RESEARCH ARTICLE OPEN ACCESS

Analytical and Exact solutions of a certain class of coupled nonlinear PDEs using Adomian-Padé method

Khadijah M. Abualnaja
Department of Mathematics and Statistics, College of Science, Taif University, Taif, KSA

ABSTRACT
The purpose of this study is to introduce a modification of the Adomian decomposition method using Padé approximation and Laplace transform to obtain a closed form of the solutions of nonlinear partial differential equations. Several test examples are given; illustrative examples and the coupled nonlinear system of Burger's equations. The obtained results ensure that this modification is capable for solving the nonlinear PDEs that have wide application in physics and engineering.
Keywords: Padé approximation; Laplace transform; Adomian decomposition method; nonlinear Burger's equations.

I. INTRODUCTION

In this paper, the Adomian decomposition method (ADM) [3] is used to solve nonlinear partial differential equations (NPDEs). It is based on the search for a solution in the form of a series and on decomposing the nonlinear operator into a series in which the terms are calculated recursively using the Adomian's polynomials ([4], [7]). In recent years, a growing interest towards the applications of this method in nonlinear problems has been devoted by engineering practice, see ([1], [2], [6], [9], [11], [12]). This method has many advantages such as, fast convergence, doesn't require discretizations of space-time variables, which gives rise to rounding off errors, and doesn't require solving the resultant nonlinear system of discrete equations. Unlike, the traditional methods, the computation in this method doesn't require a large computer memory. Also, we are going to improve the accuracy of ADM by using the Padé approximation and apply the improved method to solve some physical models of NPDEs, namely, illustrative examples and the system of Burger's equations.

II. ANALYSIS OF THE ADOMIAN DECOMPOSITION METHOD

In this section, the basic idea of ADM to solve NPDEs is introduced as follows: Consider the following nonlinear partial differential equation

$$A_{xt}\, u(x,t) + N(u) = f(x,t), \qquad (1)$$

with suitable initial and boundary conditions. Where A_{xt} is the partial differential operator and $N(u)$ is the non-linear term.

Let $A_{xt} = = L_t + L_x + R_{xt}$ where L_t is the highest partial derivative with respect to t (assume of order n), L_x is the highest partial derivative with respect to x and R_{xt} is the remainder from the operator, therefore, (1) will take the following operator form

$$L_t\, u(x,t) = f(x,t) - (L_x + R_{xt})u(x,t) - N(u). \qquad (2)$$

By applying the inverse operator on both sides of (2), we get

$$u(x,t) = \varphi(x,t) + L_t^{-1}\,[f(x,t) - (L_x + R_{xt})u(x,t) - N(u)], \qquad (3)$$

where the inverse operator is defined by:

$$L_t^{-1}(.) = \int_0^t \dots n-\ fold\ \dots \int_0^t (.)\, dt \dots dt\,,$$

and $\varphi(x,t)$ is the solution of the homogeneous differential equation $L_t\, u(x,t) = 0$ with the same initial conditions of (1).
Now, ADM suggests that the solution can be expressed as an infinite series of the following form

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)\,, \qquad (4)$$

and the nonlinear term expanded in terms of Adomian's polynomials A_n

$$N(u) = \sum_{n=0}^{\infty} A_n\,, \qquad (5)$$

where these polynomials are defined by the relation

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N\left(\sum_{i=0}^{n} \lambda^i u_i\right)\right]_{\lambda=0}, \quad n \ge 0. \qquad (6)$$

Substituting from (4) and (5) in (3), we obtain

$$\sum_{n=0}^{\infty} u_n(x,t) = \varphi(x,t) + L_t^{-1}\left[f(x,t) - (L_x + R_{xt}) \sum_{n=0}^{\infty} u_n(x,t) - \sum_{n=0}^{\infty} A_n \right],$$

equating the similar terms in both sides of the above equation, we get the following recurrence relation

$$u_0(x,t) = \varphi(x,t) + L_t^{-1}[f(x,t)]$$

$$u_n(x,t) = -(L_x + R_{xt})u_{n-1}(x,t) - A_{n-1}\,], \qquad n \ge 1. \qquad (7)$$

Now, we present some basic definitions of the Taylor series method, Padé approximation ([5], [13]), needed in the next sections of the paper.

