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Syncronization of The Photogravitational Magnetic Binary Problem With Variable Mass

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ABSTRACT
This article deals with the complete synchronization and anti synchronization behavior of the photogravitational magmetic binary problem when the charged particle has the variable mass, bigger primary is a source of radiation and smaller primary is a oblate body. Here we have designed a non linear controller based on the Lyapunov stability theory. Numerical simulations are performed to plot time series analysis graphs of the master system and the slave system which further illustrate the effectiveness of the proposed control technique.

Key words: photogravitational magnetic binary problem; complete synchronization; Lyapunov stability theory; Jean's Law; variable mass.

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I. INTRODUCTION

The last few years has been devoted to the study of nonlinear dynamical systems and their various properties. At present, there are different kinds of control methods and techniques that have been proposed for carrying out chaos control and synchronization of chaotic dynamical systems. Pecora and Carroll introduced a method to synchronize two identical chaotic systems with deferent initial conditions [17] and they demonstrated that chaotic synchronization could be achieved by driving or replacing one of the variables of a chaotic system with a variable of another similar chaotic device, the active control scheme proposed by E. W. Bai and K. E. Lonngren [2] has been received and successfully implemented in almost all the field of nonlinear sciences for synchronization for different systems with various techniques.

The synchronization problem via nonlinear control scheme is studied by Amir Abbas Emadzadeh, and Mohammad Haeri [1], M. Mossa Al-sawalha, M.S.M. Noorani in [10] and Moh. Arif [14] and [15] Chen and Han [5], Chen [6], Ju H. Park [8], .

Jeans [9] has studied the two-body problem with variable mass. Omarov [16] has also discussed the restricted problem of perturbed motion of two bodies with variable mass. Shrivastava and Ishwar [18] have studied the circular restricted three body problem with variable mass with the assumption that the mass of the infinitesimal body varies with respect to time. Singh et al. [7] has discussed the non-linear stability of equilibrium points in the restricted three body with variable mass

So many cases of the magnetic binaries problem have been studied by A. Mavragnais [11], [12] and [13], Bhatnagar et al.[3] and Bhatnagar and Mohd. Arif [4].

In this article we have discussed the complete synchronization and anti synchronization behavior of the photogravitational magnetic binary problem by taking into consideration the bigger primary is a source of radiation and smaller primary is a oblate body when the charged particle has the variable mass, here we have designed a nonlinear controller based on the Lyapunov stability in both cases. The system under consideration is chaotic for some values of parameter involved in the system. Here two systems (master and slave) are synchronized and start with deferent initial conditions. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time.

II. EQUATION OF MOTION

Two primaries in which the bigger primary is a source of radiation and smaller primary is a oblate body with magnetic fields move under the influence of gravitational forces and a charged particle P of charge q_1 and variable mass m moves in the vicinity of these primaries.

The equation of motion in the rotating coordinate system including the effect of the gravitational forces of the primaries on the charged particle P written as:

$$\ddot{x} + \frac{\dot{m}}{m}\,(\dot{x} - \omega y) - 2\,\dot{y}\,f \;= -\,\frac{1}{m}\,U_x \qquad (1)$$

$$\ddot{y} + \frac{\dot{m}}{m}\,(\dot{y} + \omega x) + 2\,\dot{x}\,f \;= -\,\frac{1}{m}\,U_y \qquad (2)$$

Where

$$\mathrm{f} = \omega - \frac{1}{m}\left(\frac{\vartheta}{\rho_1^3} + \frac{\lambda}{\rho_2^3} + \frac{\lambda I_2}{2\mu\rho_2^5}\right),\; U_x = \frac{\partial U}{\partial x} \text{ and } U_y = \frac{\partial U}{\partial y}$$

$$U = -\frac{m\,\omega^2}{2}\,(x^2+y^2) - (x^2+y^2)\,\omega\left\{\frac{\vartheta}{\rho_1^3} + \frac{\lambda}{\rho_2^3} + \frac{\lambda I_2}{2\mu\rho_2^5}\right\} - x\omega\left\{\frac{\vartheta\mu}{\rho_1^3} - \frac{\lambda(1-\mu)}{\rho_2^3} - \frac{\lambda(1-\mu)I_2}{2\mu\rho_2^5}\right\} - \frac{\vartheta m\,(1-\mu)}{\rho_1} - \frac{m\,\mu}{\rho_2} - \frac{m\,I_2}{2\,\rho_2^5} \qquad (3)$$

$\rho_1^2 = (x-\mu)^2 + y^2,\; \rho_2^2 = (x+1-\mu)^2 + y^2,\; \lambda = \frac{M_2}{M_1}$ (M_1, M_2 are the magnetic moments of the primaries which lies perpendicular to the plane of the motion). $\vartheta =$ radiation factor.

