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A Homogenization Method to Analyze and Simulate the Behaviors of the Folded Core Sandwich Panels in Case of In-plane Shear Forces

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ABSTRACT
Sandwich panels with many outstanding features have widely used in many fields especially in engineering. To use them effectively, we need to know the mechanical behaviors of these sandwich panels. This paper presents a homogenization method to determine the mechanical behaviors of folded core sandwich panels in in-plane shear loading. This method allows us to build an equivalent model (2D) for original model (3D) which helps to significantly reduce the computational time as well as time to build the geometry of model. A very good agreement is obtained between the 3D shell simulations and our homogenization model for in-plane shear problem verifying the accuracy and efficiency of our model. We can also use, of course, this method to build equivalent models for any type of folded core as well as many types of sandwich panels.

***Keywords* -** Folded core, homogenization, in-plane shear, sandwich panels

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I. INTRODUCTION

Nowadays, many types of sandwich structures are widely used in many fields especially in engineering because they have high strength to weight ratio and many outstanding features. A sandwich panel is composed of two different materials called the skins and the core. The first one is thin and has a high elastic module and strength, the second material has a quite important thickness but very low density, therefore the inertial moment of structure will increase hence the rigidity of the structure too. The variety of core geometries are used for sandwich panels including honeycomb, solid, foam, folded, truss, web cores… in which folded core with many different types were widely used. A correct use of these materials in different applications requires a better knowledge of their mechanical behavior. There have been many research studies interested in sandwich panels and many research methods were suggested in which finite element methods are regarded as being a most powerful and versatile tool for structural analysis [1]. However, its application is often restricted by demanding a large computer and long computational time as well as time to build the geometry of models.

The sandwich plate is a multilayer plate, so theory of multilayer plate can be used to calculate for this sandwich plate. Laminated plate theory is valid only in the case of continuous environment, the deformation is assumed linearly with thickness z. Homogenization is a method often used to build equivalent models that we can easily to obtain the overall stiffness [2, 3, 4]. In the case of folded core sandwich plate with many cavities, laminated plate theory should be adjusted.

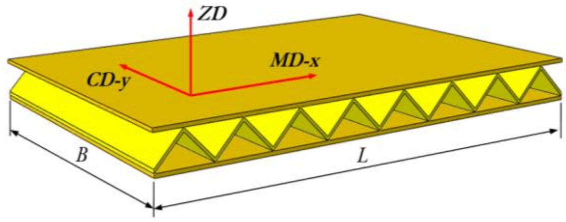

Fig. 1. The model of sandwich plate with folded core

The folded core sandwich plate is produced by a manufacturing process in which three or more layers are laminated together. The flat layers are called liners and the folded cores are referred to as flutes (Fig. 1). Folded core sandwich is one of the most used materials to make partitions or roof skin construction or automobile. The manufacturing process gives three characteristic directions: The machine direction (MD), the cross direction (CD), and the thickness direction (ZD).

This paper presents an efficient homogenization model for the mechanical behavior of a folded core sandwich composed of three layers (single flutes). The homogenization is carried out by calculating analytically the global rigidities of the folded core sandwich and then this 3D structure is replaced by an equivalent homogenized 2D (H-2D) plate. The simulations in case of in-plane shear force of Abaqus-3D and H-2D model of folded core sandwich will be studied in this article. This H-2D model is very fast and has close results comparing to the 3D model using the Abaqus shell elements.

II. RECALL OF THEORY OF LAMINATED PLATES

For a composite plate, the Mindlin theory is often used. It assumed that a right segment and perpendicular to the mean surface remains straight but not perpendicular to the medium surface after deformation. This assumption allows to consider the transverse shear deformations. The membrane forces, bending and torsion moments and transverse shear forces are obtained by integration of the constraints on the thickness.

