RESEARCH ARTICLE OPEN ACCESS

Multi – Objective Two Stage Fuzzy Transportation Problem with Hexagonal Fuzzy Numbers Using Fuzzy Geometric Programming

Dr. M. S. Annie Christi[1], Ms.R Sumitha Devi[2]
[1]*Associate Professor, Department of Mathematics,Providence College for Women, Coonoor, India.*
[2]*Department of Mathematics, Providence College for Women, Coonoor, India.*

ABSTRACT
Fuzzy geometric programming approach is used to determine the optimal solution of a multi-objective two stage fuzzy transportation problem in which supplies, demands are hexagonal fuzzy numbers and fuzzy membership of the objective function is defined. This paper aims to find out the best compromise solution among the set of feasible solutions for the multi-objective two stage transportation problem. To illustrate the proposed method, example is used.
Keywords: Transportation problem, Hexagonal fuzzy numbers, two stage fuzzy transportation problem, Multi-objective.

I. INTRODUCTION

Transportation models provide a powerful framework to meet this challenge. They ensure the efficient movement and timely availability of raw materials and finished goods. Transportation problem is a linear programming problem stemmed from a network structure consisting of a finite number of nodes and arcs attached to them. In a typical problem a production is to be transported from m sources to n destinations and their capacities are a_1 , a_2 ,...a_m and b_1,b_2 ...b_n , respectively. In addition there is a penalty C_{ij} associated with transporting unit of production from source i to destination j. This penalty may be cost or delivery time or safety of delivery etc. A variable X_{ij} represents the unknown quantity to be shipped from source *i* to destination *j*. In general the real life problems are modeled with multi-objectives, which are measured in different scales and at the same time in conflict. In some circumstances due to storage constraints designations are unable to receive the quantity in excess of their minimum demand. After consuming parts of whole of this initial shipment they are prepared to receive the excess quantity in the second stage. According to Sonia and Rita Malhotra **[23]** in such situations the product transported to the destination has two stages. Just enough of the product is shipped in stage I so that the minimum requirements of the destinations are satisfied and having done this the surplus quantities (if any) at the sources is shipped to the destinations according to cost consideration. In both the stages the transportation of the product from sources to the destination is done in parallel. Efficient algorithms **[21]** have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. However, there are cases that these parameters may not be presented in a precise manner. For example, the unit shipping cost may vary in a time frame. The supplies and demands may be uncertain due to some uncontrollable factors.

To deal quantitatively with imprecise information in making decisions, Bellman and Zadeh **[2]** and Zadeh **[28]** introduce the notion of fuzziness. Since the transportation problem is essentially a linear program, one straightforward idea is to apply the existing fuzzy linear programming techniques **[4, 5, 9, 10, 15, 19, 24]** to the fuzzy transportation problem. Unfortunately, most of the existing techniques **[4, 5, 9, 10, 24]** only provide crisp solutions. The method of Julien **[10]** and Parra et al. **[19]** is able to find the possibility distribution of the objective value provided all the inequality constraints are of ‘‘≤’’ type or ‘‘≥’’ type. However, due to the structure of the transportation problem, in some cases their method requires the refinement of the problem parameters to be able to derive the bounds of the objective value. There are also studies discussing the fuzzy transportation problem **[14].** Chanas et al. **[7]** investigate the transportation problem with fuzzy supplies and demands and solve them via the parametric programming technique in terms of the Bellman–Zadeh **[2]** criterion. Their method is to derive the solution which simultaneously satisfies the constraints and the goal to a maximal degree. Chanas and Kuchta **[6]** discuss the type of transportation problems with fuzzy cost coefficients and transform the problem to a bi-criterial transportation problem with crisp objective function. Their method is able to determine the efficient solutions of the transformed problem; nevertheless, only crisp solutions are provided. Verma et al. **[25]** apply the fuzzy programming technique with hyperbolic and exponential membership functions to solve a multi-objective transportation problem **[26],** the solution derived is a compromise solution. Similar to the method of Chanas and Kuchta **[6]**, only crisp solutions are provided. Obviously, when the cost coefficients or the supply and demand quantities are fuzzy numbers, the total transportation cost will be fuzzy as well.