$I_2 = \mu\left(\frac{R_e^2 - R_p^2}{5}\right)$ $R_e =$ equatorial radii and $R_p =$ polar radii of the primary.

We have taken a particular case when $q_1 = c$ where c is the velocity of light.

We assumed that the distance between the primaries as the unit of distance and the coordinate of one primary is $(\mu, 0, 0)$ then the other is $(\mu-1, 0, 0)$. We also assumed that the sum of their masses as the unit of mass. If mass of the one primaries μ then the mass of the other is $(1-\mu)$. The unit of time in such a way that the gravitational constant G has the value unity. Position vector of P at any time t be $\overline{\rho} = (xi + yj)$ referred to a rotating frame of reference $O(x, y)$ which is rotating with the same angular velocity ω as those the primaries.

The variation of mass of the charged particle P is given by (Jeans law)

$$\frac{dm}{dt} = -\alpha\, m^n \;\; i.e \;\; \frac{\dot{m}}{m} = -\alpha\, m^{n-1} \qquad (4)$$

Where α is a constant coefficient and $n \in [0.4,\; 4.4]$

Now introduce the space-time transformation as:

$$x = \xi\,\gamma^{-q}, \qquad y = \eta\,\gamma^{-q}, \qquad dt = \gamma^{-k} d\tau$$

$$\rho_1 = r_1\gamma^{-q}, \qquad \rho_2 = r_2\gamma^{-q}, \qquad \gamma = \frac{m}{m_0} < 1$$

Where m_0 is the mass of the charge particle P at time $t = 0$.

Differentiating x and y with respect to t twice and Putting the values of $x, y\; \dot{x}, \dot{y}, \ddot{x}, \ddot{y}, U_x, U_y$ and $\frac{\dot{m}}{m}$ in equations (1) and (2) and after some simplification we get,

$$\xi'' + \beta\xi'\,(2q-k-1)\,\gamma^{n-k-1} - \beta^2 q\,\xi\,(n-q)\,\gamma^{2(n-k-1)} - 2\,\eta'\,\gamma^{-k}\left[\omega - \frac{1}{\gamma\,m_0}\left\{\frac{\vartheta\,\gamma^{3q}}{r_1^3} + \frac{\lambda\,\gamma^{3q}}{r_2^3} + \frac{\lambda I_2\,\gamma^{5q}}{2\mu r_2^5}\right\}\right] - \beta\,\eta\,\gamma^{\frac{n-q-1}{2k-q}}\left[\omega - 2\,q\left\{\omega - \frac{1}{\gamma\,m_0}\left(\frac{\vartheta\,\gamma^{3q}}{r_1^3} + \frac{\gamma^{3q}\lambda}{r_2^3} + \frac{\gamma^{5q}\lambda\,I_2}{2\mu r_2^5}\right)\right\}\right] = -\,\frac{\gamma^{2q-2k-1}}{m_0}\,\frac{\partial U}{\partial \xi} \qquad (5)$$

$$\eta'' + \beta\,\eta'\,(2q-k-1)\,\gamma^{n-k-1} - \beta^2 q\,\eta\,(n-q)\,\gamma^{2(n-k-1)} + 2\,\xi'\,\gamma^{-k}\left[\omega - \frac{1}{\gamma\,m_0}\left\{\frac{\vartheta\,\gamma^{3q}}{r_1^3} + \frac{\lambda\,\gamma^{3q}}{r_2^3} + \frac{\lambda I_2\,\gamma^{5q}}{2\mu r_2^5}\right\}\right] + \beta\,\xi\,\gamma^{\frac{n-q-1}{2k-q}}\left[\omega - 2\,q\left\{\omega - \frac{1}{\gamma\,m_0}\left(\frac{\vartheta\,\gamma^{3q}}{r_1^3} + \frac{\lambda\,\gamma^{3q}}{r_2^3} + \frac{\lambda\,I_2\,\gamma^{5q}}{2\mu r_2^5}\right)\right\}\right] = -\,\frac{\gamma^{2q-2k-1}}{m_0}\,\frac{\partial U}{\partial \eta} \qquad (6)$$

