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Aspect of Cantilever Deflection of Bridge Deck Due to Material Nonlinearity

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ABSTRACT
When a structure is analysed for linear elastic behavior, the analysis is carried out on the premise that a proportional relationship exists between loads and displacements. This assumes a linear material stress-strain relationship and small geometric displacements. The assumption of linear behavior is effective in most of the structures. However, nonlinear analysis is necessary when stresses are excessive, or large displacements exist in the structure. Nonlinear analysis can be classified into three categories. The material nonlinear behavior are encountered when relatively big loadings are applied to a structure thereby resulting in high stresses in the range of nonlinear stress-strain relationship.

***Keywords*:** Linear, Nonlinear, Cantilever, material

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I. INTRODUCTION

A fundamental difference between elastic and plastic material behavior is that no permanent deformations occur in the structure in elastic behavior, whereas permanent or irreversible deformations occur in the structure in plastic behavior.

A structure with several components has completed in with different duration with number of construction stages. The configurations of the structure, loadings, boundary conditions and even the physical properties of structural members change during the behavior in real-time actions. Such changes in the structural system are induced a linear and nonlinear behavior.

In general the linear and nonlinear element stiffness environments are considered together with the unbalance between stresses in the model and the external applied loading. This linear set of equations are solved with resulting in a change of displacements. The total stresses in the nonlinear elements are calculated and evaluated using FEM software. If the stresses in the nonlinear elements are in appropriate arrangement with the definite nonlinear element stiffness conditions, then the result will be accepted, otherwise the respective element stiffness will be adapted and a new displacement change will be calculated and corresponding total stresses will be evaluated. So from the analysis the change in displacement and stress are evaluated for linear and nonlinear conditions.

The relationship, which is typically represented as in Fig1, widely varies with loading methods and material properties

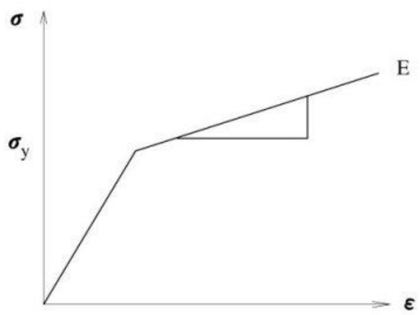

Fig 1: Stress-strain relationship used for material nonlinearity

A material nonlinear analysis is carried out when a structure undergoes large displacements and the change of its material linear range renders a nonlinear material stress-strain relationship [1]. The geometric nonlinearity may exist even in the state of linear material behavior. Cable structures such as suspension bridges are analysed for material nonlinearity. A material nonlinear analysis must be carried out if a structure exhibits significant change of its shape and cracking conditions under applied loads such that the resulting large displacements change the coordinates of the structure or additional loads like moments are induced.

II. APPROACH OF CURRENT STUDY

A. Linear static model

Linear static analysis is very beginner level of analysis. The word “linear” which is actually the computed response—stress or displacement, for example the linearly related to the applied force. The word “static” means the forces which do not change with time otherwise, that the time variation is not that influence and can be ignore safely.

An example of a static force is a building's dead load, which is comprised of the building's weight plus the weight of offices, equipment, and furniture [2]. This dead load is often expressed in terms of lb/ft2 or N/m2. Such loads are often defined using a maximum expected load with some factor of safety applied for conservatism.

In addition to the time invariant dead load described above, another example of a static load is an enforced displacement. For example, in a building part of the foundation may settle somewhat, inducing static loads. Another example of a static load is a steady-state temperature field. The applied temperatures cause thermal expansion which, in turn, causes induced forces [3].

The basic equation adopted in FEM software for linear static analysis is as follows:

[K] {U} = {P}

Where,

[K] : Stiffness matrix

{U} : Displacement vector

{P} : Load vector

B. Nonlinear static model

Material Nonlinear Analysis:

A fundamental difference between elastic and plastic material behaviour is that no permanent deformations occur in the structure in elastic behaviour, whereas permanent or irreversible deformations occur in the structure in plastic behaviour.

In the design of procedure, engineers usually perform the elastic analysis to calculate the internal forces within the structure. The dimension of reinforcement is then sized using either cracked section or uncracked section analysis or by using a non-linear theory. There has been a great deal of interest in the analysis and design of reinforced concrete structures since Nielsen and Wood published a yield criterion for an isotropic membrane and later extended it to the orthotropic reinforced case by using a lower bound plasticity theory. He developed procedure for finding the envelope of a given panel of known reinforcement and concrete strength, and the method of designing the reinforcement for a given set of forces. In this theory Nielsen assumed that the concrete alone would resist compression and no compression reinforcement would be provided [4].