In this paper two stage fuzzy transportation problems **[17]** is discussed with multi- objective constraints where the supply and demand is hexagonal fuzzy numbers. This paper aims to find out the best compromise solution among the set of feasible solutions for the multi-objective two stage transportation problem. Finally, some conclusions are drawn from the discussions. A numerical illustration is given to check the validity of the proposed method.

II. PRELIMINARIES

2.1. Definition: Fuzzy Number:

A fuzzy number **[31]** $\tilde{A}$ is a convex normalized fuzzy set on the real line R such that there exists at least one x∈R with $\mu_{\tilde{A}}(x) = 1$, where $\mu_{\tilde{A}}(x) = 1$ is piecewise continuous.

2.2. Definition: Triangular Fuzzy Number:

A fuzzy number $\tilde{A}$ is a TFN **[11]** denoted by $\tilde{A} = (a_1, a_2, a_3)$ where a_1, a_2 *and* a_3 real numbers and its membership function are given below:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \le x \le a_2 \\ 1, & \text{for } x = a_2 \\ \frac{a_3-x}{a_3-a_2}, & \text{for } a_1 \le x \le a_2 \\ 0, & \text{otherwise} \end{cases}$$

2.3. Definition: Trapezoidal Fuzzy Number:

A fuzzy number $\tilde{A}$ is a TrFN **[1]** denoted by $\tilde{A} = (a_1, a_2, a_3, a_4)$ where a_1, a_2, a_3 *and* a_4 real numbers and its membership function are given below:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \le x \le a_2 \\ 1, & \text{for } a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}, & \text{for } a_3 \le x \le a_4 \\ 0, & \text{otherwise} \end{cases}$$

2.4. Definition: Hexagonal Fuzzy Number:

A fuzzy number $\widetilde{A_H}$ is a HFN **[20]** denoted by $\widetilde{A_H} = (a_1, a_2, a_3, a_4, a_5, a_6)$ where $a_1, a_2, a_3, a_4, a_5, a_6$ real numbers and its membership function are given below:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & \text{for } a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & \text{for } a_2 \le x \le a_3 \\ 1, & \text{for } a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & \text{for } a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right), & \text{for } a_5 \le x \le a_6 \\ 0, & \text{otherwise} \end{cases}$$

2.5. Definition: Arithmetic operations on Hexagonal Fuzzy Number:

If $\widetilde{A_H} = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\widetilde{B_H} = (b_1, b_2, b_3, b_4, b_5, b_6)$ are two HFN's then the following three operations can be performed as follows:

- Addition:
  $\widetilde{A_H} + \widetilde{B_H} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6)$
- Subtraction:
  $\widetilde{A_H} - \widetilde{B_H} = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5, a_6 - b_6)$
- Multiplication:
  $\widetilde{A_H} * \widetilde{B_H} = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5, a_6 * b_6)$

2.6. Definition: Robust's Ranking Techniques:

Robust's ranking technique **[16]** which satisfy compensation, linearity and additive properties and provides results which are consistent with human intuition. If ã is a fuzzy number then the Robust's ranking index is defined by R (ā) $= \int_0^1 0.5\,(a_\alpha^L\ , a_\alpha^U)\, d\alpha$ where (a_α^L, a_α^U) is the α cut of a fuzzy number ã.

Where (a_α^L, a_α^U) = ((b-a) α+a, d-(d-c)α, (d-c) α+c, f-(f-e) α).

2.7. Definition: Compromise solution:

A feasible Vector **[12]** X^*∈S is called a **compromise solution** of P_1 iff x^*∈ E and $F(X^*) \le \wedge_{x \in s} F(X)$ where ^ stands for 'minimum' and E is the set of feasible solutions.

III. FUZZY PROGRAMMING APPROACH FOR SOLVING

MULTI-OBJECTIVE TWO STAGE FUZZY

Transportation Problem (MOTSFTP): [22]

The minimum fuzzy requirement of a homogeneous product at the Destination j is denoted by $\tilde{b_j}$ and the fuzzy availability of the same at source i is denoted by $\tilde{a_i}$. Let F^K(x)={F^1(x),F^2(x),……F^k(x)} be a vector of K objective functions and the superscript on both F^k(x) and cij^k are used to identify the number of objective functions k=1,2,3, k. Assume $a_i > 0\ \forall$ i, $b_j > 0\ \forall$ j, $cij^k >=0\ \forall$ i,j and $\sum_i a_i = \sum_j b_j$. In stage-I the Multi –objective Two-stage fuzzy Cost Minimization Transportation Problem deals with supplying the destinations their minimum requirements and in stage-II the quantity $\sum_i \bar{a_i} = \sum_j \bar{b_j}$ is supplied to the destinations from the sources which have surplus quantity left after the completion of stage-I.

