Rewa (M.P.) India*
[2]*Gramyanchal P.G. College, Haidergarh, Barabanki (UP)*
[3]*APS University, Rewa (M.P.) India*
Corresponding Author: R. K. Dubey

ABSTRACT
Here we investigated Bianchi type III string cosmological models with bulk viscosity. To get a deterministic, it is assumed that $\theta \propto \frac{1}{\eta}$ and $\Lambda \propto H$, Where η is the coefficient of bulk viscosity, θ is the expansion scalar, H is Hubble parameter, Λ is cosmological constant. The Physical and geometrical behaviours of the models are also discussed.
Kewords: Bianchi type III space time, string cosmological model, expansion scalar, bulk viscosity, cosmology.

Date of Submission: 12-10-2017 Date of acceptance: 11-11-2017

I. INTRODUCTION:

Cosmology is the scientific study of large scale properties of the universe as a whole cosmology is study of the motion of crystalline objects The origin of the universe is greatest cosmological mystery even today. The recent observations that $\Lambda \sim 10^{-55}\,\mathrm{Cm}^{-2}$ while the particle physics predication for Λ is the greater than this value by a factor of order 10^{120}. The present universe is both spatially homogeneous and isotropic. The basic problem in cosmology is to find the cosmological models of universe and to compare the resulting models with the present day universe using astronomical data. However bulk viscosity is expected to play an important role at certain stages of an expanding universe.

In the last few years the study of cosmic strings has attracted considerable interest as they are believed to play an, important role during early stages of the universe. The idea was that particles like the photon and the neutron could be regarded as waves on a string. The presence of strings of the early universe is a by product of Grant Unified theories (GUT) Roy and Banerjee [1] have investigated some LRS Bianchi type II string cosmological models which represent geometrical and massive strings. Some cosmological solution of massive strings for Bianchi type I space time in presence and absence of magnetic field have investigated by Banerjee et. al. [2] Bali et al. [3-6] have a investigated Bianchi types I, V, IV string cosmological models in General relativity Wang [7-10] has investigated and discussed some cosmological models and their physical implication in some Bianchi type space times. At the early stayes of the evolution of the universe, when radiation is in the form of photons as well as neutrino decoupled, the matter by haved like a viscous fluid. Bulk viscosity could arise in many circumstances and could lead to an effective mechanism of galaxy formation. Murphy[11] constructed isotropic homogeneous spatially flat cosmological model with a fluid containing bulk viscosity alone because the shear viscosity can not exist due to assumption of isotropy. He observed that the 'Big-Bang singularity of finite past may be avoided by introduction of bulk viscosity. Bali and Dave [12] investigated the Bianchi type-III string cosmological model with bulk viscosity Recently Bali paradhan [13] investigated the Bianchi type –III string cosmological model with time dependent bulk viscosity. Recent interest in the cosmological constant term Λ has received considerable attention among researchers for various concepts other researchers like Zeldovich [14], Bertolumi [15,16], Ozer and Taha[17], Weinberg[18], Carroll et al. [19] Calberg et al. [20], Friemann and Waga [21] Pradhan et al. [22] Investigated more significant cosmological models with Cosmological constant Λ in this paper Bianchi type III Bulk viscous string cosmological model shear free and time dependent cosmological constant in modified general relativity. To obtain a determine cosmological model we assume that the coefficient of the viscosity in inversely proportion to the expansion scalar and for the cosmological constant $\Lambda(t)$, we assume that $\Lambda \propto H$.