Classification by the method of defining the effective plastic strain

Strain hardening: The effective plastic strain in strain hardening is defined as follows

$d\mathcal{E}p=\sqrt{(2/3\ (d\mathcal{E}p)T\ d\mathcal{E}p)} \quad = \sqrt{(2/3\ aT\ as)}\ \lambda$-Eq1

The effective plastic strain is derived from transforming the norm of plastic strains to conform to uniaxial strain with the assumption that there is no volumetric plastic deformation. Although this is applicable in principle only to Tresca or von Mises, it is often applied to other cases because of numerical convenience.

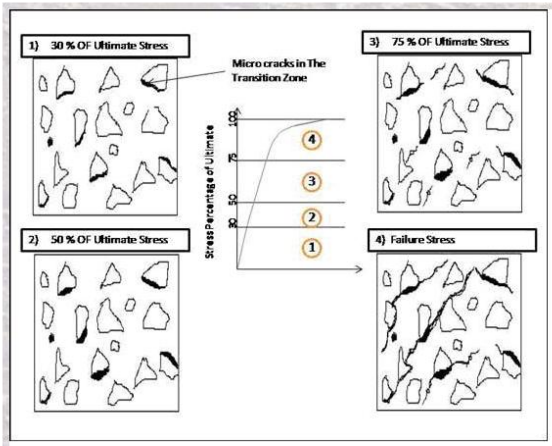

Fig 2. Stress strain relationship for ordinary concrete

When a metal is stressed beyond its elastic limit it enters the plastic region (The region in which residual strain remains upon unloading). When the load is increased further (a kind of rearrangement occurs at atom level and the mobility of the dislocation decreases) dislocation density’ increases that makes the material harder and stronger through the resulting plastic deformation.

It means, it’s more difficult to deform the material as the strain increases and hence it’s called “strain hardening”. This tends to increase the strength of the metal and decrease its ductility.

C. Tresca model

The Tresca yield criterion is taken to be the work of Henri Tresca. It is also known as the maximum shear stress theory (MSST) and the Tresca criterion. In terms of the principal stresses the Tresca criterion is expressed as;

$1/2 \quad max(|\sigma\_1-\sigma\_2 \quad |,|\sigma\_2-\sigma\_3 \quad |,|\sigma\_3-\sigma\_1 \quad |)=S\_sy=1/2\ S\_y$

Where Ssy is the yield strength in shear, and Sy is the tensile yield strength.

Assumption: The assumption that plastic deformation of a material begins when the difference between the maximum and minimum principal stresses equals twice the yield stress in shear.

Fig 1 shows the Tresca–Guest yield surface in the three-dimensional space of principal stresses. It is a prism of six sides and having infinite length. This means that the material remains elastic when all three principal stresses are linearly comparable (a hydrostatic pressure), nonsubstance how much it is compressed or stretched. However, when one of the principal stresses becomes smaller or larger than the others the material is subject to shearing. In such situations, if the shear stress reaches the yield limit then the material enters the plastic domain.

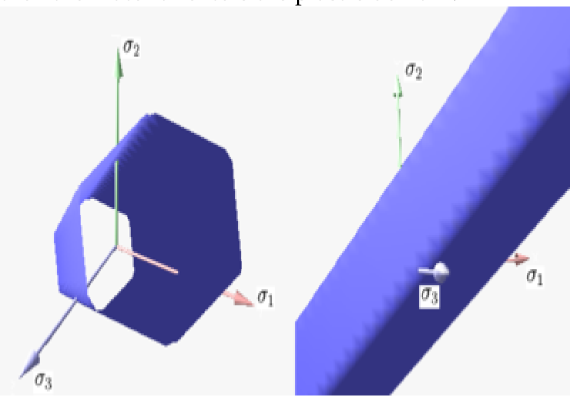

Fig 3. Tresca – Guest yield surface in 3D space principal stresses

III. MODEL DESCRIPTION

The purpose of an FEM analysis is to obtain wanted results, and this is what the post-processing step is for the typically, various components or measures of stress, strain, and displacement at any given location in the structure are available for putout. Additional quantities for output may include factory of safety, energy norm error, contact pressure, reaction force, strain energy density, etc. The way a quantity is outputted depends on the FEM software.