III. THE PADÉ APPROXIMANTS ON THE SERIES SOLUTION

The general setup in approximation theory is that a function f is given and that one wants to approximate it with a simpler function g but in such a way that the difference between *f* and *g* is small. The advantage is that the simpler function *g* can be handled without too many difficulties, but the disadvantage is that one loses some information since *f* and *g* are different.

Definition 1.
When we obtain the truncated series solution of order at least *L+M* in t by ADM, we will use it to obtain $PA\left[\frac{L}{M}\right](x,t)$ Padé approximation for the solution $u(x,t)$. The Padé approximation ([4], [25]) are a particular type of rational fraction approximation to the value of the function. The idea is to match the Taylor series expansion as far as possible.

We denote the $PA\left[\frac{L}{M}\right]$ to $R(x) = \sum_{i=0}^{\infty} a_i x^i$ by:

$$PA\left[\frac{L}{M}\right](x) = \frac{P_L(x)}{Q_M(x)}, \qquad (8)$$

where $P_L(x)$ is a polynomial of degree at most L and $Q_M(x)$ is a polynomial of degree at most M.

$$P_L(x) = p_0 + p_1 x + p_2 x^2 + \dots + p_L x^L, \quad Q_M(x) = 1 + q_1 x + q_2 x^2 + \dots + q_M x^M.$$
(9)

To determine the coefficients of $P_L(x)$ and $Q_M(x)$, we may multiply (8) by $Q_M(x)$, which linearizes the coefficient equations. We can write out (8) in more detail as:

$$\begin{aligned} &a_{L+1} + a_L q_1 + \ldots + a_{L-M+1} q_M = 0, \\ &a_{L+2} + a_{L+1} q_1 + \ldots + a_{L-M+2} q_M = 0, \\ &\qquad\ldots \\ &a_{L+M} + a_{L+M-1} q_1 + \ldots + a_L q_M = 0, \end{aligned} \tag{10}$$

$$\begin{aligned} &a_0 = p_0, \\ &a_1 + a_0 q_1 = p_1, \\ &a_2 + a_1 q_1 + a_0 q_2 = p_2, \\ &\qquad\ldots. \\ &a_L + a_{L-1} q_1 + \ldots + a_0 q_L = p_L. \end{aligned} \tag{11}$$

To solve these equations, we start with Eq.(10), which is a set of linear equations for all the unknown q's. Once the q's are known, then Eq.(11) gives an explicit formula for the unknown p's, which complete the solution. Each choice of L, degree of the numerator and M, degree of the denominator, leads to an approximant. The major difficulty in applying the technique is how to direct the choice in order to obtain the best approximant. This needs the use of a criterion for the choice depending on the shape of the solution. A criterion which has worked well here is the choice of $\left[\frac{L}{M}\right]$ approximation such that *L=M*. We construct the approximation by built-in utilities of Mathematica in the following sections. If ADM truncated Taylor series of the exact solution with enough terms and the solution can be written as the ratio of two polynomials with no common factors, then the Padé approximation for the truncated series provide the exact solution. Even when the exact solution cannot be expressed as the ratio of two polynomials, the Padé approximation for the ADM truncated series usually greatly improve the accuracy and enlarge the convergence domain of the solutions.

IV. THE MODIFIED ALGORITHM OF ADM

In spite of the advantages of ADM, it has some drawbacks. By using ADM, we get a series, in practice a truncated series solution. The series often coincides with the Taylor expansion of the true solution at point $x = 0$, in the initial value case. Although the series can be rapidly convergent in a very small region, it has very slow convergent rate in the wider region we examine and the truncated series solution is an inaccurate solution in that region, which will greatly restrict the application area of the method. All the truncated series solutions have the same problem. Many examples given can be used to support this assertion [12].

Padé approximation [5] approximates any function by a ratio of two polynomials. The coefficients of the powers occurring in the polynomials are determined by the coefficients in the Taylor expansion. Generally, the Pade approximation can enlarge the convergence domain of the truncated Taylor series and can improve greatly the convergent rate of the truncated Taylor series [10].

The suggested modification of ADM can be done by using the following algorithm.

Algorithm

Step 1. Solve the differential equation using the standard ADM;

Step 2. Truncate the obtained series solution by ADM;

Step 3. Take the Laplace transform of the truncated series;

Step 4. Find the Padé approximation of the previous step;

Step 5. Take the inverse Laplace transform.

