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Ranking of octagonal fuzzy numbers for solving fuzzy linear Programming problems

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ABSTRACT
Fuzzy linear programming problem performs an vital role in modeling process, as we can produce uncertainty arising in real life problems. The paper aims at solving linear programming problems in which the parameters are octagonal fuzzy numbers with the help of robust ranking method. The fuzzy linear programming problems can be transformed into crisp value problem to obtain a optimal solution using simplex method. Further the obtain result is compared with a graphical method.

***Keywords*:** Octagonal fuzzy number, Linear Programming Problem, Ranking technique, Simplex method, graphical method.

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I. INTRODUCTION

Linear programming is a leading technique used in optimization techniques. Tanoak et al was the first one to propose the notion of fuzzy linear programming problems. Zimmerman introduced fuzzy linear programming in fuzzy environment. The aim of the paper is to solve Linear Programming Problem in which decision variable cost coefficient involving in objective function and the right hand side coefficient in the constraints are octagonal fuzzy numbers with the help of robust ranking method.

Fuzzy Linear Programing problem is an application of fuzzy set theory in linear decision making problem and most of these problems are related to linear programming with fuzzy variable. In this paper a new method for solve fuzzy variable linear programming problem directly using linear ranking function is proposed. Linear programming is one of the most frequently applied operations research technique. As Linear Programming model representing real world situations, involves a lot of parameters whose values are assigned by experts and in the conventional approach, they are required to fix a exact value to the aforementioned parameters. If exact values are suggested this are only statistical inference from past data and their stability is doubtful. So the parameter of the problem are usually denoted by the decision makers is an uncertained way. Therefore it is useful to consider. Malaki.et.al proposed method for solving fuzzy numbers linear programming problem using the concept of ranking of fuzzy numbers. Nasseri and Ardil developed simplex method to FLP problems by using certain ranking function. This method uses simplex table which is used for solving linear programming problem in crisp environment before. Several authors have used ranking function for solving fuzzy linear programming problem.

II. PRELIMINARIES

2.1 Fuzzy Set [3]

If X is a collection of objects denoted generically by x, then the fuzzy set Á in X is defined to be a set of ordered pairs. Where$\mu_{\tilde{A}}$(x) is called the membership function for the fuzzy set. The membership function maps each element of x to a value between (0,1).

2.2 Fuzzy number [3]

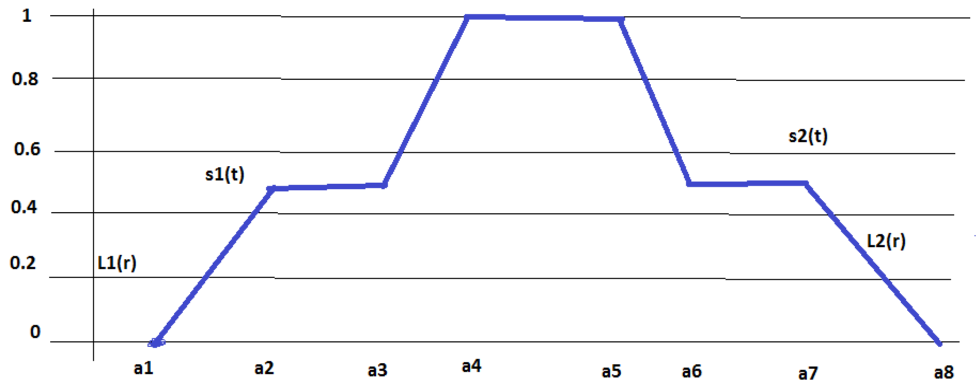
A Fuzzy number Á in the real line R is a fuzzy set$\mu_{\tilde{A}}$(x): R→ (0,1) that satisfies the following properties.

i) There exists an at least one x∈R with $\mu_{\tilde{A}}$(x)=1
ii) $\mu_{\tilde{A}}$(x)is piece wise continuous

2.3 Octagonal fuzzy number [5]

A fuzzy number is the normal Octagonal fuzzy number is denoted by $(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)$ where $a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6 \le a_7 \le a_8$ are real numbers and its membership function $\mu_{\tilde{A}}$(x)is given below