If we consider a composite panel consisting of several layers, the resulting forces may be combined in layers. In the theory of laminated, after the integration along the thickness (t), we obtain the overall stiffness matrix that links the generalized deformations with resultant forces [5]:

$$\begin{bmatrix} \{N\} \\ \{M\} \\ \{T\} \end{bmatrix} = \begin{bmatrix} [A] & [B] & [0] \\ [B] & [D] & [0] \\ [0] & [0] & [F] \end{bmatrix} \begin{bmatrix} \{\varepsilon_m\} \\ \{\kappa\} \\ \{\gamma_s\} \end{bmatrix} \quad (1)$$

where $\{N\}$, $\{T\}$ and $\{M\}$ are the internal forces and moments; $[A]$, $[D]$, $[B]$ and $[F]$ are the stiffness matrices related to the membrane forces, the bending-torsion moments, the bending-torsion-membrane coupling effects and the transverse shear forces respectively; $\{\varepsilon_m\}$ is the membrane strain vector, $\{\kappa\}$ is the curvature vector and $\{\gamma_s\}$ is the transverse shear strain vector.

The components of the stiffness matrices above are defined by:

$$A_{ij} = \sum_{k=1}^{n}\left[h^k - h^{k-1}\right] Q_{ij}^k = \sum_{k=1}^{n} Q_{ij}^k t^k$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n}\left[\left(h^k\right)^2 - \left(h^{k-1}\right)^2\right] Q_{ij}^k = \sum_{k=1}^{n} Q_{ij}^k t^k z^k$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n}\left[\left(h^k\right)^3 - \left(h^{k-1}\right)^3\right] Q_{ij}^k = \quad (2)$$

$$= \sum_{k=1}^{n} Q_{ij}^k \left[t^k \left(z^k\right)^2 + \frac{\left(t^k\right)^3}{12} \right]$$

$$F_{ij} = \sum_{k=1}^{n}\left[h^k - h^{k-1}\right] C_{ij}^k = \sum_{k=1}^{n} C_{ij}^k t^k$$

with

$$[Q] = \begin{bmatrix} \dfrac{E_x}{1-\nu_{xy}\nu_{yx}} & \dfrac{\nu_{xy}E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ \dfrac{\nu_{yx}E_x}{1-\nu_{xy}\nu_{yx}} & \dfrac{E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \quad (3)$$

where $\dfrac{\nu_{xy}}{E_x} = \dfrac{\nu_{yx}}{E_y}$ (4)

$$[C] = \begin{bmatrix} G_{xz} & 0 \\ 0 & G_{yz} \end{bmatrix} \quad (5)$$

III. HOMOGENIZATION MODEL FOR FOLDED CORE SANDWICH

The sandwiches with folded cores are made of multilayer including face-sheets and cores that have fluting cores and the cavities between layers. To apply the calculation method of the theory of plates, the matrix (1) obtained by the theory of laminated plates should be modified [6, 7]. The geometry of folded core plate is shown in Fig. 2.

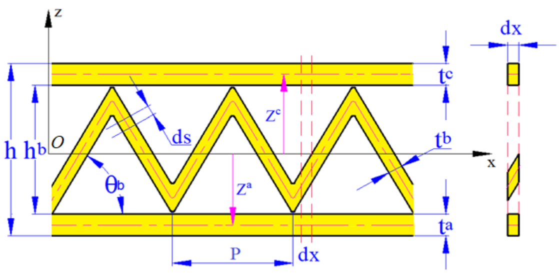

Fig. 2. Geometry of folded core plates

To homogenize a folded core sandwich plate, we consider a volumetric element represents (VER). That volumetric element must be small enough comparing to the size of the entire plate. According to the structure of the plate, we take out a period length of the core as a VER. We calculate the average mechanical properties of VER and use them to model this 3D structure by a homogeneous 2D plate. However, the folded core has a vertical position varies with x direction so we cut it into very little vertical slices (thickness dx) and integrate along the thickness (or sum up the participants of three layers) on each slice.