This modification often gets the closed form of the exact solution of the differential equation. Now, we implement this algorithm to some examples of linear and nonlinear differential equations to illustrate our modification.

V. ILLUSTRATIVE EXAMPLES

Example 1:

Consider the following differential equation

$$\frac{d^2u}{dt^2} + u = 0, \qquad (12)$$

subject to the initial conditions $u(0) = 0,\ \dot{u}(0) = 1$. We can rewrite (12) in an operator form as follows

$$L_t u(t) + u(t) = 0, \qquad (13)$$

where the differential operator L_t is $L_t = \frac{d^2}{dt^2}$.

By applying the inverse operator on (13) and using the initial conditions, we can derive

$$u(t) = u(0) + tu_t(0) - L_t^{-1} u(t), \qquad (14)$$

where the operator L_t^{-1} is an integral operator and given by $L_t^{-1}(.) = \int_0^t \int_0^t (.)\, dtdt$.

The ADM [3] assumes that the unknown solution can be expressed by an infinite series of the following form

$$u(t) = \sum_{n=0}^{\infty} u_n(t). \qquad (15)$$

Substituting from (15), into (14) and equating the similar terms in both sides, we get the following recurrence relation

$$u_0(t) = u(0) + tu_t(0), \qquad u_{n+1}(t) = -L_t^{-1} u_n(t), \qquad n \geq 0. \qquad (16)$$

Now, from the recurrence relationship (16) we obtain the following components of the solution u(t)

$$u_0(t) = u(0) + tu_t(0) = t,$$

$$u_1(t) = -L_t^{-1} u_0(t) = \frac{-t^3}{3!},$$

$$u_2(t) = -L_t^{-1} u_1(t) = \frac{t^5}{5!}, \ldots$$

$$u_{n+1}(t) = -L_t^{-1} u_n(t) = \frac{(-1)^n t^{2n+1}}{(2n+1)!}.$$

By the same procedure we can obtain the components of the solution. Therefore, the approximate solution can be readily obtained by

$$u(t) \cong \varphi_n(t) = \sum_{k=0}^{n} \frac{(-1)^k t^{2k+1}}{(2k+1)!}. \qquad (17)$$

Which is the partial sum of the Taylor series of the solution u(t) at t=0. Figure 1 shows the error between the exact solution u(t) and $\varphi_5(t)$, from this figure we can find that the error at $t \in [0, 5]$ is nearly to 0, but at $t \in [5,10]$ the error takes large values.

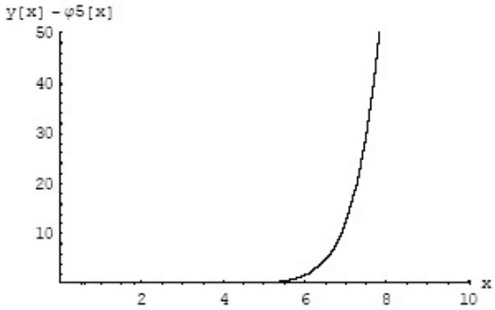

Figure 1: The error between the exact solution u(t) and the solution by ADM, $\varphi_5(t)$.

Now, because (12) is an oscillatory system, here we can apply Laplace transform [8] to $\varphi_n(t)$ which yields

$$\pounds[\varphi_n(t)] = \frac{1}{s^2} - \frac{1}{s^4} + \frac{1}{s^6} - \ldots + \frac{(-1)^{n+1}}{s^{2n+2}}.$$

For the sake simplicity, let $s = \frac{1}{t}$ then:

$$\pounds[\varphi_n(t)] = t^2 - t^4 + t^6 - \ldots + (-1)^{n+1} t^{2n+2}.$$

Its $[\frac{n+1}{n+1}]$ Padé approximations with n>0 yields $[\frac{n+1}{n+1}] = \frac{t^2}{1+t^2}$.

Replace $t = \frac{1}{s}$, we obtain $[\frac{n+1}{n+1}]$ in terms of s as follows:

$$[\frac{n+1}{n+1}] = \frac{1}{1+s^2},$$

by using the inverse Laplace transform to $[\frac{n+1}{n+1}]$ we obtain the true solution of (12):

$$u(t) = \sin(t).$$

Example 2:

Consider the following nonlinear partial differential equation

$u_{tt} - u_{xx} + 2uu_t = g(x,t) = -2\sin^2(x)\sin(t)\cos(t),$ (18) with the initial conditions $u(x, 0) = \sin(x)$, $u_t(x,0) = 0$.