The Metric and field equation:

We consider the Bianchi type space time in the metric form.

$$ds^2 = -dt^2 + \alpha_1^2\, dx_1^2 + \alpha_2^2\,(e^{2x} dx_2^2 + dx_3^2) \qquad (1)$$

Where α_1 and α_2 are functions of time 't' only

The energy momentum tensor for a cloud string with bulk viscous fluid of string is given by.

$$T_{\alpha\beta} = \rho v_\alpha v_\beta - \mu x_\alpha x_\beta - \eta\theta\left(v_\alpha v_\beta + g_{\alpha\beta}\right) \qquad (2)$$

Where v_α and x_α satisfy the relations.

$$v^\alpha v_\alpha = -x^\alpha x_\alpha = -1,\ v^\alpha v_\alpha = 0 \qquad (3)$$

In equ (2) ρ is the proper density for a cloud string with particle, attached them, η is the coefficient of bulk viscosity, μ is the string tension density of particles, v^α is the cloud four velocity vector and x^α is a unit space, like vector representing the direction fo string. $\theta = v^l_{;l}$ is the scalar of expansion. If the particle density of the configuration is denoted by ρ_p then we get.

$$\rho = \rho_p + \mu \qquad (4)$$

In co-moving co-ordinate system we get.

$$v^\alpha = (0,0,0,1),\ x^\alpha = (\alpha_1^{-1},0,0,0) \qquad (5)$$

The physical quantities of the expansion scalar θ and shear tensor σ^2 are defined as

$$\theta = v^l_{;l} = \frac{\dot{\alpha}_1}{\alpha_1} + \frac{2\,\dot{\alpha}_2}{\alpha_2} = 3H \qquad (6)$$

$$\sigma^2 = \frac{1}{2}\sigma_{\alpha\beta}\sigma^{\alpha\beta} = \frac{1}{3}\left(\frac{\dot{\alpha}_1^2}{\alpha_1^2} + \frac{\dot{\alpha}_2^2}{\alpha_2^2} - \frac{2\dot{\alpha}_1\dot{\alpha}_2}{\alpha_1\alpha_2}\right) \qquad (7)$$

Where H is Hubble parameter.

The Hubble parameter defined as

$$H = \frac{\dot{\xi}}{\xi} = \frac{1}{3}\left(\frac{\dot{\alpha}_1}{\alpha_1} + \frac{2\dot{\alpha}_2}{\alpha_2}\right) \qquad (8)$$

The average scale factor $\xi(t)$ is defined as

$$\xi(t) = (\alpha_1\alpha_2^2)^{\frac{1}{3}} \qquad (9)$$

The Einstein field equation with gravitation units 8 ⊠ $G = 1$ (⊠ = squared times) read as

$$R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = T_{\alpha\beta} - \Lambda g_{\alpha\beta} \qquad (10)$$

For the metric (1), Einstein field equations can be written as

$$\frac{2\ddot{\alpha}_2}{\alpha_2} + \frac{\dot{\alpha}_2^2}{\alpha_2^2} = \eta\theta - \Lambda \qquad (11)$$

$$\frac{\ddot{\alpha}_1}{\alpha_1} + \frac{\ddot{\alpha}_2}{\alpha_2} + \frac{\dot{\alpha}_1\dot{\alpha}_2}{\alpha_1\alpha_2} = \eta\theta - \Lambda \qquad (12)$$

$$\frac{\ddot{\alpha}_1}{\alpha_1} + \frac{\ddot{\alpha}_2}{\alpha_2} + \frac{\dot{\alpha}_1\dot{\alpha}_2}{\alpha_1\alpha_2} - \frac{1}{\alpha_1^2} = \mu + \eta\theta - \Lambda \qquad (13)$$

$$\frac{2\dot{\alpha}_1\dot{\alpha}_2}{\alpha_1\alpha_1} + \frac{\dot{\alpha}_2^2}{\alpha_2^2} - \frac{1}{\alpha_1^2} = \rho + \Lambda \qquad (14)$$

$$\frac{\dot{\alpha}_1}{\alpha_1} - \frac{\dot{\alpha}_2}{\alpha_2} = 0 \qquad (15)$$