Noting that the mechanical properties of the core achieved by experiments are valid only in its face. Thus, we need to calculate the local coordinate system. Once the overall stiffness of each slice obtained by integrating over the thickness of the plate, homogenization along x will be performed to calculate the average stiffness of all slices in one period [7]:

$$[A]=\frac{1}{P}\int_0^P [A(x)]\,dx\ ;\qquad [B]=\frac{1}{P}\int_0^P [B(x)]\,dx;\qquad [D]=\frac{1}{P}\int_0^P [D(x)]\,dx;\qquad [F]=\frac{1}{P}\int_0^P [F(x)]\,dx \tag{6}$$

Considering a folded core sandwich panel and using *a*, *b*, and *c* to represent the lower liner, the folded core and the upper liner (Fig. 2). The rigidities of plate related to traction, bending and in-plane shear is defined as follow:

3.1. Traction and bending stiffnesses related to N_x, M_x, N_y, M_y

The vertical position (z) of a groove portion (ds) is a function of x and a thickness over its vertical section is a function of the angle of inclination of the groove. Equation (2) can be write:

$$\begin{aligned}
A_{ij} &= Q_{ij}^a t^a + Q_{ij}^b \frac{t^b}{\cos\theta^b} + Q_{ij}^c t^c \\
B_{ij} &= Q_{ij}^a t^a z^a + Q_{ij}^b \frac{t^b}{\cos\theta^b} z^b + Q_{ij}^c t^c z^c \\
D_{ij} &= Q_{ij}^a \left[t^a \left(z^a\right)^2 + \frac{1}{12}\left(t^a\right)^3 \right] + \\
&+ Q_{ij}^b \left[\frac{t^b}{\cos\theta^b}\left(z^b\right)^2 + \frac{1}{12}\left(\frac{t^b}{\cos\theta^b}\right)^3 \right] + \\
&+ Q_{ij}^c \left[t^c \left(z^c\right)^2 + \frac{1}{12}\left(t^c\right)^3 \right]
\end{aligned} \tag{7}$$

where

$$h = t^a + h^b + t^c$$

$$z^a = -\frac{h}{2} + \frac{t^a}{2};\quad z^c = \frac{h}{2} - \frac{t^c}{2}$$

$$z^b(x) = -\frac{h}{2} + t^a + \frac{2h}{P}x$$

$$\cos\theta^b = \frac{P}{2l^b}\quad ;\quad l^b = \sqrt{\left(h^b\right)^2 + \frac{P^2}{4}}$$

3.2. Shear stiffnesses in plane related to N_{xy}, N_{yx}

In a laminated composite plate, the integration through the thickness is used to calculate shear stiffness in the plane. This process consists in summing the product of the shear modulus and the thickness of all layers. However, it is no longer valid for folded core panel because of the cavities.

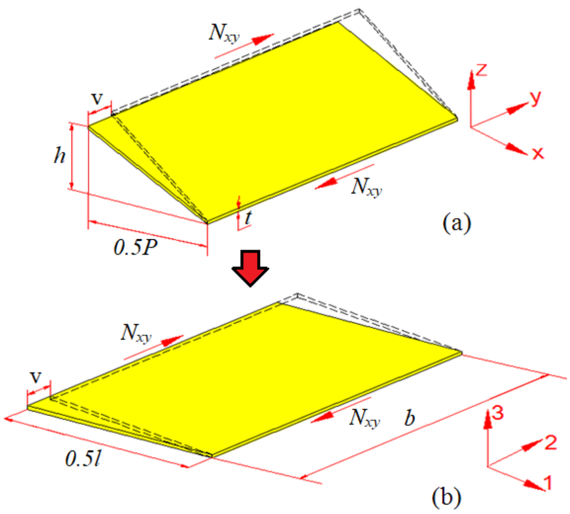

Fig. 3. Equivalent model of the folded core in xy plane shear force

Considering the folded core of a panel with length P/2 (along x) and width b (along y) (Fig. 3). A pair of shear forces per unit of width N_{xy} (along y) applied to the section MD gives a displacement v. The shearing of the folded core can be easily processed by calculating in the plane of the folded core (Fig. 3b). In this model, we have:

$$\tau_{12} = G_{12}\gamma_{12} \quad \Rightarrow \quad \frac{N_{xy}}{t} = G_{12}\frac{\mathrm{v}}{0.5l} \tag{8}$$