To eliminate the non-variational factor from equations (5) and (6) we assume

$2q-k-1 = 0,\; n-k-1 = 0,\; n = 1, k = 0,\; q = \frac{1}{2},\; \beta = \alpha.$

Thus we have

$$\xi'' - 2\,\eta'\left[\omega - \frac{\sqrt{\gamma}}{m_0}\left\{\frac{\vartheta}{r_1^3} + \frac{\lambda}{r_2^3} + \frac{\lambda\,I_2\gamma}{2\mu r_2^5}\right\}\right] = \frac{\beta^2\,\xi}{4} - \frac{\beta\,\eta\,\gamma^{\frac{3}{2}}}{m_0}\left(\frac{\vartheta}{r_1^3} + \frac{\lambda}{r_2^3} + \frac{\gamma\lambda\,I_2}{2\mu r_2^5}\right) - \frac{1}{m_0}\,\frac{\partial U}{\partial \xi} \qquad (7)$$

$$\eta'' + 2\,\xi'\left[\omega - \frac{\sqrt{\gamma}}{m_0}\left\{\frac{\vartheta}{r_1^3} + \frac{\lambda}{r_2^3} + \frac{\lambda\,I_2\gamma}{2\mu r_2^5}\right\}\right] = \frac{\beta^2\eta}{4} + \frac{\beta\,\xi\,\gamma^{\frac{3}{2}}}{m_0}\left(\frac{\vartheta}{r_1^3} + \frac{\lambda}{r_2^3} + \frac{\gamma\lambda\,I_2}{2\mu r_2^5}\right) - \frac{1}{m_0}\,\frac{\partial U}{\partial \eta} \qquad (8)$$

Where

$$U = -\frac{m_0\,\omega^2}{2}\,(\xi^2+\eta^2) - (\xi^2+\eta^2)\omega\left\{\frac{1}{r_1^3} + \frac{\lambda}{r_2^3}\right\}\gamma^{\frac{1}{2}} - \gamma\,\xi\omega\left\{\frac{\mu}{r_1^3} - \frac{\lambda(1-\mu)}{r_2^3}\right\} - \gamma^{\frac{3}{2}}\left(\frac{m_0\,(1-\mu)}{r_1} + \frac{m_0\,\mu}{r_2}\right) \qquad (9)$$

III. COMPLETE SYNCRONIZATION

Let
$\xi = \xi_1,\ \xi' = \xi_2,\ \eta = \xi_3,\ \eta' = \xi_4$
Then the equation (7) and (8) can be written as:

$$\xi_1' = \xi_2 \tag{10}$$
$$\xi_2' = 2\xi_4\,\omega + \xi_1\left(\frac{\beta^2}{4} - \omega^2\right) + A_1 \tag{11}$$
$$\xi_3' = \xi_4 \tag{12}$$
$$\xi_4' = -2\xi_2\omega + \xi_3\left(\frac{\beta^2}{4} - \omega^2\right) + A_2 \tag{13}$$

Where

$$A_1 = -\frac{1}{m_0}\left(\beta\,\xi_3\,\gamma^{\frac{3}{2}} + 2\omega\,\xi_1\sqrt{\gamma}\right)\left(\frac{\vartheta}{r_1^3} + \frac{\lambda}{r_2^3} + \frac{\gamma\lambda\,I_2}{2\square\,r_2^5}\right) + \frac{\omega\sqrt{\gamma}}{m_0}\left({\xi_1}^2 + {\xi_3}^2\right)\left\{\frac{3\,\vartheta\,(\xi_1 - \square\sqrt{\gamma})}{r_1^5} + \frac{3\,\lambda(\xi_1 + \sqrt{\gamma} - \square\sqrt{\gamma})}{r_2^5} +\right.$$