The FEM simulation is analogous to building the structure or making the specimen in physical testing. Several sub-steps involved in model processing are geometry creation, material property assignment, boundary condition specification, and mesh generation.

The geometry of the structure to be analysed is defined in the geometry creation step. After the solid geometry is created, the material properties of the solid are specified in the material property assignment step. The material required for the FEM analysis depends on the type of analysis. For example, in the elastic deformation analysis of an isotropic material under isothermal condition, only the modulus of elasticity and the Poisson's ratio are needed.

For most beginner users of FEM, the boundary condition specification step is probably the most challenging of all pre-processing steps. Two types of boundary conditions are possible. The first is prescribed displacement boundary condition which is analogous to holding or supporting the specimen in physical testing. [5] The second is applied force boundary condition which is analogous to loading the specimen.

Although validation is formal part of the FEM analysis and it is important too. The validation usually involves comparing FEM results at one or more selected positions with exact or approximate solutions using classical approaches such as elasticity or mechanics of materials. Going through validation strengthens conceptual understanding and enhances learning.

Solid elements are generally used to model voluminous structures such as concrete foundations, car engines, thick walls, rubbers, etc. A solid element may be a tetrahedron, pentahedron or hexahedron. The solid element can be used for both static (linear & nonlinear) and dynamic analyses. The solid element has the following stress and strain tensors

The solid element is formulated as an isoparametric element, and the Incompatible Modes theory is used for 8-node hexahedron and 6-node pentahedron. Solid elements only have translational displacements u, v and w in the ECS x, y and z directions in each node.

Ui={ui vi wi}

In the element the coordinates x, y and z and displacements u, v and w can be expressed as follows:

$X=\sum_{(i=1)}^{n} N_i x_i$; $Y=\sum_{(i=1)}^{n} N_i Y_i$;
$Z=\sum_{(i=1)}^{n} N_i Z_i$
$u=\sum_{(i=1)}^{n} N_i u_i$; $v=\sum_{(i=1)}^{n} N_i v_i$;
$w=\sum_{(i=1)}^{n} N_i w_i$

The matrix D represents the relationship between the stress and the strain tensor. For isotropic materials, D.

$$D=\frac{E}{(1+v)(1-2v)}\begin{bmatrix} 1-\gamma & \gamma & \gamma & 0 & 0 & 0 \\ \gamma & 1-\gamma & \gamma & 0 & 0 & 0 \\ \gamma & \gamma & 1-\gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\gamma}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\gamma}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\gamma}{2} \end{bmatrix}$$

Where E is the Young"s modulus and ν is the Poisson ratio. Orthotropic materials can be defined in the Material Coordinate System MCS.

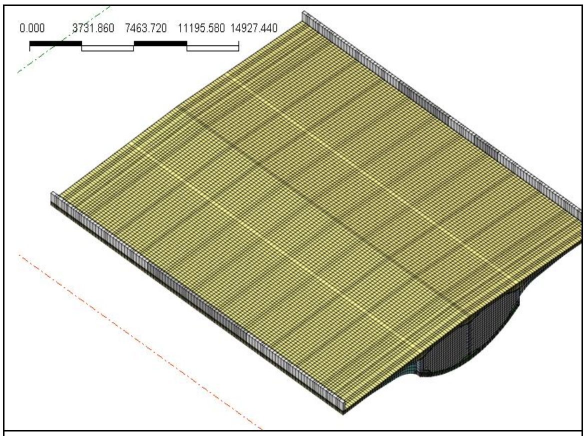

Fig 4. 3D FEM model for a single span

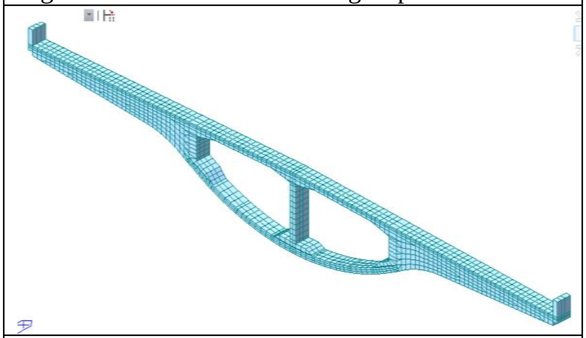

Fig 5. 3D FEM model for 1m strip for nonlinear deflection

IV. LINEAR ANALYSIS MODEL

When a structure is subjected to external loads, the corresponding structural response may exhibit material nonlinearity to a certain extent. However, in most structural analyses for design purposes, structures behave almost linearly provided that the member stresses remain within the limits of design codes. Material nonlinearity thus is rarely considered in practice.