The stage-I problem can be formulated as below:

Min F^k (x) = $\min_{X\in S_1}[\max_{|X|}(C_{ij}^k\,(X_{ij}))]$ (1)

Where the set S_1 is given by

$$S_1 = \begin{cases} \sum_{j=1}^{n} x_{ij} \le \bar{a_i} & i = 1,2\ldots\ldots m \\ \sum_{i=1}^{m} x_{ij} = \bar{b_j} & j = 1,2\ldots\ldots n \end{cases}$$

$x_{ij} \ge 0, \forall$ (i, j) , corresponding to a feasible solution X = (x_{ij}) of the stage-I problem, the set S_2 = { $\bar{X}$ = (x_{ij})} of feasible solution of the stage-II problem is given by

$$S_2 = \begin{cases} \sum_{j=1}^{n} x_{ij} \le \bar{a_i} & i = 1,2\ldots\ldots m \\ \sum_{i=1}^{m} x_{ij} \ge \bar{b_j} & j = 1,2\ldots\ldots n \end{cases}$$

$x_{ij} \ge 0, \forall$ (*i j*) , where $\bar{a_i}$ is the quantity available at the i^{th} source on completion so the stage-I, that is $\bar{a_i} = \bar{a_i} - \sum_j X_{ij}$. Clearly

$\sum_i \overline{a_i} = \sum_i \overline{a_i} - \sum_j \bar{b_j}$.

Thus the state-II problem would be mathematically formulated as:

min F^k (x) = $\min_{X\in S_2}[\max_{|X|}(C_{ij}^k\,(X_{ij}))]$ (2)

The feasible solution X =(X $_{ij}$) *of* the stage-I problem corresponding to which the optimal cost for stage-II is such that the sum of the shipment is the least. The Multi-objective two stage fuzzy cost minimizing transportation problem **[8]** can, therefore, be stated as,

min F^k (x) = $\min_{X\in S_1}[C_1^K(x) + [\min_{X\in S_2} C_2^K X]$ (3)

Also from a feasible solution of the problem (3) can be obtained. Further the problem (3) can be solved by solving following fuzzy cost minimizing Transportation problem

P1: min F^k (x) = $\min_{X\in S_2}[\max_{|X|}[C_{ij}^k\,[X_{ij}]]]$ (4)

where S_2, the set of feasible solutions of (3), is defined as follows

$$S_2 = \begin{cases} \sum_{j=1}^{n} x_{ij} = \tilde{a_i} & i = 1,2\ldots\ldots m \\ \sum_{i=1}^{m} x_{ij} = \tilde{b_j} & j = 1,2\ldots\ldots n \end{cases}$$

$X_{ij} \ge 0\forall$ (i, j)

where $\tilde{a_i}$, and $\tilde{b_j}$, represent fuzzy parameters involved in the constraints with their membership functions for $\mu_{\tilde{a}}$ a certain degree α together with the concept of α level set **[13]** of the fuzzy numbers $\tilde{a_i}, \tilde{b_j}$. Therefore the problem of Two stage MOFCMTP can be understood as following non fuzzy α -general Two stage transportation problem (α -two stage MOFCMTP).

$$S = \begin{cases} \sum_{j=1}^{n} x_{ij} = \tilde{a_i} & i = 1,2\ldots\ldots m \\ \sum_{i=1}^{m} x_{ij} = \tilde{b_j} & j = 1,2\ldots\ldots n \end{cases}$$

A point X*∈ X $(\tilde{a_i}, \tilde{b_j})$is said to be α -optimal solution (α -Two stage FCMTP), if and only if there does not exist another *x, y* ∈ *x (a,b)*, , such that C_{ij} $x_{ij} \le C_{ij} x_{ij}^*$ with strict inequality holding for the at least one C_{ij}. **[6]**

The problem (α -Two stage MOFCMTP) can be re written in the following equivalent form (α′-Two stage MOFCMTP)

$$S = \begin{cases} \sum_{j=1}^{n} x_{ij} = \tilde{a_i} & i = 1,2\ldots\ldots m \\ \sum_{i=1}^{m} x_{ij} = \tilde{b_j} & j = 1,2\ldots\ldots n \end{cases}$$