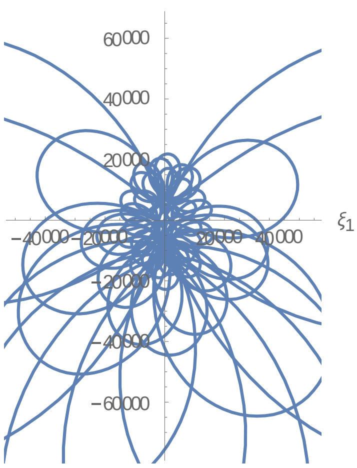
$5\,\lambda\,\gamma\xi1{+}\gamma{-}\square\gamma I22\square r27 - \gamma\omega\, m0\vartheta\square r13{-}\ \lambda 1{-}\mu r23{-}\ \lambda\,\gamma 1{-}\mu I22\square r25{+}\ \gamma\omega\,\xi 1\, m0$

$$\left\{\frac{3\,\square\,\vartheta(\xi_1 - \square\sqrt{\gamma})}{r_1^5} + \frac{3\,\lambda(1-\square)(\xi_1 + \sqrt{\gamma} - \square\sqrt{\gamma})}{r_2^5} + \frac{5\,\gamma I_2\,\lambda(1-\square)(\xi_1 + \sqrt{\gamma} - \square\sqrt{\gamma})}{2\square r_2^7}\right\} + \gamma^{\frac{3}{2}}\left\{\frac{(1-\square)(\xi_1 - \square\sqrt{\gamma})\vartheta}{r_1^3}\right\} +$$
$$+\gamma^{\frac{3}{2}}\left\{\frac{\square(\xi_1 + \sqrt{\gamma} - \square\sqrt{\gamma}))}{r_2^3} + \frac{3\,\gamma\,I_2\,(\xi_1 + \sqrt{\gamma} - \square\sqrt{\gamma}))}{2\,r_2^5}\right\} - \frac{2\,\xi_4\sqrt{\gamma}}{m_0}\left(\frac{\vartheta}{r_1^3} + \frac{\lambda}{r_2^3} + \frac{\gamma\lambda\,I_2}{2\square\,r_2^5}\right).$$

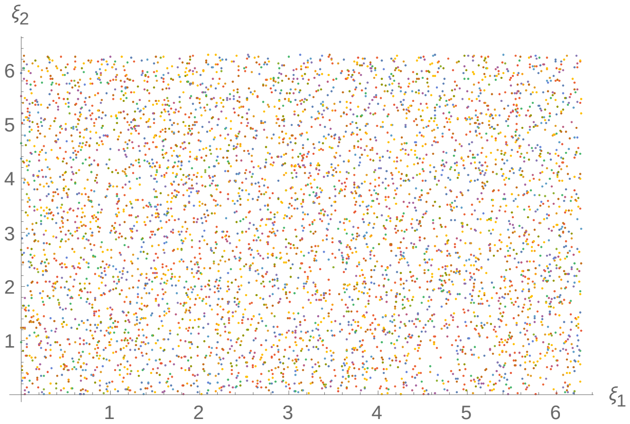
$$A_2 = \frac{1}{m_0}\left(\beta\,\xi_1\,\gamma^{\frac{3}{2}} + 2\omega\sqrt{\gamma}\xi_2 - 2\,\xi_3\sqrt{\gamma}\right)\left(\frac{\vartheta}{r_1^3} + \frac{\lambda}{r_2^3} + \frac{\gamma\lambda\,I_2}{2\square\,r_2^5}\right) + \frac{\omega\,\xi_3\sqrt{\gamma}}{m_0}\left({\xi_1}^2 + {\xi_3}^2\right)$$
$$\left\{\frac{3\ \vartheta}{r_1^5} + \frac{3\,\lambda}{r_2^5} + \frac{5\,\gamma I_2\,\lambda}{2\square r_2^7}\right\} + \frac{\gamma\,\omega\,\xi_1\xi_3}{m_0}\left\{\frac{3\,\square\,\vartheta}{r_1^5} - \frac{3\,\lambda(1-\square)}{r_2^5} - \frac{5\,\gamma I_2\,\lambda(1-\square)}{2\square r_2^7}\right\} +$$
$$+\gamma^{\frac{3}{2}}\,\xi_3\left\{\vartheta\,\frac{(1-\square)}{r_1^3} + \frac{\square}{r_2^3} + \frac{3\,\gamma\,I_2\,)}{2\,r_2^5}\right\}.$$

$r_1^2 = (\xi_1 - \square)^2 + {\xi_3}^2,\quad r_2^2 = (\xi_1 + 1 - \square)^2 + {\xi_3}^2.$

The system (10,11,12 and 13) is the master system. The state orbits and the surface of section of this system are shown in Figure (1) and Figure (2) respectively these figures shows that the system is chaotic.