Moreover, the recent quantum leaps in computational power have greatly raised the expectations on the analyst, who is under increasing pressure to provide highly accurate results [6]. At the very least, he/she will face the situation of having to choose a suitable nonlinear method -from literally hundreds which best matches his/her particular problem.

• A localized nonlinearity in a structure can have a significant global impact while leaving some areas mostly unaffected. Examples of local nonlinearities are joints, geometric discontinuities, regions undergoing large displacements, in this investigation the displacement is considered as the nonlinear parameter with material nonlinearity.

• The nonlinear effects are usually confined to just a few modes and coordinates, while the rest behave linearly [7].

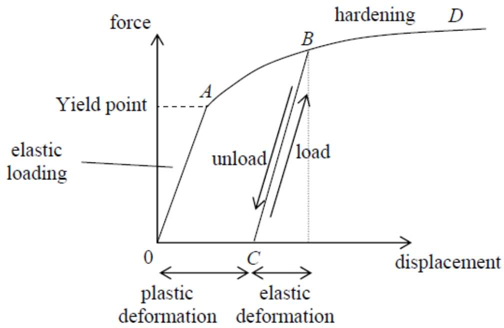

Fig. 6. Force / Displacement curve

Table 1. Straining hardening values

STRAIN HARDENING	
PLASTIC STRAIN	YIELD STRESS
0.0000	0.0000
0.0018	50.0000
0.0035	50.0000

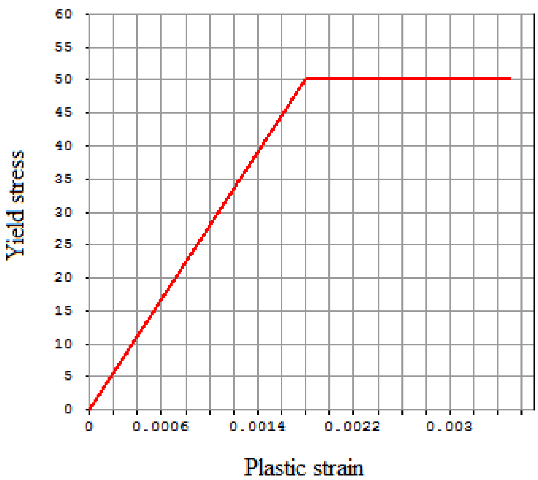

Fig 7. Strain Hardening Graph

V. RESULTS SUMMARY

i. Deflection component (Dz)

Physical source. The behavior of the material depends on current deformation condition and possibly the history of the deformation. Structures which is undergoing nonlinear elastic condition, plasticity, viscoelasticity, creep, or inelastic rate effects.

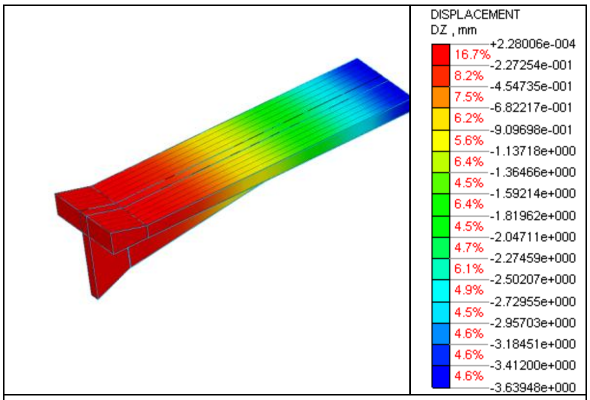

Fig 8. Linear analysis deflection (Dz)

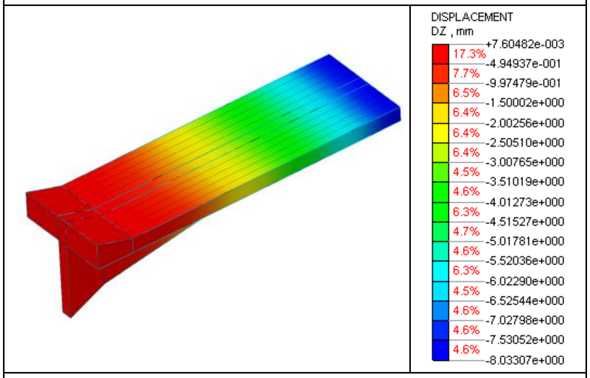

Fig 9. Non-Linear analysis deflection (Dz)

ii. Stress component:

Von mises stress: The widely used yield criterion is Von Mises criterion for metallic materials. It depends on distortional strain energy and the yield function.