$\square_i^0 \le a_i \le H_i^0, \square_j^0 \le b_j \le H_j^0$

$x_{ij} \ge 0\ \forall$ i, j

The constraint (a_i, b_j ∈ L$\alpha(\tilde{a_i}, \tilde{b_j})$) has been replaced by the Constraint $\square_i^0 \le a_i \le H_i^0$ and $\square_j^0 \le b_j \le H_j^0$ where $\square_i^0$ *and* H_i^0 *and* $\square_j^0$ *and* H_j^0 are lower and upper bounds and a_i, b_j are constants. **[9]**

The parametric study **[18]** of the problem (α ' - Two stage MOFCMTP) where and $\square_i^0, H_i^0$ are assumed to be parameters rather than constants and (renamed h_i, H_i and h_j, H_j) can be understood as follows.

Let *X* (*h*, *H*) denotes the decision space of problem (α ' - Two Stage MOFCMTP), defined by

$X(h, H) = (x_{ij}, a_i, b_j) \in R^{n(n+1)} \mid a_i - \sum_j x_{ij} \geq 0$

$b_j - \sum_i x_{ij} \geq 0,\ H_i - a_i \geq 0, H_j - b_j \geq 0,$

$a_i - h_i \geq 0, b_j - h_j \geq 0, x_{ij} \geq 0, i \in I, j \in J$

IV. SOLUTION ALGORITHM [22]

Step 1: Construct the Transportation problem

Step 2: Supply and demand are hexagonal fuzzy numbers (a_1, a_2, a_3, a_4, a_5, a_6) and (b_1, b_2, b_3, b_4, b_5, b_6) respectively in the formulation problem (Two Stage MOFCMTP).

Step 3: Convert the problem (α -Two Stage MOFCMTP) in the form of the problem (α ' - Two stage MOFCMTP)

Step 4: Formulate the problem (α ' - Two stage FCMTP) in the parametric form.

Step 5: Apply VAM to get the basic feasible solution.

V. VOGEL APPROXIMATION METHOD: (VAM)

VAM is an improved version of the least cost method that generally, but not always, produces better starting solutions.

Step 1: For each row (column), determine a penalty measure by subtracting the smallest unit cost element in the row (column) from the next smallest unit cost element in the same row (column).

Step 2: Identify the row or column with the largest penalty. Break ties arbitrarily. Allocate as much as possible to the variable with the least unit cost in the selected row or column. Adjust the supply and demand, and cross out the satisfied row or column. If a row and a column are satisfied simultaneously, only one of the two is crossed out, and the remaining row (column) is assigned zero supply (demand).

Step 3:

(a). If exactly one row or column with zero supply or demand remains uncrossed out, stop.

(b). If one row (column) with positive supply (demand) remains uncrossed out, determine the basic variables in the row (column) by the least cost method. Stop.

(c). If all the uncrossed out rows and columns have (remaining) zero supply and demand, determine the zero basic variables by the least cost method. Stop.

(d). Otherwise, go to step 1.

VI. GEOMETRIC PROGRAMMING APPROACH FOR SOLVING MOTP

In 1970, Bellman and Zadeh **[2]** introduced three basic concepts; fuzzy goal (G), fuzzy constraints (C), and fuzzy decision (D) and explored the applications of these concepts to decision making under fuzziness. The fuzzy decision is defined by,

$D = G \cap C$

This problem is characterized by the membership functions **[27]:**

$\mu_D(x) = \min(\mu_G(x), \mu_C(x))$

let L_k ,U_k be the lower and upper bounds of the objective functions $F^k(x)$. To define the membership function of MOTP problem, these values are determined as follows: consider a single objective transportation problem in that the individual minimum of each objective function subject to the given set of constraints are calculated. The optimal solutions for the K different transportation problems is given by X^1, X^2 ,....X^k . Evaluate each objective function at all these k optimal solutions. Assume that at least two of these solutions are different for which the k^{th} objective function has different bounded values. Find the lower bound (minimum value) L_k and the upper bound (maximum value) U_k for each objective function $F^k(x)$. On the basis of definitions L_k and k U_k, Biswal **[3]** gives a membership function of a multi-objective geometric programming problem which can be implemented for the MOTP problem as follows:

$$U_k F^k(x) = \begin{cases} 1 & if\ F^k(x) \leq L_k \\ \frac{U_k - F^k(x)}{U_k - L_k} & if\ L_k < F^k(x) < U_k \\ 0 & if\ F^k(x) \geq U_k \end{cases}$$

where $L_k \neq U_k$, k = 1,2,....,k. If $L_k = U_k$ then $\mu_k(F^k(x)) = 1$ for any value of k.