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ x < a_1 \\ k\left[\frac{x-a_1}{a_2-a_1}\right] & for\ a_1 \le x \le a_2 \\ k & for\ a_2 \le x \le a_3 \\ k+(1-k)\left(\frac{x-a_3}{a_4-a_3}\right) & for\ a_3 \le x \le a_4 \\ 1 & for\ a_4 \le x \le a_5 \\ k+(1-k)\left(\frac{a_6-x}{a_6-a_5}\right) & for\ a_5 \le x \le a_6 \\ k & for\ a_6 \le x \le a_7 \\ k\left(\frac{a_8-x}{a_8-a_7}\right) & for\ a_7 \le x \le a_8 \\ 0 & for\ x \ge a_8 \end{cases}$$



2.4 Ranking technique

To provide results which are consistent with human intuition,robust ranking technique is used and its satisfies compensation,linearity and additive properties.If $\tilde{a}$ is a convex fuzzy number, the robust ranking index is defined by

$R(\tilde{a}) = \int_0^1 (0.5) \quad (a_\alpha^L\ ,\ a_\alpha^U)\ d\alpha$

Where $(a_\alpha^L\ ,\ a_\alpha^U) = [\{(b-a)\alpha + a, d-(d-c)\alpha, f-e\alpha+e, h-(h-g)\alpha$

is the α- level cut of a fuzzy number $\tilde{a}$. Here this method is proposed for ranking the objective values. The representative value of fuzzy number $\tilde{a}$ is given by Robust ranking index $R(\tilde{a})$.

2.5 Working rule

Step 1: Using the Robust ranking method, the given decision variable and cost coefficient are Converted in to a crisp value.

Step 2: Now find a basic feasible solution by using simplex method.

Step 3: using the optimality condition,the entering variable will be selected and Select a leaving Variableusing the feasibility condition.

Step 4: use this condition until finding the optimal solution.

III. NUMERICAL EXAMPLE

A company produces two products P_1 and P_2 .These products are processed on two different machines M_1 and M_2.The products P_1 and details for the products P_2 are as follows,

1) The time taken by the machine M_1 to produce the unit quantity of the products P_1 and P_2 of the products are represented by octagonal fuzzy number (6, 8, 9, 11, 15, 17, 18, 20) and (1,2, 4, 5, 8, 10, 12, 14) respectively Similarly the time taken by the second machine M_2 to produce the unit quantity of products P_1 and P_2 are (2, 3, 6, 7, 8, 10, 11, 13) and (2, 3, 6, 7, 8, 10, 11, 13) respectively.

2)The profits on per unit of P_1 and P_2 are represented by octagonal fuzzy numbers Rs.(-2, -1, 0, 1, 2, 3, 6, 7) and Rs.(1, 2, 3, 4, 5, 6, 9, 10) respectively.

3) The total available time for machine M_1 and M_2 are represented by octagonal fuzzy numbers (90,100,120,130,150,160,170,180) and (115,120,130,135,135,145,145,155) respectively.

How many approximate units of P_1 and P_2 should be produced to maximize the profit.

Solution:

Let approximate x_1& $x_2$units of products P_1 and P_2should be produced. Then the above problem may be formulated as follows:

Consider Max $Z=C_1x_1+C_2x_2$

Subject to,

$a_{11}x_1+a_{12}x_2 \le b_1$

$a_{21}x_1+a_{22}x_2 \le b_2$

subject to the constraints $x_1,x_2 \ge 0$

Where,

C_1= (-2, -1, 0, 1, 2, 3, 6, 7)

C_2= (1, 2, 3, 4, 5, 6, 9, 10)

a_{11}= (6, 8, 9, 11,15, 17,18,20)
a_{12}= (1, 2, 4, 5, 8,10,12,14)
a_{21} = (2,3,6,7,8,10,11,13)
a_{22} = (2,3,6,7,8,10,11,13)
b_1= (90,100,120,130,150,160,170,180)
b_2 = (115,120,130,135,135,145,145,155)

Step (1)

Using robust ranking technique. The given octagonal fuzzy number can be converted into a crisp value.

$R(\tilde{a}) = \int_0^1 (0.5) \quad (a_\alpha^L\ ,\ a_\alpha^U)\ d\alpha$

$R(6,8,9,11,15,17,18,20)=\int_0^1(0.5)(2\alpha\ + 6 + 11 - 2\alpha + 2\alpha + 15 + 20 - 2\alpha)$

$= \int_0^1(0.5)52\ d\alpha = 26.$

Similarly, R(-2, -1, 0, 1, 2, 3, 6, 7) = 4
R.(1, 2, 3, 4, 5, 6, 9, 10) = 10
R(1,2,4,5,8,10,12,14) = 14
R(2,3,6,7,8,10,11,13) = 15
R(2,3,6,7,8,10,11,13) = 19
R(90,100,120,130,150,160,170,180) =275
R(115,120,130,135,135,145,145,155) = 270