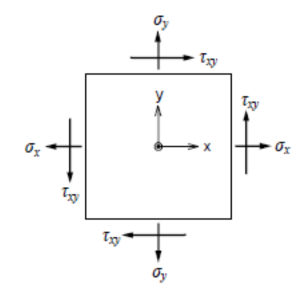

Fig 10.1. Axial and shear components

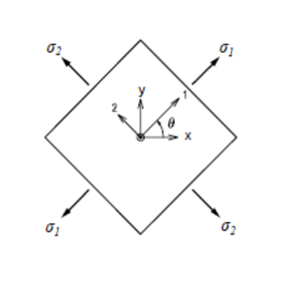

Fig 10. 2. principal stress components

σx : Axial stress in x direction

σx : Axial stress in y direction

τxy : Shear stress in the x-y plane

Maximum principal stress,

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Minimum principal stress,

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Maximum shear stress,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

θ is angle between the x axis and principal axis, 1

Von- Mises Stress, $\sigma_{eff} = \sqrt{(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)}$

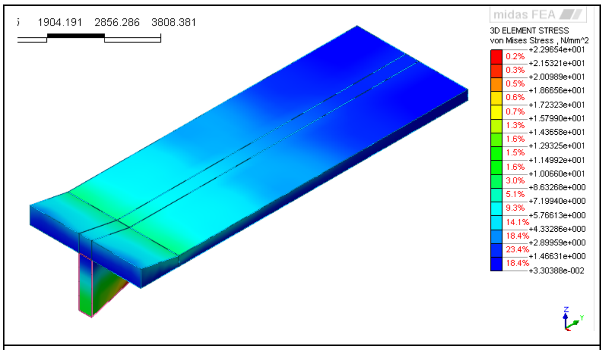

Fig 11.1. Linear Von mises stress

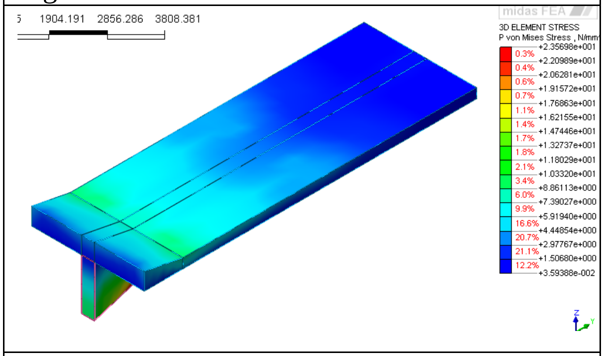

Fig 11.2. Non Linear Von mises stress

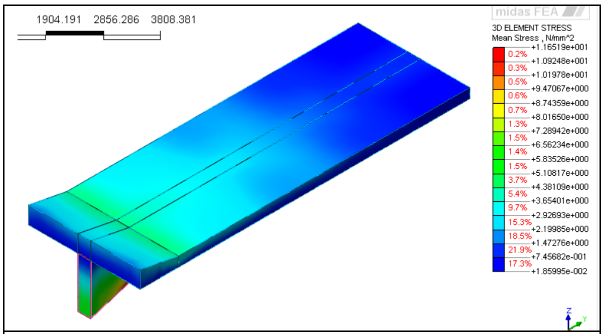

Fig 12.1. Linear mean shear stress

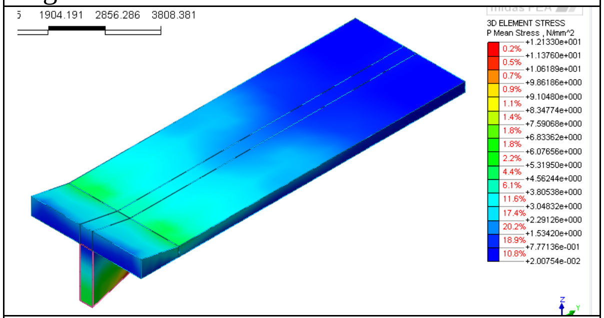

Fig 12.2 Nonlinear mean shear stress

iv. Strain component:

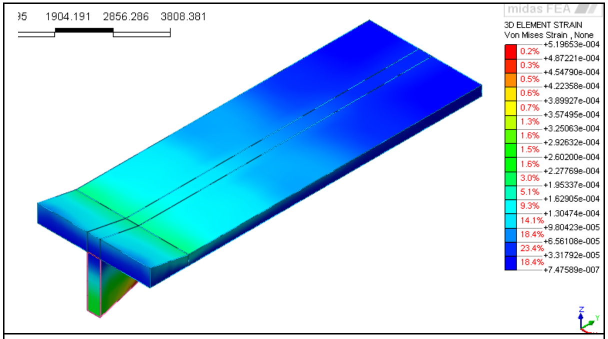

Fig 13.1. Linear Von-mises strain

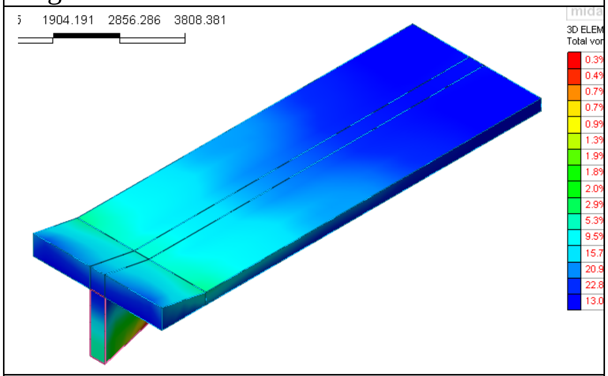

Fig 13.2. Nonlinear Von-mises strain

a. Volumetric strain

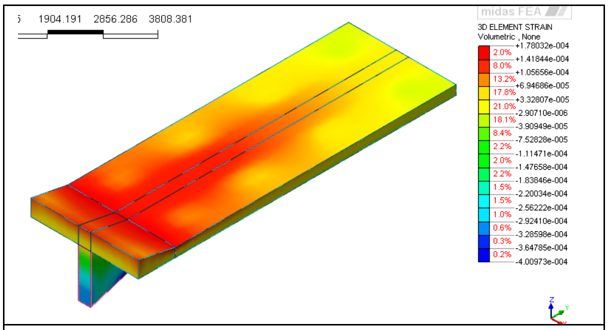

Fig 14.1. Linear volumetric strain

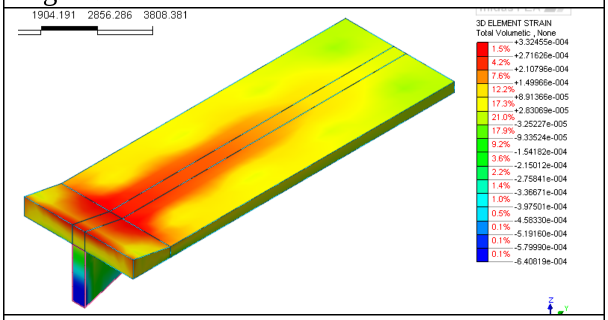

Fig 14.2. Nonlinear volumetric strain

VI. CONCLUSION:

A response diagram characterizes only the gross behavior of a structure, which might be observed as merely as conducting an experiment on a mechanical testing machine. Further insight into the source of nonlinearity is required to capture such physical behavior with mathematical and computational models for computer simulation.

The following conclusions from various case studies are presented here:

- The outcomes of the conducted finite element analysis, obtained from the calculations using material nonlinear provisions regarding the load-deflection relationship, and the crack initiation and propagation.
- The material nonlinear approach gives a very good to estimate the material behavior for most practical applications. The reinforcement thus computed can be further optimized if a sophisticated layering meshing technique is used instead of a simple mesh type which makes a bonding the concrete and steel.
- The concrete Tresca and von-mises model and steel bilinear model computed by the plastic theory is optimum for most practical cases wherein the serviceability criterion is not required.
- Where serviceability criterion is required regarding crack width, max service stress in concrete, and minimum compression depths, etc., the reinforcement required will be much higher than computed using plastic theory.
- The ratios of steel required from SLS to ULS condition varies with the loading organization and could be in the order of 80 to 100% for some cases.
- The initial cracking in the finite element model appropriated place in the form of vertical flexure. Subsequent flexural shear cracks appeared in the model as the load was increased beyond the cracking moment strength and until steel reinforcement yielding.
- The load-deflection relationship is linear elastic up to the cracking moment strength then the curve inclines more towards the horizontal. After steel reinforcement yielding the curve inclines appreciably towards the horizontal.

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