In which, G_{12} is the shear module in the plane of folded core; l is the length of the folded core. Shear deformation (in the plane of panel) can be obtained by:

$$\gamma_{xy} = \frac{\mathrm{v}}{0.5P} \tag{9}$$

thus we have:

$$N_{xy} = \frac{G_{12}Pt}{l}\gamma_{xy} \tag{10}$$

It can be shown that the average of the shear force on the CD section is equal to the shear force (constant) on the MD section. In fact, according to the reciprocity theorem, the flux of shear stress along the folded core on CD is equal to that on MD ($\tau_{yx} = \tau_{xy} = N_{xy}/t = const$); the component force along x of this flux gives the shear force N_{yx}:

$$\begin{aligned} N_{yx} &= \frac{1}{0.5P}\int_0^{0.5P} \tau_{yx}\ t\ \cos\theta\ ds = \\ &= \frac{1}{0.5P}\int_0^{0.5P} N_{xy}\ dx = N_{xy} \end{aligned} \tag{11}$$

So the relation $N_{yx} = N_{xy}$ on MD and CD is proved and the shear stiffness is unique even if the two sections are very different.

For a folded core sandwich panel, the shear stiffness in the plane of the panel is given by the sum of the rigidities of the 3 layers:

$$N_{xy} = A_{33}\ \gamma_{xy};\ \ A_{33} = G_{xy}^a t^a + \frac{G_{12}^b P^b t^b}{l^b} + G_{xy}^c t^c \tag{12}$$

IV. VALIDATION OF HOMOGENZATION MODEL

To validate our homogenization model (H-Model), a folded core sandwich panel with the dimension L =160 mm and B = 150 mm is used. We first discretize the three layers of folded core plate by shell elements S4R of Abaqus to obtain the Abaqus-3D model. Then, we discretize the middle surface of folded core plate by shell elements S4R of Abaqus combined with our H-Model (using "user's subroutine UGENS" [7]) to obtain H-2D model. The comparison of the results allows us to evaluate the efficiency and accuracy of our homogenization model.

The calculations and comparisons are made on a folded core sandwich panel having CD section illustrated in Fig. 2, the geometrical parameters are: t^a = 0.2 mm, t^b = 0.15 mm, t^c = 0.2 mm, h = 4 mm and P = 8 mm. The properties of material are given in the Table 1. The rigidities of 2D equivalent plate are calculated as shown in Table 2.

Table 1. The material properties of three layers formed folded core plate

Layers	E_{11} (MPa)	E_{22} (MPa)	G_{12} (MPa)	ν_{12}
a	2372.6	704.2	493.1	0.377
b	1094.7	856.4	165.9	0.421
c	2372.6	704.2	493.1	0.377

Table 2. Rigidities of the 2D equivalent plate

Rigidities	A_{11} (N/mm)	A_{12} (N/mm)	A_{22} (N/mm)	A_{33} (N/mm)
Values	990.8	110.9	475.8	214.8

Table 3. Comparison between Abaqus-3D and H-2D model under shear force

Force	Direction		Abaqus-3D	H-2D Model	Error (%)
F = 200N	MD	Displacement U_1 (mm)	6.055	5.952	-1.7
		CPU time (s)	5.1	1.4	3.6 times
	CD	Displacement U_2 (mm)	5.352	5.474	+2.27
		CPU time (s)	6.6	1.7	3.8 times

The results and compassions are shown in Fig.4, Fig.5 and Table 3. In both types of simulations (Abaqus-3D model and H-2D model), a rigid plate is bonded to the MD or CD section at the right end of the sandwich panel to better apply forces or moments (Fig. 4, 5). The calculations by our H-2D model are very fast while calculations by Abaqus-3D are longer especially for large size panels. The comparisons of results obtained by the two models and the percentages of error in H-2D model compared to Abaqus-3D results for the in-plane shear are presented in Table 3. We note that the numerical results given by the two models are very close.

