Fluctuation of LQ45 index and BCA stock price at Indonesian Stock Exchange IDX

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ABSTRACT
The financial market can be considered as a complex system in physics. The moving stock price can be regarded as moving colloid particle that obey stochastic process. In this research, the stock is assumed to obey Geometric Brownian Motion and the variance obeys Ornstein-Uhlenbeck stochastic process. The theoretical probability density of return in this model is in accordance with the empirical probability density of return that taken from LQ45 dan BCA data series at their peaks and their tails. The theoretical probability densities of the return in this model is compared with the Gaussian distribution, Power law, and exponential distribution.

Keywords: BCA, fluctuation, LQ45, probability density, stochastic process.

I. INTRODUCTION
The financial market can be regarded as a complex system in physics system [1,2], so we can apply some appropriate physical laws to consider the market. Moreover, the financial market provides abundant data so that it is possible to use the physical laws to analyse the structure of financial market [3,4]. One component of financial structure to be analysed is the probability density of stock price and the variance. The distribution of stock price probability and its variance will reflect the process in the system. Empirical properties of stock price fluctuation have been researched in a number of studies as in [4-8]. The previous studies have shown that the fluctuation of the distributions are close to the Gaussian distribution, but the distributions have more fat tail than that of Gaussian. The models to be applied to analyse the fat tail of the distributions are exponential and power-law model. In this work, we study a stochastic variance model to explain the distribution and compare with the Gaussian, exponential and power-law distribution.

In this work, we regard the stock price moving as the colloid particle moving. A particle in a colloid will obey a stochastic process because of the collision with other colloid particle. Like in the case of the motion of a colloid particle, the stock price (or index) and the variance follow a kind of stochastic process. Because both of the stock price and the variance process are diffusion process, the probability density can be calculated from Fokker-Planck equation for their process. The theoretical probability density calculated using our model will be fitted with empirical probability density of LQ45 index and BCA stock prices. LQ45 is an index in IDX and BCA is a company that listed in IDX. The theoretical probability density will compare with Gaussian, exponential and Power-law distribution.

II. METHODS
In this work, the fluctuation of the stock price is measured by the return such as \( r = \frac{S(t)}{S(0)} - 1 \). \( S(t) \) is the stock price at time \( t \), \( S(0) \) is the stock price at \( t=0 \). The stock price assumed to follow the Geometric Brownian process

\[
dS(t) = \phi S(t)dt + \sigma S(t)dW_s .
\]

Where \( dW_s \) is a standard Wiener process for the stock price. The Lavengin equation (or differential equation) for the return that obtained from Ito calculus is

\[
dr = -\frac{\nu}{2}dt + \sqrt{\nu}dW_s .
\]

If the variance and the volatility of stock price are constant, the return density probability will have a Gaussian distribution. The empirical distributions have the tails that are fatter than that of the Gaussian. In this work we assumed that the variance \( \nu = \sigma^2 \) follows a diffusion process namely Ornstein-Uhlenbeck process [9]

\[
dv(t) = -\mu(v - \gamma)dt + \kappa \sqrt{v}dW_v .
\]

Where \( \nu \) is variance and \( dW_v \) is a standard Wiener process for the variance.

The correlation between \( dW_v \) and \( dW_s \) is

\[
dW_v(t) = \rho dW_s(t) + \sqrt{1 - \rho^2}Z(t) , \text{ with } \rho \in [-1,1].
\]
Eq. (3) says that variance of stock price tends to revert to $\gamma$, the variance that is attained at a long time. The speed of the revert is $\mu$ and the volatility of variance is $\kappa$. The Fokker-Planck for density probability transition of the return is [10]

$$\frac{\partial p}{\partial t} = \mu \frac{\partial}{\partial v} [v - \gamma] p + \frac{\kappa}{2} \frac{\partial^2}{\partial v^2} (v p) + \rho x \frac{\partial^2}{\partial v^2} (v p) + \frac{1}{2} \frac{\partial^2}{\partial v^2} (\kappa v^2)$$

where $p = p(z,v_i)$ is the probability density of the transition from initial state at $r = 0$ with variance $v_i$ to jump to a state at arbitrary $z$ with variance $v$. The variance or volatility does not seem explicitly from data series. We can obtain the probability density of the variance at stationary state from Eq. 2 as [11]

$$\Pi_i (v_i) = \frac{\theta^{\alpha+1}}{\Gamma (\alpha+1)} v_i^{-\alpha} \exp(-\theta v_i)$$

where $\theta = 2\mu\kappa^2$ and $\alpha = \theta \eta - 1$, whereas $\Gamma$ is Gamma function.