The exact solution of this equation is $u(x, t) = \sin(x)\cos(t)$.

First, to overcome the complicated excitation from the term $g(x,t)$, which can cause difficult integrations and proliferation of terms, we use the Taylor expansion of the function $g(x, t)$ at $t = 0$, in the following form

$$g(x,t) = \sum_{k=0}^{\infty} g_k(x)t^k = -2\sin^2(x)(t - \frac{2}{3}t^3 + \frac{2}{15}t^5 - \frac{4}{315}t^7 + \ldots),$$

in this case, (18) reduces to the form

$$u_{tt} - u_{xx} + 2uu_t = = \sum_{k=0}^{\infty} g_k(x)t^k. \qquad (19)$$

We can rewrite (19) in an operator form as follows

$$L_t u - L_x u + N(u) = \sum_{k=0}^{\infty} g_k(x)t^k, \qquad (20)$$

where the differential operator $L_t = \frac{\partial^2}{\partial t^2}$ the nonlinear term is $N(u) = 2uu_t$. By applying the inverse operator on (20) and using the initial conditions, we can derive

$$u(x,t) = u(x,0) + tu_t(x,0) + L_t^{-1}[u_{xx} - N(u) + \sum_{k=0}^{\infty} g_k(x)t^k]. \qquad (21)$$

The ADM [3] assumes that the unknown solution can be expressed by an infinite series of the form (4) and the nonlinear operator term can be decomposed by an infinite series of polynomials, given by (5), where the components $u_n(x, t)$, $n \geq 0$ will be determined recurrently and A_n are the so-called Adomian's polynomials of $u_0, u_1, u_2, \ldots$ defined by (6), these polynomials can be constructed for all nonlinearity according to algorithm set by Adomian.
Substituting from (4), (5) into (21) and equating the similar terms in both sides of the equation, we get the following recurrence relation

$$u_0(x,t) = u(x,0), \qquad u_{n+1}(x,t) = L_t^{-1}[u_{nxx} - A_n + g_n(x)\, t^n], \quad n \geq 0, \qquad (22)$$

where the first A_n Adomian's polynomials that represent the nonlinear term $N(u) = 2uu_t$ are given by

$$A_0 = 2u_0 u_{0t},$$
$$A_1 = 2(u_0 u_{1t} + u_1 u_{0t}),$$
$$A_2 = 2(u_2 u_{0t} + u_1 u_{1t} + u_0 u_{2t}),$$
$$A_3 = 2(u_3 u_{0t} + u_2 u_{1t} + u_0 u_{3t} + u_1 u_{2t}), \ldots$$

other polynomials can be generated in a like manner.
Now from the recurrence relationship (22) we obtain the following resulting components

$$u_0(x,t) = \sin(x), \qquad u_1(x,t) = \frac{-t^2}{2!}\sin(x),$$
$$u_2(x,t) = \frac{t^4}{4!}\sin(x), \qquad u_3(x,t) = \frac{-t^6}{6!}\sin(x),$$
$$u_4(x,t) = \frac{t^8}{8!}\sin(x), \qquad u_5(x,t) = \frac{-t^{10}}{10!}\sin(x).$$

Having $u_i(x,t)$, $i = 0,1,2,\ldots,n$, the solution is as follows

$$u(x,t) \cong \varphi_n(x,t) = \sum_{i=0}^{n-1} u_i(x,t). \qquad (23)$$

In our calculations we use the truncated series $\hat{\varphi}_4(x,t)$ to order t^8, i.e.,

$$\hat{\varphi}_4(x,t) = (1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \frac{t^8}{8!})\sin(x), \qquad (24)$$

which coincides with the first five terms of the Taylor series of the solution $u(x,t)$ at $t = 0$. Here, we apply Laplace transformation to $\hat{\varphi}_4(x,t)$, which yields

$$£[\hat{\varphi}_4(x,t)] = (\frac{1}{s} - \frac{1}{s^3} + \frac{1}{s^5} - \frac{1}{s^7} + \frac{1}{s^9})\sin(x).$$

For the sake simplicity, let $s = \frac{1}{t}$ then:

$£[\hat{\varphi}_4(x,t)] = (t - t^3 + t^5 - t^7 + t^9)\sin(x).$

All of the $[\frac{L}{M}]$ Padé approximants of the above equation with $L > 0$, $M > 1$ and $L+M < 10$ yields

$[\frac{L}{M}] = \frac{t}{1+t^2}\sin(x)$. Replace $t = \frac{1}{s}$, we obtain $[\frac{L}{M}]$ in terms of s as follows:

$$[\frac{L}{M}] = \frac{s}{1+s^2}\sin(x).$$

By using the inverse Laplace transform to $[\frac{L}{M}]$ we obtain the true solution of (18)

$$u(x,t) = \sin(x)\cos(t).$$

VI. IMPLEMENTATION THE MODIFICATION OF ADM TO SYSTEM OF BURGERS' EQUATIONS

Consider the following coupled nonlinear system of Burgers' equations

$$u_t - u_{xx} - 2uu_x + (uv)_x = 0, \quad (25)$$

$$v_t - v_{xx} - 2vv_x + (uv)_x = 0, \quad (26)$$

subject to the following initial conditions

$$u(x,0) = v(x,0) = \sin(x). \qquad (27)$$

Now, we will solve the system (25)-(27) by using the above algorithm as follows:

Step 1. Solve the system (25)-(27) by using ADM

In this step, we apply ADM to the system (25)-(27), so, we rewrite it in the following operator form

$$L_t u - u_{xx} + N_1(u, v) = 0, \qquad (28)$$

$$L_t v - v_{xx} + N_2(u, v) = 0. \qquad (29)$$

By using the inverse operator, we can write the above system (28)-(29) in the following form

$$u(x,t) = u(x,0) + L_t^{-1}[u_{xx}] - L_t^{-1}[N_1(u,v)], \quad (30)$$

$$v(x,t) = v(x,0) + L_t^{-1}[v_{xx}] - L_t^{-1}[N_2(u,v)], \quad (31)$$

where the inverse operator is defined by $L_t^{-1}(.) = \int_0^t (.)\,dt$, and the nonlinear terms $N_1(u, v)$ and $N_2(u, v)$ are defined by

$$N_1(u, v) = -2uu_x + (uv)_x, \quad N_2(u, v) = -2vv_x + (uv)_x. \qquad (32)$$

The ADM suggests that the solutions $u(x,t)$ and $v(x,t)$ be decomposed by an infinite series of components

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \qquad v(x,t) = \sum_{n=0}^{\infty} v_n(x,t), \qquad (33)$$

and the nonlinear terms defined in (32) are decomposed by the infinite series:

$$N_1(u, v) = \sum_{m=0}^{\infty} A_{1m}, \qquad N_2(u, v) = \sum_{m=0}^{\infty} A_{2m}, \qquad (33)$$

where $u_k(x,t)$ and $v_k(x,t)$, $k \geq 0$ are the components of $u(x,t)$ and $v(x,t)$ that will be elegantly determined, A_{kn} are called Adomian's polynomials and defined by

$$A_{km} = \frac{1}{m!}\left[\frac{d^m}{d\lambda^m} N_k\left(\sum_{i=0}^{\infty} \lambda^i u_i), \sum_{i=0}^{\infty} \lambda^i v_i\right)\right]_{\lambda=0}, \qquad k = 1,2, \quad m = 0,1,2,...$$

From the above considerations, the decomposition method defines the components $u_k(x,t)$ and $v_k(x,t)$ for $k \geq 0$, by the following recursive relations

$$u_0(x,t) = u(x,0), \qquad u_{n+1}(x,t) = L_t^{-1}[u_{nxx}] - L_t^{-1}[A_{1n}], \tag{34}$$

$$v_0(x,t) = v(x,0), v_{n+1}(x,t) = L_t^{-1}[v_{nxx}] - L_t^{-1}[A_{2n}]. \tag{35}$$

This will enable us to determine the components $u_n(x,t)$ and $v_n(x,t)$ recurrently. However, in many cases, the exact solution in a closed form may be obtained.
For numerical comparison purpose, we construct the solutions $u(x,t)$ and $v(x,t)$

$$u(x,t) = \lim_{n\to\infty} \Psi_n(x,t), \qquad v(x,t) = \lim_{n\to\infty} \Theta_n(x,t),$$

where

$$\Psi_n(x,t) = \sum_{i=0}^{n-1} u_i(x,t), \qquad \Theta_n(x,t) = \sum_{i=0}^{n-1} v_i(x,t), \qquad n \geq 1. \tag{36}$$