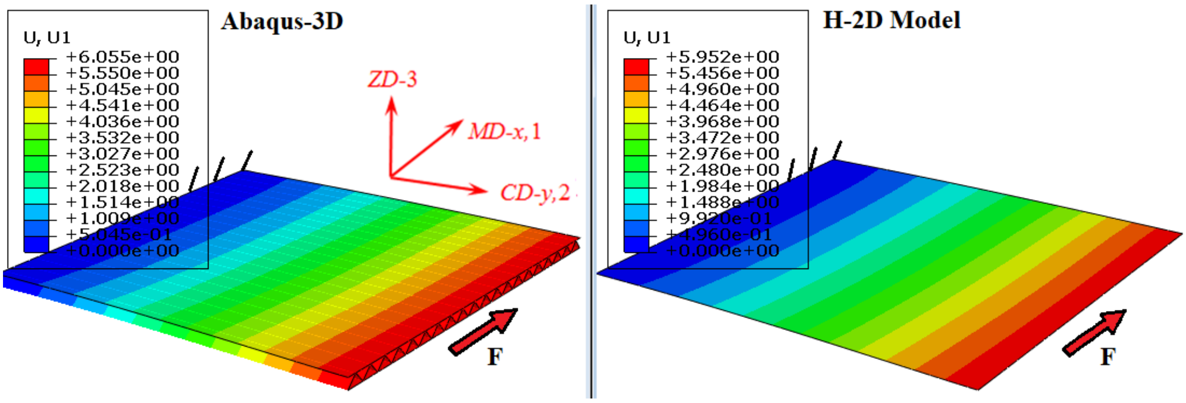

Fig.4. Simulation of Abaqus-3D and H-2D Model in MD shear force direction

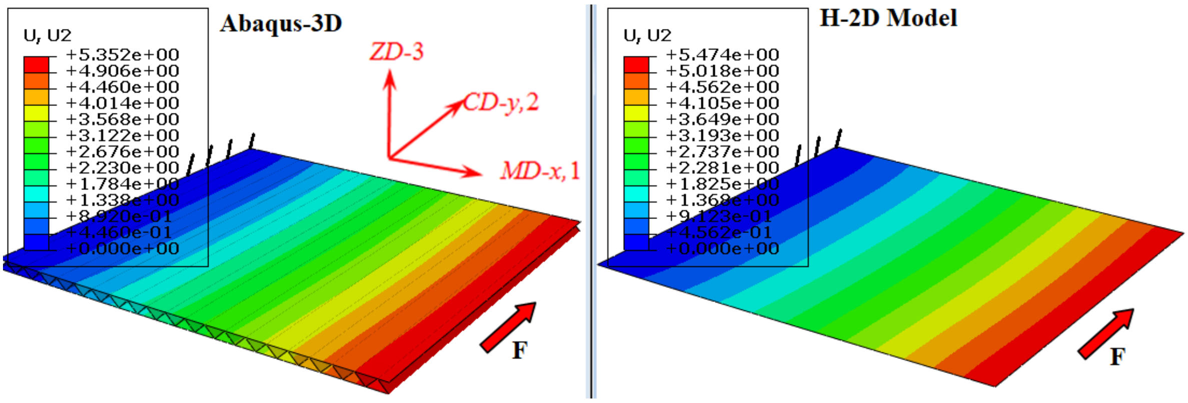

Fig.5. Simulation of Abaqus-3D and H-2D Model in CD shear force direction

V. CONCLUSIONS

In this paper, we have proposed an analytic homogenization model for the in-plane shear problem of a folded core sandwich panel. The comparison of the results obtained by the Abaqus 3D simulations and the Abaqus–Ugens H-2D simulations have proved the validation of the present homogenization model for in-plane shear problems. The present H-model allows us to largely reduce not only the time for the geometry creation and FEM calculation, but also the computational hardware requirements for the large sandwich panels. This homogenization model can be used not only for corrugated cardboard plates, but also for naval and aeronautic composite structures.

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