Fig(1)



Fig(2)

Corresponding to master system (10,11,12 and 13), the identical slave system is defined as:

$$\zeta_1' = \zeta_2 + u_1 \tag{14}$$
$$\zeta_2' = 2\omega\zeta_4 + \zeta_1\left(\frac{\beta^2}{4} - \omega^2\right) + B_1 + u_2 \tag{15}$$
$$\zeta_3' = \zeta_4 + u_3 \tag{16}$$
$$\zeta_4' = -2\omega\zeta_2 + \zeta_3\left(\frac{\beta^2}{4} - \omega^2\right) + B_2 + u_4 \tag{17}$$

Where
Where

$$B_1 = -\frac{1}{m_0}\left(\beta\,\zeta_3\,\gamma^{\frac{3}{2}} + 2\omega\,\zeta_1\sqrt{\gamma}\right)\left(\frac{\vartheta}{r_{11}^3} + \frac{\lambda}{r_{12}^3} + \frac{\gamma\lambda\,I_2}{2\square\,r_{12}^5}\right) + \frac{\omega\sqrt{\gamma}}{m_0}\left({\zeta_1}^2 + {\zeta_3}^2\right)\left\{\frac{3\,\vartheta(\zeta_1 - \square\sqrt{\gamma})}{r_{11}^5} + \frac{3\,\lambda(\zeta_1 + \sqrt{\gamma} - \square\sqrt{\gamma})}{r_{12}^5} +\right.$$

$5\,\lambda\,\gamma\zeta 1{+}\gamma{-}\square\gamma I22\square r127 - \gamma\omega\, m0\vartheta\square r113{-}\ \lambda 1{-}\mu r123{-}\ \lambda\,\gamma 1{-}\mu I22\square r125{+}\ \gamma\,\zeta 1\, m0$

$$\left\{\frac{3\,\square\,\vartheta(\zeta_1-\square\sqrt{\gamma})}{\mathrm{r}_{11}^5}+\frac{3\,\lambda(1-\square)(\zeta_1+\sqrt{\gamma}-\square\sqrt{\gamma})}{\mathrm{r}_{12}^5}+\frac{5\,\gamma I_2\,\lambda(1-\square)(\zeta_1+\sqrt{\gamma}-\square\sqrt{\gamma})}{2\square\mathrm{r}_{12}^7}\right\}+\gamma^{\frac{3}{2}}\left\{\frac{(1-\square)(\zeta_1-\square\sqrt{\gamma})\vartheta}{\mathrm{r}_{11}^3}\right\}+$$
$$+\gamma^{\frac{3}{2}}\left\{\frac{\square(\zeta_1+\sqrt{\gamma}-\square\sqrt{\gamma}))}{\mathrm{r}_{12}^3}+\frac{3\,\gamma\,I_2\,(\zeta_1+\sqrt{\gamma}-\square\sqrt{\gamma}))}{2\,\mathrm{r}_{12}^5}\right\}-\frac{2\,\zeta_4\sqrt{\gamma}}{m_0}\left(\frac{\vartheta}{\mathrm{r}_{11}^3}+\frac{\lambda}{r_{12}^3}+\frac{\gamma\lambda\,I_2}{2\square\,r_{12}^5}\right).$$
$$B_2=\frac{1}{m_0}\left(\beta\,\zeta_1\,\gamma^{\frac{3}{2}}+2\omega\sqrt{\gamma}\zeta_2-2\,\zeta_3\,\sqrt{\gamma}\right)\left(\frac{\vartheta}{\mathrm{r}_{11}^3}+\frac{\lambda}{r_{12}^3}+\frac{\gamma\lambda\,I_2}{2\square\,r_{12}^5}\right)+\frac{\zeta_3\sqrt{\gamma}}{m_0}\left({\zeta_1}^2+{\zeta_3}^2\right)$$
$$\left\{\frac{3\,\vartheta}{\mathrm{r}_{11}^5}+\frac{3\,\lambda}{\mathrm{r}_{12}^5}+\frac{5\,\gamma I_2\,\lambda}{2\square\mathrm{r}_{12}^7}\right\}+\frac{\gamma\zeta_1\zeta_3}{m_0}\left\{\frac{3\,\square\,\vartheta}{\mathrm{r}_{11}^5}-\frac{3\,\lambda(1-\square)}{\mathrm{r}_{12}^5}-\frac{5\,\gamma I_2\,\lambda(1-\square)}{2\square\mathrm{r}_{12}^7}\right\}+$$
$$+\gamma^{\frac{3}{2}}\zeta_3\left\{\vartheta\,\frac{(1-\square)}{\mathrm{r}_{11}^3}+\frac{\square}{\mathrm{r}_{12}^3}+\frac{3\,\gamma\,I_2\,)}{2\,\mathrm{r}_{12}^5}\right\}.$$
$$r_{11}^2=(\zeta_1-\square)^2+{\zeta_3}^2,\quad r_{12}^2=(\zeta_1+1-\square)^2+{\zeta_3}^2.$$