Now, by using the above procedure to the system (25)-(26), we can derive the solution of this system as follows

$$u_0(x,t) = \sin(x), \qquad v_0(x,t) = \sin(x),$$
$$u_1(x,t) = -t\sin(x), \qquad v_1(x,t) = -t\sin(x),$$
$$u_2(x,t) = \frac{t^2}{2!}\sin(x), \qquad v_2(x,t) = \frac{t^2}{2!}\sin(x).$$

The rest of components of the iterative formulas (34)-(35) are obtained in the same manner using the Mathematica package version 5. Where the first Adomian's polynomials of $A_{in},\ i = 1,2$ are given by

$$A_{10} = 2u_0u_{0x} - (u_0v_{0x} + v_0u_{0x}),$$
$$A_{20} = 2v_0v_{0x} - (u_0v_{0x} + v_0u_{0x}),$$
$$A_{11} = 2(u_0u_{1x} + u_1u_{0x}) - (u_0v_{1x} + u_1v_{0x} + v_0u_{1x} + v_1u_{0x}),$$
$$A_{21} = 2(v_0v_{1x} + v_1v_{0x}) - (u_0v_{1x} + u_1v_{0x} + v_0u_{1x} + v_1u_{0x}).$$

In the same manner, we can compute other components of A_{in}, $i = 1,2$.

Step 2. Truncate the series solution obtained by ADM

We have applied ADM by using the fourth order approximation only, i.e., the approximate solutions is

$$u(x,t) \cong U(x,t) = \sum_{i=0}^{n-1} u_i(x,t) = \left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!}\right)\sin(x), \tag{37}$$

$$v(x,t) \cong V(x,t) = \sum_{i=0}^{n-1} v_i(x,t) = \left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!}\right)\sin(x). \tag{38}$$

The behavior of the error between the exact solution and the solution obtained by ADM in the regions $0 \leq x \leq 1$ and $0 \leq t \leq 5$ is shown in the figures (2 and 3). The numerical results are obtained by using fourth order approximation only from the formulas (34)-(35). From these figures, we achieved a very good approximation for the solution of the system at the small values of time t, but at the large values of the time t, the error takes large values.

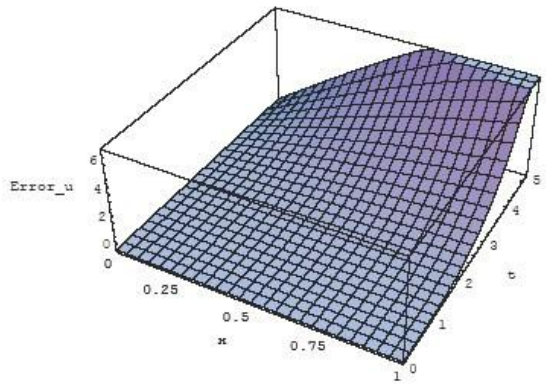

Figure 2: The error$-u = |u(x,t) - U(x,t)|$.

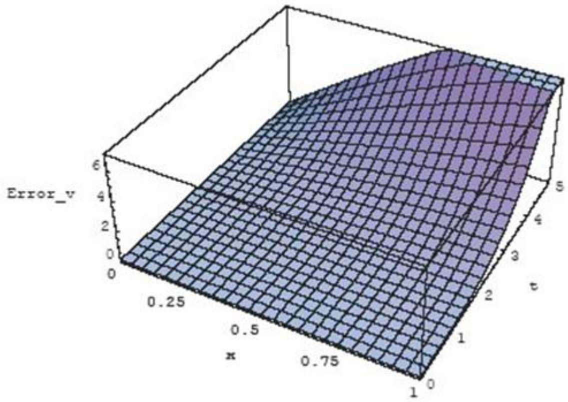

Figure 3: The error$-v = |v(x,t) - V(x,t)|$.