Following the fuzzy decision of Bellman and Zadeh **[2]** together with the linear membership function (5), a fuzzy optimization model of MOTP problem can be written as follows.

P2 : Max $\min_{k=1,2,....,k} \mu_k(F^k(x))$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1,2\dots\dots m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j = 1,2\dots\dots n$$

$$x_{ij} \geq 0, \quad i = 1, 2\dots m$$
$$j = 1, 2\dots n$$

By introducing an auxiliary variable β , problem P2 can be transformed into the following equivalent conventional linear programming (LP) problem **[30]**.

P3 : Max β

Subject to

$$\beta \leq \mu_k(F^k(x)),\ k = 1,2,\dots k$$

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1,2\dots\dots m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j = 1,2\dots\dots n$$

$$0 \leq \beta \leq 1,$$
$$x_{ij} \geq 0, \quad \forall\, i, j$$

In problem P3, constraint (1) can be reduced to the following form.

$\beta\,(U_k - L_k) \leq (U_k - F^k(x))$,

$\beta\,(U_k - L_k) + F^k(x) \leq U_k$

$\beta\,(U_k - L_k)/U_k + (1/U_k)\,F^k(x) \leq 1$

Then, the solution procedure of the MOTP problem is summarized in the following steps.

Step 1: Consider the first objective function and solve it as a single objective transportation problem subject to the constraints (2) – (4). Continue this process *K* times for *K* different objective functions. If all the solutions (i.e. $X^1 = X^2 = \dots = X^k = \{x_{ij}\}$, $i = 1,2,\dots m$, $j = 1,2,\dots, n$) are the same, then one of them is the optimal compromise solution **[21]** and go to step 6. Otherwise, go to step 2

Step 2: Evaluate the kth objective function at the k optimal solutions (k = 1, 2,...,K). In accordance to the set of optimal solutions, determine its lower and upper bounds (L_k and U_k) for each objective function.

Step 3: Define the membership function as mentioned in Eq. (5)

Step 4: Construct the fuzzy programming problem **[29]** P2 and find its equivalent LP problem P3

Step 5: Solve P3 by using an integer programming technique to get an integer optimal solution and evaluate the K objective functions at this optimal compromise solution. Combining stage 1 and stage 2, we get an optimal solution.

Step 6: Stop to construct the membership function of the MOTP problem (step 3) this solution procedure requires the determination of upper and lower bounds of each objective (step 2). After that, Zadeh's min-operator **[28]** is used to develop a linear compromise problem (P3) which is solved by using any integer programming technique.

VII. NUMERICAL EXAMPLE

Consider the following multi – objective two stage cost minimizing transportation problem. Here supplies & demands are hexagonal fuzzy numbers.

a_1 = (7, 9, 11, 13, 16, 20); a_2 = (6, 8, 11, 14, 19, 25) ; a_3 = (9, 11, 13, 15, 18, 20);

b_1 = (6, 9, 12, 15, 20, 25); b_2 = (6, 7, 9, 11, 13, 16); b_3 = (10, 12, 14, 16, 20, 24)

$$C^1 = \begin{bmatrix} (3,7,11,15,19,24) & (3,5,7,9,10,12) & (11,14,17,21,25,30) \\ (3,5,7,9,10,15) & (5,7,10,13,17,21) & (7,9,11,14,18,22) \\ (7,9,11,14,18,24) & (2,3,4,6,7,9) & (5,7,8,11,14,17) \end{bmatrix}$$

$$C^2 = \begin{bmatrix} (4,8,10,12,14,16) & (6,8,10,12,14,15) & (10,12,14,16,18,20) \\ (4,5,7,9,12,15) & (3,5,8,10,12,14) & (5,7,9,12,14,16) \\ (7,9,11,13,15,17) & (9,12,15,19,20,22) & (8,10,12,14,16,18) \end{bmatrix}$$