Step (2) The crisp valued Linear Programming problem can be formulated as

Max $Z=4x_1+10x_2$

Subject to constraints,

$26x_1+14x_2 \leq 275$

$15x_1+19x_2 \leq 270$

Step (3) Using Algorithm, the formulated problem can be written as

Let

$Z=4x_1+10x_2+0s_1+0s_2$

Subject to$26x_1+14x_2+s_1=275$

$15x_1+19x_2+s_2=270$

and $x_1, x_2, s_1, s_2 \geq 0$ (s_1, s_2 are fuzzy slack variables)

Step (4) Simplex Table

Initial Table

C_B	Y_B	X_B	x_1	x_2	s_1	s_2	Ratio
0	s_1	275	26	14	1	0	19.6
0	s_2	270	15	**19**	0	1	**14.2←**

Z_j-C_j -4 **-10**↑ 0 0

Since s_2 leaves the basis and x2 enter the basis.

First Iteration: The New simplex table is

C_B	Y_B	X_B	x_1	x_2	s_1	s_2	Ratio
0	s_1	76.2	14.2	0	1	-0.74	0
10	x_2	14.2	0.79	1	0	0.053	14.2

Zj-Cj 142 3.9 0 0 0.53

There is no entering variable. The current crisp optimal solution to the LPP is
x_1=0, x_2=14.2 and Maximize Z=142

IV. NUMERICAL EXAMPLE

As Previous Example, We compared with Graphical method of Linear Programming Problem

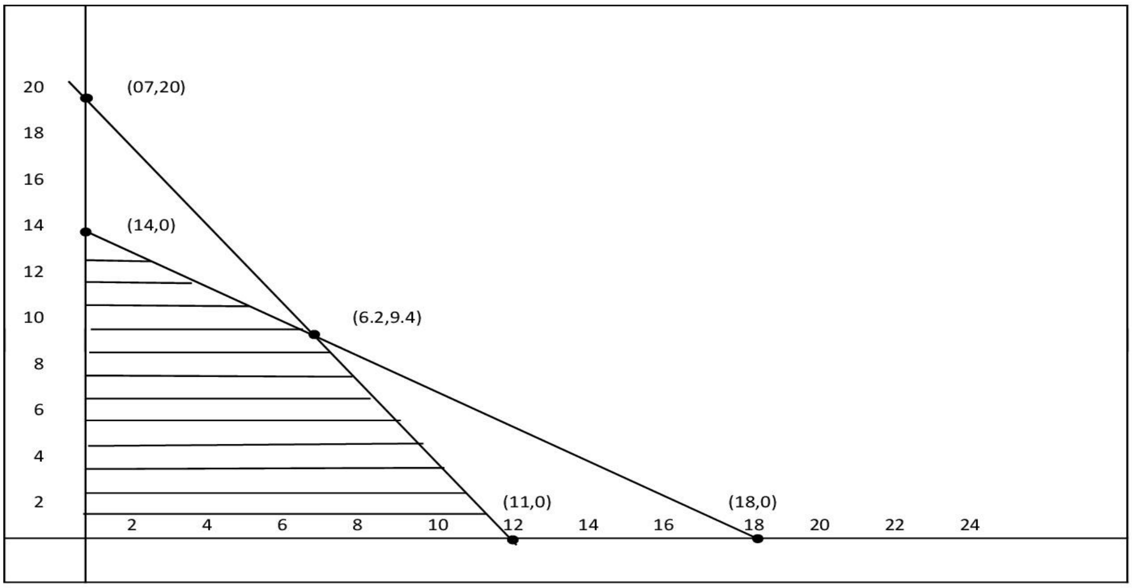


Fig. Graphical Method

Extreme Points	Coordinates (x_1,x_2)	Objective function Max $Z=4x_1+10x_2$
O	(0,0)	0
A	**(0,14)**	**140**
B	(6.2,9.4)	118.8
C	(11,0)	44

Comparison of result obtained by using simplex method and Graphical method

Simplex method	Graphical method
Maximize Z=142 at x_1=0 and x_2=14.2	Maximize Z=140 at x_1=0 and x_2=14

V. CONCLUSION

In this paper, we found a novel method for solving fuzzy Linear Programming Problem, octagonal fuzzy numbers by converting them into a crisp value using robust ranking method technique to derive a fuzzy optimal solution.

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