Step 3. Take the Laplace transform of the equations (37)-(38)

$$£[U(x,t)] = \left(\frac{1}{s} - \frac{1}{s^2} + \frac{1}{s^3} - \frac{1}{s^4} + \frac{1}{s^5}\right)\sin(x), \qquad (39)$$

$$£[V(x,t)] = \left(\frac{1}{s} - \frac{1}{s^2} + \frac{1}{s^3} - \frac{1}{s^4} + \frac{1}{s^5}\right)\sin(x), \qquad (40)$$

For the sake simplicity, let $s = \frac{1}{t}$ then

$$£[U(x,t)] = (t - t^2 + t^3 - t^4 + t^5)\sin(x), \qquad (41)$$

$$£[V(x,t)] = (t - t^2 + t^3 - t^4 + t^5)\sin(x). \qquad (42)$$

Step 4. Find the Padé approximation of the equations (41)-(42)

All of the $[\frac{L}{M}]$ Padé approximants of the above equation with $L > 0$, $M > 0$ yields

$$[\frac{L}{M}] = \frac{t}{1+t}\sin(x).$$

Replace $t = \frac{1}{s}$, we obtain $[\frac{L}{M}]$ in terms of s as follows:

$$[\frac{L}{M}] = \frac{1}{1+s}\sin(x).$$

Step 5. Take the inverse Laplace transform

By using the inverse Laplace transform to $[\frac{L}{M}]$, we obtain the true solution

$$u(x,t) = e^{-t}\sin(x), \qquad v(x,t) = e^{-t}\sin(x),$$

which are the same exact solution of the system (25)-(26).

VII. SUMMARY AND CONCLUSIONS

In this paper, we presented a modification of ADM, this modification considerably capable for solving a wide-range class of linear and nonlinear differential equations; especially the ones of high nonlinearity order in engineering and physics problems. This purpose was satisfied by solving physical model of nonlinear coupled system of PDEs. From the obtained results, we can conclude that the application of Padé approximants to the truncated series solution (from ADM) greatly improve the convergence domain and accuracy of the solution. Also, we can conclude that the approximate solutions of the presented problems are excellent agreement with the exact solution. Finally, we point out that the corresponding analytical and numerical solutions are obtained according to the iteration equations using Mathematica 5.

REFERENCES

[1]. . M. Abualnaja, A block procedure with linear multi-step methods using Legendre polynomials for solving ODEs, *Applied Mathematics*, **6**, p.(717-723), (2015).

[2]. . M. Abualnaja and M. M. Khader, A computational solution of the multi-term nonlinear ODEswith variable coefficients using the integral-collocation-approach based on Legendre polynomials, *Journal of Progressive Research in Mathematics*, **9(3)**, p.(1406-1410), 2016.

[3]. . Adomian, *Solving Frontier Problems of Physics, The Decomposition Method*, Kluwer, (1994).

[4]. . Adomian, A review of the decomposition method in applied mathematics, *Math. Anal. App.*, **135**, p.(501-544), (1988).

[5]. . A. Jr. Baker, *Essentials of Padé Approximants*, Academic Press, (1975).

[6]. S. M. El-Sayed and D. Kaya, On the numerical solution of the system of two-dimensional Burger's equations by the decomposition method, *Applied Mathematics and Computation*, **158**, p.(101-109), (2004).

[7]. . Kaya and I. E. Inan, A convergence analysis of the ADM and an application, *Applied Mathematics and Computation*, **161**, p.(1015-1025), (2005).

[8]. . S. Murray, *Laplace Transform*, New York, Schaum's Outline Series, (1981).

[9]. . H. Sweilam, Harmonic wave generation in non-linear thermo-elasticity by variational iteration method and Adomian's method, *J. Comput. Appl. Math.*, **207**, p.(64-72), (2007).

[10]. . H. Sweilam and M. M. Khader, Exact solutions of some coupled nonlinear partial differential equations using the homotopy perturbation method, Accepted in, *Computers and Mathematics with Applications*, **58**, P.(2134-2141) (2009).

[11]. . H. Sweilam, M. M. Khader and R. F. Al-Bar, Nonlinear focusing Manakov systems by variational iteration method and Adomian decomposition method, *Journal of Physics: Conference Series*, **96**, p.(1-7), (2008).

[12]. . M. Wazwaz, A comparison between Adomian decomposition method and Taylor series method in the series solution, *Appl. Math. Comput.*, **97**, p.(37-44), (1998).

[13]. P. Yang, Y. Chen and Zhi-Bin Li, ADM-Padé technique for the nonlinear lattice equations, *Applied Mathematics and Computation*, 210, p.(362-375), (2009).

K

K

G

G

G