Using Robust ranking technique.

$$R(H) = \int_0^1 (0.5)\{(b-a)\alpha + a + d - (d-c)\alpha + (d-c)\alpha + c + f - (f-e)\alpha\}$$

a_1 = 25; a_2 = 27; a_3 = 28.5

b_1 = 28.5; b_2 = 20.5; b_3 = 31.5

$$C^1 = \begin{bmatrix} 26.25 & 15.5 & 39 \\ 16.25 & 24 & 26.5 \\ 27 & 10.25 & 20.25 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} 21.5 & 21.75 & 30 \\ 17 & 17.5 & 21 \\ 24 & 32.75 & 26 \end{bmatrix}$$

STAGE I:

We take a_1=12, a_2=13, a_3=14.5

b_1=14.5, b_2=10, b_3=15,

With respect to C^1, applying VAM, we get

x_{11} = 2; x_{12} = 10; x_{21} = 12.5; x_{23} = 0.5; x_{33} = 14.5

min z = 717.5 .

With respect to C^2, applying VAM we get

$x_{11} = 2; x_{12} = 10; x_{23} = 13; x_{31} = 12.5; x_{33} = 2$
min z = 885.5

$F^1 (X^1) = 717.5; F^1 (X^2) = 930$
$F^2 (X^1) = 865; F^2 (X^2) = 885.5$
i.e. $717.5 \le F^1 \le 930$
$865 \le F^2 \le 885.5$

The member ship function of both $F^1(x)$ and $F^2(x)$ are

$$\mu_1(F^1(x)) = \frac{930 - F^1(x)}{930 - 717.5} = \frac{930 - F^1(x)}{212.5}$$

$$\mu_2(F^2(x)) = \frac{885.5 - F^2(x)}{885.5 - 865} = \frac{885.5 - F^2(x)}{20.5}$$

Now Solve Max β
S. to
$x_{11} + x_{12} + x_{13} = 12$
$x_{21} + x_{22} + x_{23} = 13$
$x_{31} + x_{32} + x_{33} = 14.5$
$x_{11} + x_{21} + x_{31} = 14.5$
$x_{12} + x_{22} + x_{32} = 10$
$x_{13} + x_{23} + x_{33} = 15$

$0.0282\, x_{11} + 0.0167 x_{12} + 0.0419 x_{13} + 0.0175\, x_{21} + 0.0258\, x_{22} + 0.0285\, x_{23} + 0.0290 x_{31} + 0.0110 x_{32} + 0.0218\, x_{33} + 0.2285\beta \le 1$

$0.0243\, x_{11} + 0.0246 x_{12} + 0.0339 x_{13} + 0.0192\, x_{21} + 0.0198 x_{22} + 0.0237 x_{23} + 0.0271 x_{31} + 0.0370\, x_{32} + 0.0294\, x_{33} + 0.0232\beta \le 1$

$x_{ij} \ge 0$ and integer, $\forall\, i, j$

The optimal compromise solution X*
$x_{11} = 2$; $x_{12} = 10$; $x_{21} = 12.5$; $x_{23} = 0.5$; $x_{33} = 14.5$;
The overall satisfaction β = 0.9956
The optimum values of the objective functions after stage I are
$F^1(X^*) = 717.5$
$F^2(X^*) = 860.5$

Stage II:
We take $a_1=13$, $a_2=14$, $a_3=14$
$b_1=14$, $b_2=10.5$, $b_3=16.5$,
With respect to C^1, applying VAM, we get
$x_{11} = 2.5; x_{12} = 10.5; x_{21} = 11.5; x_{23} = 2.5; x_{33} = 14$
min z = 765.
With respect to C^2, applying VAM we get
$x_{11} = 2.5; x_{12} = 10.5; x_{23} = 14; x_{31} = 11.5; x_{33} = 2.5$
min z = 917