where $u_i(t)$; i =1 ,2,3,4 are control functions to be determined. Let $e_i = \zeta_i - \xi_i$; i = 1, 2, 3, 4 be the synchronization errors. From (10) to (17), we obtain the error dynamics as follows:

$$e_1^{'}=e_2+u_1 \tag{18}$$
$$e_2^{'}=2\omega e_4+\left(\frac{\beta^2}{4}-\omega^2\right)e_1+B_1-A_1+u_2 \tag{19}$$
$$e_3^{'}=e_4+u_3 \tag{20}$$
$$e_4^{'}=-2\omega e_2+\left(\frac{\beta^2}{4}-\omega^2\right)e_3+B_2-A_2+u_4 \tag{21}$$

Lyapunov stability theory state that when controller satisfies the assumption with $V(e)=\frac{1}{2}\,e^t\,e$ a positive definite function and the first derivative of this function $V^{'}<0$ the chaos synchronization of two identical systems (master and slave) for different initial conditions is achieved.

The first derivative of $V(e)$ Will be

$$V^{'}=e_1(e_2+u_1)+e_2\left\{2\omega e_4+\left(\frac{\beta^2}{4}-\omega^2\right)e_1+B_1-A_1+u_2\right\}+e_3(e_4+u_3)+$$
$$+e_4\left\{-2\omega e_2+\left(\frac{\beta^2}{4}-\omega^2\right)e_3+B_2-A_2+u_4\right\}$$

Therefore, if we choose the controller u as follows,

$$u_1=-e_1-\frac{\beta^2}{4}\,e_2-e_2+\omega^2 e_2 \tag{22}$$
$$u_2=-e_2-B_1+A_1-2\omega e_4 \tag{23}$$
$$u_3=-e_3-e_4 \tag{24}$$
$$u_4=2\omega e_2-e_4-B_2+A_2-\frac{\beta^2}{4}\,e_3+\omega^2 e_3 \tag{25}$$

Then

$$V^{'}=-{e_1}^2-{e_2}^2-{e_3}^2-{e_4}^2<0 \tag{26}$$

Hence the error state

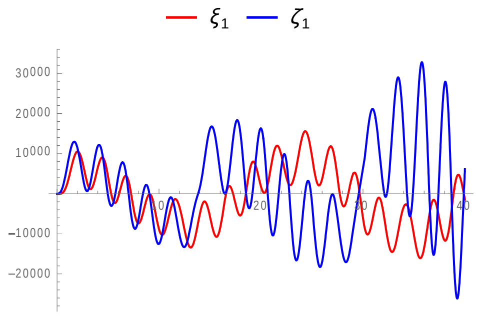
$$\lim_{t\to\infty}\|e(t)\|=0$$

which gives asymptotic stability of the system. This means that the controlled chaotic systems (master and slave) are synchronized for deferent initial conditions.

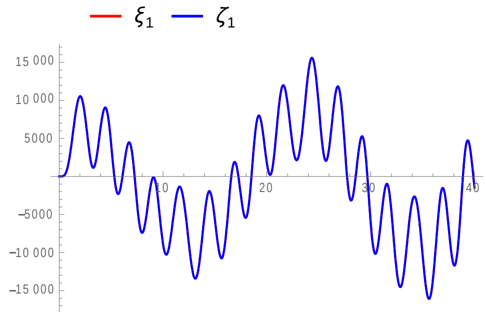
III. NUMERICAL SIMULATION

We select the parameters μ =.1, γ =.45, β =.1 and λ=1, with the different initial conditions for master and slave systems. Simulation results for uncoupled system are presented in figures. 3,5,7 and 9 and that of controlled system are shown in figures.4,6,8 and 10 respectively..

