RESEARCH ARTICLE OPEN ACCESS

On The Ternary Quadratic Equation $3x^2+3y^2-2xy=24z^2$

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ABSTRACT
The ternary Quadratic Diophantine equation $3x^2+3y^2-2xy=24z^2$ is analyzed for its non – zero distinct integer solutions. Six different patterns of non-zero distinct integer solutions to the equation under consideration are obtained. A few interesting relation between the solutions and special numbers are exhibited.
Keywords: Integral solutions, Ternary quadratic, polygonal number, Oblong number

Date of Submission: 07-10-2017 Date of acceptance: 31-10-2017

I. INTRODUCTION

The Ternary Quadratic Diophantine equation offers an unlimited field for research because of their variety [1-2]. For an extensive review of various problems, one may refer [3-10]. This communication concerns with yet another interesting Also a few interesting relations among the solutions have been presented..

II. NOTATIONS

obl $_n$- Oblong number of rank 'n' $t_{m,n}$- Polygonal number of rank 'n' with sides'm'

III. METHOD OF ANALYSIS

The ternary quadratic equation to be solved in integers is

$$3x^2+3y^2-2xy=24z^2 \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Now, introducing the linear transformations

$$x=u+v\,;\; y=u-v\;, \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

We have $u^2+2v^2=6z^2$ ……………… (3)

6 can be written as

$$6=(2+i\sqrt{2})(2-i\sqrt{2}) \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Assume $z=a^2+b^2$ ……………… (5)

Using (4) and (5) in (3) and applying the method of factorization, define

$$6[\,a^2+b^2\,]^2=(u+i\sqrt{2}v)(u-i\sqrt{2}v)$$

Equating real and imaginary parts

$$u=u(a,b)=2a^2-4b^2-4ab$$
$$v=v(a,b)=a^2-2b^2+4ab$$

Substitute u and v in (2)

$$x=x(a,b)=3a^2-6b^2$$
$$y=y(a,b)=a^2-2b^2-8ab$$
$$z=z(a,b)=a^2+2b^2$$

Properties:

(i) $x(1,n)+y(1,n)+8obl_n \equiv 0(\text{mod}\,4)$

(ii) $y(1,n)-3z(1,n)+8obl_n \equiv 0(\text{mod}\,2)$

(iii) $x(1,n)+z(n,1)\equiv 0(\text{mod}\,5)$

(iv) $y(1,n)+z(1,n)+8n\equiv 0(\text{mod}\,2)$

PATTERN:II

(3) Can be written as

$$u^2+2v^2=2z^2+4z^2$$
$$\frac{u+2z}{z-v}=\frac{2(z+v)}{u-2z}=\frac{A}{B}$$

Using cross multiplication, we get

$$x=x(a,b)=-3a^2+6b^2$$
$$y=y(a,b)=-a^2+2b^2-8ab$$
$$z=z(a,b)=a^2-2b^2$$

From which we get

$u = 6ab;$

$v = -a^2 + 3b^2;$ …… (6)

$z = 3b^2 + a^2$

Using (5) in (2), we obtain the integer solutions to (1) as given below:

$x = x(a,b) = -a^2 + 3b^2 + 6ab$

$y = y(a,b) = a^2 - 3b^2 + 6ab$

$z = z(a,b) = -a^2 + 3b^2$

PROPERTIES

(i) $x(1,n) + y(1,n) \equiv 0 (\bmod 2)$

(ii) $y(1,n) + z(1,n) - 6obl_n \equiv 0 (\bmod 6)$

(iii) $x(1,n) - z(1,n) \equiv 0 (\bmod 6)$

(iv) $3x(1,n) - 6y(1,n) + 9n(n-2) \equiv 0 (\bmod 9)$

(v) $4y(1,n) - 2z(1,n) \equiv 0 (\bmod 6)$

It is observed that (3) may also be written in the following three ways

WAY : 1

$x = x(a,b) = 3b^2 - a^2 - 6ab$

$y = y(a,b) = a^2 - 3b^2 - 6ab$

$z = z(a,b) = -a^2 - 3b^2$

PROPERTIES

(i) $x(1,n) - z(1,n) + 6obl_n + 0$

(ii) $x(1,n) + y(1,n) \equiv 0 (\bmod 12)$

(iii) $3y(1,n) + z(1,n) + 6obl_n + 0$

(iv) $6x(1,n) + 3y(1,n) \equiv 0 (\bmod 3)$

(v) $x(1,n) - z(1,n) + 12n = 6obl_n$

WAY : 2

$x(a,b) = b^2 - 3a^2 + 6ab$

$y(a,b) = -b^2 + 3a^2 + 6ab$

$z(a,b) = 3a^2 + b^2$

PROPERTIES

(i) $x(1,n) + y(1,n) \equiv 0 (\bmod 6)$

(ii) $x(n,1) + 3z(n,1) - 6n \equiv 0 (\bmod 10)$

(iii) $6x(n,1) - 2y(n,1) \equiv 0 (\bmod 6)$

(iv) $3x(n,1) + 3nz(n,1) = 18obl_n$

(v) $y(n,1) + z(n,1) - 6obl_n \equiv 0 (\bmod 6)$

WAY: 3

$x = x(a,b) = -3a^2 + b^2 - 6ab$

$y = y(a,b) = 3a^2 - b^2 - 6ab$

$z = z(a,b) = -3a^2 - b^2$

PROPERTIES

(i) $x(1,n) - y(1,n) + 2n^2 \equiv 0 (\bmod 6)$

(ii) $y(1,n) + z(1,n) + 2t_{4,n} \equiv 0 (\bmod 6)$

(iii) $x(1,n) + z(1,n) + 2t_{4,n} \equiv 0 (\bmod 5)$

(iv) $y(1,n) + z(1,n) + 2n(n+3) = 0$

PATTERN:2

Assume $u^2 + 3v^2 = 3z^2$ (7)

Also write 4 as

$$3 = (i\sqrt{3}) * (-i\sqrt{3}) \quad (8)$$

$$z = a^2 + 3b^2 \quad (9)$$

Substitute (8) and (9) in (7) and employing the method of factorization, define

$u + i\sqrt{3}v = (i\sqrt{3})[a + i\sqrt{3}b]^2$

Equating the real and imaginary parts, we get

$u = -6ab$

$v = a^2 - 3b^2$

$z = a^2 + 3b^2$

and hence

x = x(a,b) = $a^2 - 3b^2 - 6ab$

y = y(a,b) = $-a^2 + 3b^2 - 6ab$

z = z(a,b) = $a^2 + 3b^2$

IV. CONCLUSION

To conclude, one may search for other patterns of solutions to the equation under consideration.

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ACKNOWLEDGEMENTS

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Dr.R.Anbuselvi . "On The Ternary Quadratic Equation ." International Journal of Engineering Research and Applications (IJERA) , vol. 7, no. 10, 2017, pp. 01–03.