$F^1 (X^1) = 765; F^1 (X^2) = 960.5$
$F^2 (X^1) = 894; F^2 (X^2) = 917$
i.e. $765 \le F^1 \le 960.5$
$894 \le F^2 \le 917$
The member ship function of both $F^1(x)$ and $F^2(x)$ are

$$\mu_1(F^1(x)) = \frac{960.5 - F^1(x)}{960.5 - 765} = \frac{960.5 - F^1(x)}{195.5}$$

$$\mu_2(F^2(x)) = \frac{917 - F^2(x)}{917 - 894} = \frac{917 - F^2(x)}{23}$$

Now Solve Max β
S. to
$x_{11} + x_{12} + x_{13} = 13$
$x_{21} + x_{22} + x_{23} = 14$
$x_{31} + x_{32} + x_{33} = 14$
$x_{11} + x_{21} + x_{31} = 14$
$x_{12} + x_{22} + x_{32} = 10.5$
$x_{13} + x_{23} + x_{33} = 16.5$

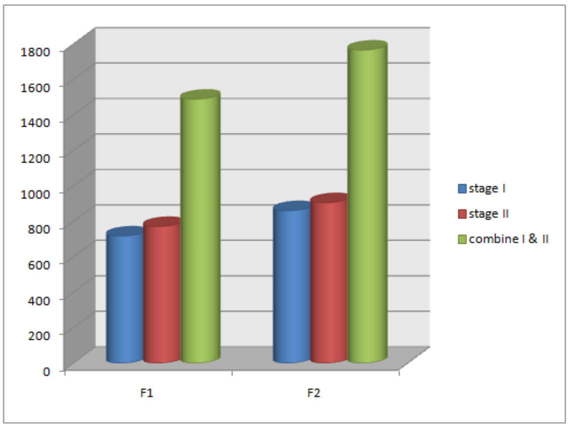
$0.0273\, x_{11} + 0.0161 x_{12} + 0.0406 x_{13} + 0.0169\, x_{21} + 0.0250\, x_{22} + 0.0276\, x_{23} + 0.0281 x_{31} + 0.0107 x_{32} + 0.0211\, x_{33} + 0.2035\beta \le 1$

$0.0234\, x_{11} + 0.0237 x_{12} + 0.0327 x_{13} + 0.0185\, x_{21} + 0.0191 x_{22} + 0.0229 x_{23} + 0.0262 x_{31} + 0.0357\, x_{32} + 0.0284\, x_{33} + 0.0251\beta \le 1$

$x_{ij} \ge 0$ and integer, $\forall\, i, j$
The optimal compromise solution X*
$x_{12} = 10.5$; $x_{13} = 2.5$; $x_{21} = 14$; $x_{33} = 14$;
The overall satisfaction β = 0.8026
The optimum values of the objective functions after stage II are
$F^1(X^*) = 771.25$
$F^2(X^*) = 905.4$
The optimal values of the objective functions combining stage I and stage II are
$F^1(X^*) = 717.5 + 771.25 = 1489$
$F^2(X^*) = 860.5 + 905.4 = 1766$

Table:

	Stage I	Stage II	Combine I & II
$F^1(X^*)$	717.5	771.25	1489
$F^2(X^*)$	860.5	905.4	1766



VIII. CONCLUSION

Transportation models have wide applications in logistics and supply chain for reducing problems. In this study , Fuzzy geometric programming approach is used to determine the optimal compromise solution of a multi-objective two stage fuzzy transportation problem, in which supplies, demands are Hexagonal fuzzy numbers and fuzzy membership of the objective function is defined.