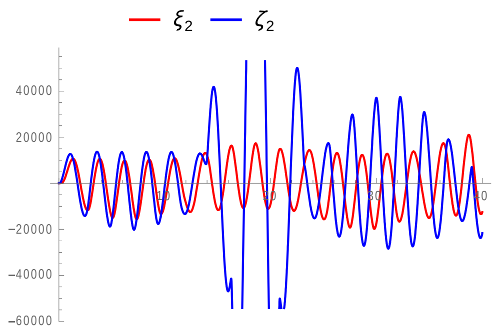
It can be seen from the figures that the chaos-synchronization errors converge to zero rapidly.



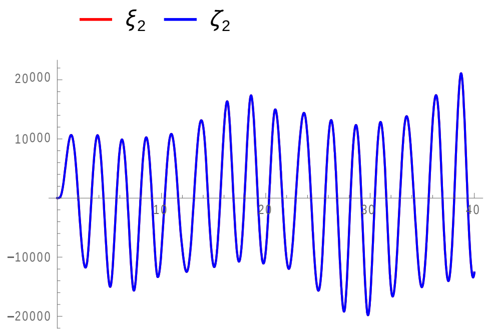
Fig(3)



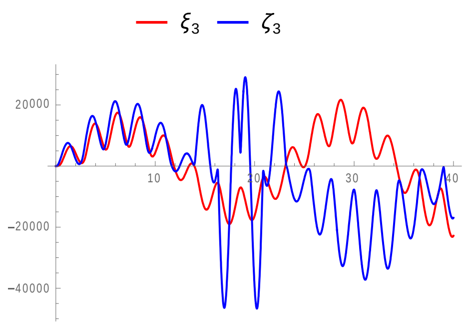
Fig(4)



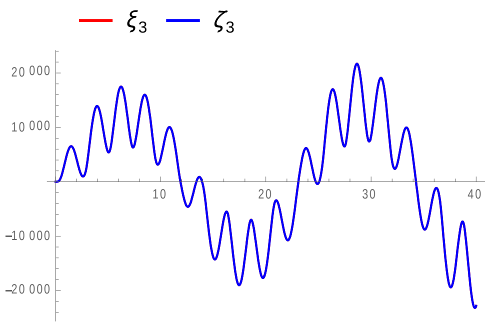
Fig(5)



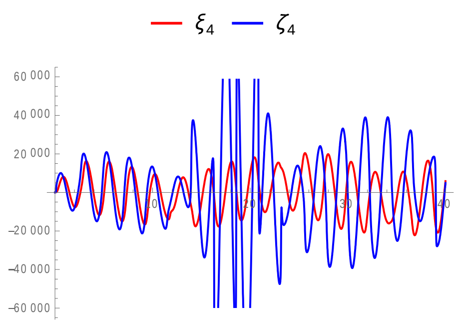
Fig(6)



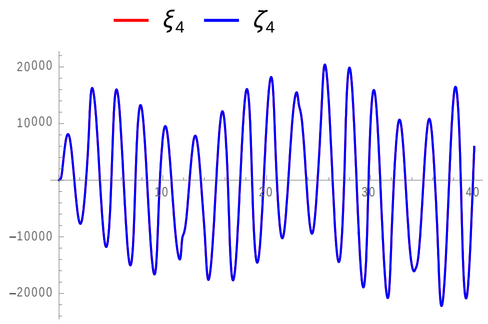
Fig(7)



Fig(8)



Fig(9)



Fig(10)

IV. ANTI SYNCRONIZATION

To observe anti-synchronization between the master and the slave system, let $E_i = \zeta_i + \xi_i$; i = 1, 2, 3, 4 be the synchronization errors. Now from (10) to (17), we obtain the error dynamics as.

$$E_1' = E_2 + u_{11} \tag{27}$$
$$E_2' = 2\omega E_4 + \left(\frac{\beta^2}{4} - \omega^2\right) E_1 + B_1 + A_1 + u_{12} \tag{28}$$
$$E_3' = E_4 + u_{13} \tag{29}$$
$$E_4' = -2\omega E_2 + \left(\frac{\beta^2}{4} - \omega^2\right) E_3 + B_2 + A_2 + u_{14} \tag{30}$$