Under the assumption that the probability density of variance is in a stationary state, the probability density that the value of the return is $r$ for each value of variance is given by [11,12]

$$p(r) = \frac{1}{(2\pi)^{\alpha/2}} \int_0^\infty dp e^{i\theta p r} \left( \frac{\mu^2}{\kappa^2} \ln \left( \frac{(\omega^2 - x^2)^2 + 2\mu x \sinh \left(\frac{\omega}{2}\right) + \cosh \left(\frac{\omega}{2}\right)}{2\mu x} \right) \right)$$

We can obtain the parameter $\mu$, $\gamma$, $\kappa$ by fitting the empirical probability density from data series with (6). $p(r)$ at (6) can be calculated numerically. In this work, we have calculated (6) with $\tau=1$ day and $\rho=0$. We use LQ45 data series for index data series and BCA stock price for company stock price. LQ45 is an index at IDX that consists of 45 most liquid company in Indonesia and BCA is a company listed at IDX. The data series are taken at time interval 2004-2012 containing 2187 data point. Each data point is index or stock price at closing.

### III. RESULT AND DISCUSSION

The LQ45 return probability density daily is shown in Fig.1, Fig.2, and Fig.3. We can see that theoretical return probability density is accordance with the empirical distribution than the Gaussian distribution. The return probability is not symmetric as in Gaussian. Fig.2 and Fig.3 show that the tails of theoretical probability close to the empirical, the power law, and exponential. The theoretical probability in accordance with the empirical at the tail as well as at the peak of distribution, whereas the power law and exponential agrees only at the tail. The theoretic probability close to the Gaussian just at the peak.

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**Figure 1.** The return probability density of LQ45 index.

**Figure 2.** The left tail of return probability density for LQ45 index.
The BCA return probability density is shown in Fig. 4, Fig. 5 and Fig. 6. We can see that theoretical return probability density agree with the empirical than Gaussian distribution. The BCA return probability is not symmetric as in the Gaussian. The empirical probability close to Gaussian just at the peak. Fig. 5 shows the left tail of the probability density. The empirical is close to theoretical and the exponential, but it is not close to Power-law and Gaussian distribution. Fig. 6 shows the right tail probability density. As in Fig. 5, the empirical is close to theoretical and the exponential, but it is not close to power-law and Gaussian distribution.

The LQ45 and BCA theoretical probability are in accordance with the empirical at the tail and the peak of distribution, whereas the power law and or the exponential agree only at the tail of distribution. The Gaussian distribution is close to the empirical only at the peak probability density. We can conclude that the model can be accepted model for financial market. Index and Stock price
at IDX, especially LQ45 and BCA, have behaviour as colloid particles. The variance is not constant but it obeys an Ornstein-Uhlenbeck stochastic process so that the tail of return density probability decay more slowly than the Gaussian.

The parameter $\mu$, $\gamma$, $\kappa$ for LQ45 is respectively 0.00051, 0.00082 and 0.0001. The LQ45 index variance at a long time is 0.00082, and the speed to revert is 0.00051 /day, whereas the parameter $\mu$, $\gamma$, $\kappa$ for LQ45 is respectively 0.00051, 0.002 and 0.0002. The BCA stock price variance at a long time is 0.002, the speed to revert is 0.0051 IDR/day. The BCA stock price is more fluctuation than the LQ45 index. It can be understood because the LQ45 consist some companies, the index of fluctuation indicates the average behaviour of the fluctuation price fluctuation of the companies consisting them.

IV. CONCLUSION

The variance of stock price, especially LQ45 index and BCA stock price, is not constant but obeys Ornstein-Uhlenbeck stochastic process. The process can explain why the tail of return density probability decays more slowly than that of Gaussian. The return probability density according to the model is acceptable with empirical return probability density for both the peak and the tail.

REFERENCES