REFERENCES

[1] Abhinav Bansal, “Trapezoidal Fuzzy Numbers (a, b, c, d): Arithmetic behavior”, International Journal of Physical and Mathematical Sciences, ISSN- (39-44) (2010-1791),(2011).
[2] Bellman R.Zadeh L.A, “Decision Making in a Fuzzy Environment”, Management Sci.17(B) ,141-164, (1970).
[3] Biswal M.P., “Fuzzy Programming Technique to Solve Multi-objective Geometric Programming Problem”, Fuzzy sets and systems 51, 67-71, (1992).
[4] Buckly J.J., “Possibilistic Linear Programming with Triangular Fuzzy Numbers”, Fuzzy Sets and Systems, 26 ,135–138, (1988).
[5] Buckly J.J., “Solving Possibilistic Programming Problems”, Fuzzy Sets and Systems, 31 ,329–341, (1988).
[6] Chanas S., Kuchta D., “A Concept of the Optimal Solution of the Transportation Problem with Fuzzy Cost Coefficients”, Fuzzy Sets and Systems 82, 299– 305, (1996).
[7] Chanas S., Kolodziejczyk W., Machaj A., “A Fuzzy Approach to the Transportation Problem”, Fuzzy Sets and Systems 13 (1984)
[8] Diaz J.A., “Solving Multi-objective Transportation Problems”, Ekonomicko-Matemarcky Obzor(15), 267-274, (1976).
[9] Fang S.C., Hu C.F., Wang H.F., Wu S.Y., “Linear Programming with Fuzzy Coefficients in Constraints”, Computers and Mathematics with Applications 37, 63–76, (1999).
[10] Julien B., “An Extension to Possibilistic Linear Programming”, Fuzzy Sets and Systems 64 , 195–206(1994).
[11] R. Jhon Paul Antony, S. Johnson Savarimuthu and T.Pathinathan, “Method for solving Transportation Problem Using Triangular Intuitionistic Fuzzy Number”,International Journal of Computing Algorithm,03, 590-605, (2014),
[12] Leberling H., “On Finding Compromise Solution in Multi-criteria Problems using the Fuzzy Min operator”, Fuzzy Sets and Systems 6, 105-118, (1981).
[13] Lingo User_s Guide, LINDO Systems Inc., Chicago, (1999).
[14] Liu S.T., Kao C. / European Journal of Operational Research 153, 661–674(2004).
[15] Luhandjula M.K., “ Linear Programming with a possibilistic objective function”, European Journal of Operational Research, 31 ,110–117, (1987).
[16] Nagarajan.R. and Solairaju.A. “A Computing improved fuzzy optimal Hungarian. Assignment Problem with fuzzy cost under Robust ranking technique”, International Journal of Computer Application Volume 6,No.4. pp 6-13, (2010).
[17] Nagoor Gani A., and Abdul Razak K., “Two Stage Fuzzy Transportation Problem”, Journal of Physical Sciences, Vol. 10, 63-69, (2006),.
[18] Omar M.Saad and Samir A.Abbas, “A Parametric study on Transportation problem under Fuzzy Environment”, The Journal of Fuzzy Mathematics 11, No.1, 115 -124, (2003).
[19] Parra M.A., Terol A.B., Uria M.V.R., “Solving the Multi-objective Possibilistic Linear Programming Problem”, European Journal of Operational Research, 117,
[20] 175–182, (1999).
[21] Rajarajeswari.P, A.Sahaya Sudha and R.Karthika, “A New Operation on Hexagonal Fuzzy Number”, International Journal of Fuzzy Logic Systems, 3(3), 15-26, (2013).
[22] Reklaitis G.V., Ravindran A., Ragsdell K.M., “Engineering Optimization”, John Wiley & Sons, NY, (1983).
[23] Ritha.W and Merline Vinotha.J ., “Multi-objective Two Stage Fuzzy Transportation Problem”,Journal of Physical Sciences, Vol. 13, 107-120, (2009).
[24] Sonia and Rita Malhotra, “A Polynomial Algorithm for a Two Stage Time Minimising Transportation Problem”, OPSEARCH, 39, No.5&6, 251-266, (2003).
[25] Tanaka H., Ichihashi H., Asai K., “A Formulation of Fuzzy Linear Programming based on Comparison of Fuzzy Numbers”, Control and Cybernetics 13,
[26] 185– 194, (1984).

[27] Verma R., Biswal M., Bisawas A., “Fuzzy programming technique to solve Multiple objective Transportation Problems with some Nonlinear Membership functions”, Fuzzy Sets and Systems, 91, 37–43, (1997).

[28] Waiel F.Abd El-wahed, “A Multi-objective Transportation Problem under Fuzziness”, Fuzzy Sets and Systems, 117 ,27-33, (2001).

[29] Yager R.R., “A Characterization of the Extension Principle”, Fuzzy Sets and Systems, 18, 205–217, (1986).

[30] Zadeh L.A., “Fuzzy Sets as a basis for a theory of possibility”, Fuzzy Sets and Systems, 1, 3–28, (1978).

[31] Zimmermann H.J., “Fuzzy Set Theory and Its Applications”, third ed., Kluwer-Nijhoff, Boston, (1996).

[32] Zimmermann H.J., “Fuzzy Programming and Linear Programming with several objective functions”, Fuzzy Sets and System 1,45-55, (1978).

[33] Zeleny M, “Multiple Criteria Decision Making”, McGraw-Hill ,New York.(1982)