Now the first derivative of $V(e)$ Will be

$$V' = E_1(E_2 + u_{11}) + E_2\left\{2\omega E_4 + \left(\frac{\beta^2}{4} - \omega^2\right) E_1 + B_1 + A_1 + u_{12}\right\} + E_3(E_4 + u_{13}) +$$
$$+E_4\left\{-2\omega E_2 + \left(\frac{\beta^2}{4} - \omega^2\right) E_3 + B_2 + A_2 + u_{14}\right\}$$

Therefore, if we choose the controller u as follows,

$$u_{11} = -E_1 - \frac{\beta^2}{4} E_2 - E_2 + \omega^2 E_2 \tag{31}$$
$$u_{12} = -E_2 - B_1 - A_1 - 2\omega E_4 \tag{32}$$
$$u_{13} = -E_3 - E_4 \tag{33}$$
$$u_{14} = 2\omega E_2 - E_4 - B_2 - A_2 - \frac{\beta^2}{4} E_3 + \omega^2 E_3 \tag{34}$$

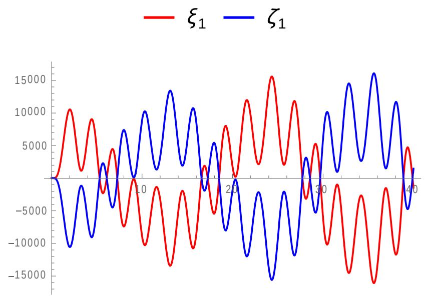
Then
$$V' = -{E_1}^2 - {E_2}^2 - {E_3}^2 - {E_4}^2 < 0 \tag{35}$$

Hence the error state
$$\lim_{t\to\infty} \|E(t)\| = 0$$

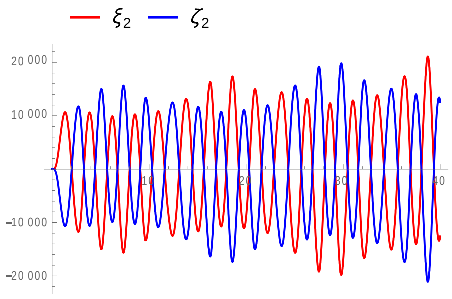
which gives asymptotic stability of the system. This means that the controlled chaotic systems (master and slave) are Anti synchronized for deferent initial conditions.

V. NUMERICAL SIMULATION

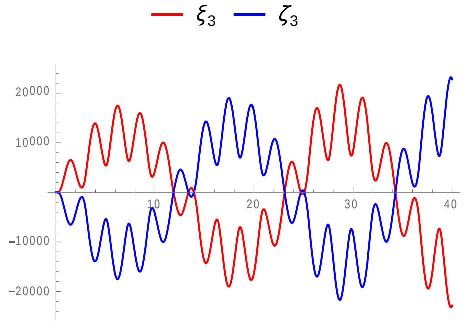
We select the parameters μ =.1, γ =.45, β =.1 and λ=1, with the different initial conditions for master and slave systems and anti synchronization is achieved between the master and slave systems. Time Series Analysis graphs of the above are shown next to each via figures 11 to 14.



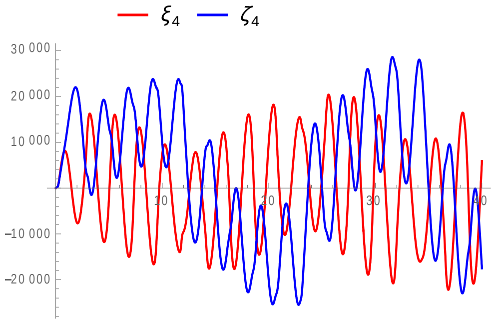
Fig(11)



Fig(12)



Fig(13)



Fig(14)

VI. CONCLUSION

An investigation on complete synchronization and anti synchronization behavior of the photogravitational magmatic binary problem by taking into consideration the bigger primary is a source of radiation and smaller primary is a oblate body when the charged particle has the variable mass via non linear controller based on the Lyapunov stability theory have been made. Here two systems (master and slave) are compete synchronized and start with deferent initial conditions. This problem may be treated as the design of control laws for chaotic slave system using the known information of the master system so as to ensure that the controlled receiver synchronizes with master system. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time. The results were validated by numerical simulations using Mathematica.

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