Where dots '.' on α_1 and α_2 represent the ordinary differentiation w.r.t. 't' From equ (14) we get

$$\alpha_1 = n\,\alpha_2 \qquad (16)$$

Where m is an integrating constant

From equ (14) we take $n = 1$

$$\alpha_1 = (1)\alpha_2$$

$$\alpha_1 = \alpha_2 \qquad (17)$$

We assume that the coefficient of bulk viscosity η is inversely proportional to the expansion scalar θ i.e. $\eta \propto \frac{1}{\theta}$ (18)

$$\eta = \frac{P_1}{\theta}$$

$$\eta\theta = P_1 \qquad (19)$$

Where P_1 is a positive constant Therefore

$$\theta = \frac{3\dot{\alpha}_2}{\alpha_1}$$

$$\theta = 3H$$

_______________(20)

Using equ (16), Equ (11) – (14) become,

$$\frac{2\ddot{\alpha}_2}{\alpha_2} + \frac{\dot{\alpha}_2^2}{\alpha_2^2} = \eta\theta - \Lambda$$

_______________(21)

$$\frac{2\ddot{\alpha}_2}{\alpha_2} + \frac{\dot{\alpha}_2^2}{\alpha_2^2} - \frac{1}{\alpha_1^2} = \mu + \eta\theta - \Lambda$$

_______________(22)

$$\frac{3\dot{\alpha}_2^2}{\alpha_2^2} - \frac{1}{\alpha_2^2} = \rho + \Lambda$$

_______________(23)

In equ (21) – (23) five unknown parameters $\alpha_2, \eta, \Lambda, \rho,$ and μ Therefore, to solve the system of equations completely we apply the condition.

$\Lambda \propto H$

$$\Lambda = lH$$

_______________(24)

Where l is constant.

Substituting equ (19) and (24) in equ (22) we have

$$\frac{2\ddot{\alpha}_2}{\alpha_2} + \frac{\dot{\alpha}_2^2}{\alpha_2^2} = P_1 - lH$$

_______________(25)

$$\frac{\ddot{\alpha}_2}{\dot{\alpha}_2} = \frac{1}{2}\left(P_1 \frac{\alpha_2}{\dot{\alpha}_2} - l - \frac{\dot{\alpha}_2}{\alpha_2}\right)$$

_______________(26)

Integrating equ (26) we get

$$\dot{\alpha}_2 = n_1 e^{-\frac{lt}{2}} \alpha_2^{-\left(\frac{P_1+1}{2}\right)}$$

_______________(27)

Again integrating we get.

$$\alpha_2 = \left(\frac{P_1+3}{2}\right)^{\frac{2}{P_1+3}} \left(-\frac{2}{l}.e^{-\frac{lt}{2}}\ n_1 + n_2\right)^{\frac{2}{P_1+3}}$$

_______________(28)

Where n_1 and n_2 are constants of integration thus

$$\alpha_1 = \left(\frac{P_1+3}{2}\right)^{\frac{2}{P_1+3}} \left(-\frac{2}{l}.e^{-\frac{lt}{2}}\ n_1 + n_2\right)^{\frac{2}{P_1+3}}$$

_______________(29)

Therefore the line element (1) can be be written us.

$$ds^2 = -dt^2 + \left[\left(\frac{P_1+3}{2}\right)\left(-\frac{2}{l}.e^{-\frac{lt}{2}}\ n_1 + n_2\right)\right]^{\frac{4}{P_1+3}} \left(dx_1^2 + e^{2x}dx_2^2 + dx_3^2\right)$$

_______________(30)

$$\frac{\dot{\alpha}_2}{\alpha_2} = e^{-\frac{lt}{2}}\ n_1 \left[\left(\frac{P_1+3}{2}\right)^{-1}\left(-\frac{2}{l}.e^{-\frac{lt}{2}}\ n_1 + n_2\right)^{-1}\right]$$

_______________(31)

Some physical and Geometrical properties

The every density $\rho,$ the string tension density $\mu,$ the scalar of expansion $\theta,$ Hubble parameter H, cosmological term Λ and the shear scalar σ^2 are respectively given by

$$\rho = 3n_1^2\, e^{-lt}\left(\frac{P_1+3}{2}\right)^{-2}\left(-\frac{2}{l}.e^{-\frac{lt}{2}}\ n_1 + n_2\right)^{-1}$$

_______________(32)

$$\mu = -\left[\left(\frac{P_1+3}{2}\right)^{\frac{-4}{P_1+3}}\left(-\frac{2}{l}.e^{-\frac{lt}{2}}\ n_1 + n_2\right)^{\frac{-4}{P_1+3}}\right]$$

_______________(33)

$$\theta = 3e^{-\frac{lt}{2}}\ n_1 \left[\left(\frac{P_1+3}{2}\right)^{-1}\left(-\frac{2}{l}.e^{-\frac{lt}{2}}\ n_1 + n_2\right)^{-1}\right]$$

_______________(34)

$$H = 9e^{-\frac{lt}{2}}\ n_1 \left[\left(\frac{P_1+3}{2}\right)^{-1}\left(-\frac{2}{l}.e^{-\frac{lt}{2}}\ n_1 + n_2\right)^{-1}\right]$$

_______________(35)

$$\Lambda = 9le^{-\frac{lt}{2}}\ n_1 \left[\left(\frac{P_1+3}{2}\right)^{-1}\left(-\frac{2}{l}.e^{-\frac{lt}{2}}\ n_1 + n_2\right)^{-1}\right]$$

_______________(36)

$$\sigma^2 = 0$$

$$\sigma = 0$$

_______________(37)

REFERENCES

[1]. S.R. Roy and S.K. Banerji, Class Quantum Grav 11, 1943 (1995).

[2]. A Banerjee, A.K. Sanyual, and S. Chakraborty, Pramana – J.Phys. 34,.1 (1990)

[3]. R. Bali and S.Dave, Praman- J. Phys. 56, 513 (2001)

[4]. R. Bali and R.D. Upadhya, Astrophysics and space – science 283, 97 (2003)

[5]. R. Bali and S. Dave, Astrophysics and space science 288, 503 (2003)

[6]. R. Bali and D.K singh, Astrophysics and space science 300, 387 (2005)

[7]. X.X. Wang, Astrophysics and space- science 293, 933 (2004)

[8]. X.X. Wang Chia. Phys. Lett. 21, 1205; (2004).
[9]. X.X. Wang, Chin. Phys. Lett 22, 29 (2005).
[10]. X.X. Wang. Chin. Phys. Lett 23, 170 (2006).
[11]. Murphy, G.L.: 1973, Phys. Rev. D8, 4232 .
[12]. Bali, R. and Dave S; Astrophys. space sci. 282, 461 (2002).
[13]. Bali, R. and Prathana, A; Chi Phys lett. 24, 5 85 (2007).
[14]. Ya . B. Zeldovich (1968) 50v. PHys. Uspekhi 11. 381.
[15]. O. Bertolami (1986) Nuovo Cimento B 93 36.
[16]. O. Bertolami (1986) Fortschr. Phys. 34829.
[17]. M. Ozer. and M.o Taha (1987) Nucl. Phys. B287 776
[18]. S. Weinberg (1989) Rev. Mod. Phys. 611.
[19]. S.M. Carroll, W. H. Press, and E.L. Turner (1992) Annu Rev. Astron. Astrophys 30499

International Journal of Engineering Research and Applications (IJERA) is **UGC approved** Journal with Sl. No. 4525, Journal no. 47088. Indexed in Cross Ref, Index Copernicus (ICV 80.82), NASA, Ads, Researcher Id Thomson Reuters, DOAJ.

R. K. Dubey. "Bianchi type III Bulk viscous string cosmological model with shear free and time dependent cosmological constant in modified general relativity." International Journal of Engineering Research and Applications (IJERA) , vol. 7, no. 11, 2017, pp. 56–59.