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Optimization to Mitigate Voltage Harmonics of Cascaded Multilevel Converters

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ABSTRACT—The use of pulse width modulation with selective harmonic elimination (SHE-PWM) in cascaded H-bridges (CHB) multilevel converters allows the reduction of the output current distortion by the elimination of low order harmonic components. However, voltages produced by CHB modules to synthesize the SHE-PWM pattern is an issue that deserves deeper investigation. CHB systems fed by sources with current source characteristics, as photovoltaic strings operating with MPPT, may feature dif- ferent source capacities. Thus, it is important that the energy provided by each module is proportional to the available energy, so input voltages remain balanced. This paper presents the development of a genetic algorithm (GA) to determine optimal waveforms to be synthesized by the CHB modules that ensures the equalization of the fundamental components, or even that a specific relationship between between them is achieved. Simula- tion results for five and nine-levels demonstrate the applicability of the technique, allowing the voltage balance of the DC buses tobeaddressedeveninthemodulationstage.

Index Terms—Genetic algorithm, modulation, optimization, multilevel inverters, harmonic elimination, THD.

I. INTRODUCTION

Applicationsinvolvinghighlevelsofpowerandvoltageprovide additional constructive challenges to power electronics. The cost and the reduced availability of low-loss semiconductors for these applications still imposes major restrictions on its implementation. In these systems, multilevel converters are shown as ideal candidates for reducing voltage or current efforts imposed to semiconductor[1].

Among available topologies, a commonly employed is the cascaded H-bridge (CHB), which is formed by the series connection of two or more full-bridge converters fed by independent DC buses [2]. Recently these converters have been also employed by systems fed by renewable sources, where each source may be implemented by an array of photovoltaic modules [3]. Under MPPT operation most of PV array exhibit the behavior of a current source, which leads to the CHB system depicted by Fig. 1. In these cases, the DC-bus voltage is a function of circuit currents. For symmetric cases, where the DC-bus voltages are equal, each H-bridge can produce $+V_{dc}$, V_{dc} and 0V on the output. Then, a CHB arrangement of N converter is able to synthesize $2N + 1$ voltage levels. On the other hand, for asymmetric cases the number of levels can reach 3^N, according to the input voltage configuration [4].

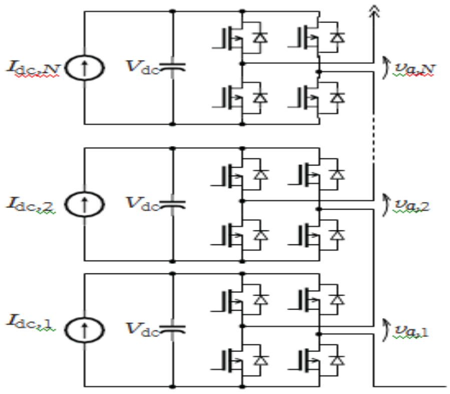

Fig. 1. Basic structure for one phase of a multilevel cascaded H-bridges with N-levels

One of the strategies that may be applied to CHB multilevel converter is the pulse width modulation with selective harmonic elimination (SHE-PWM). Proposed by Patel and Hoft [5], [6], this technique is based on the switching of DC bus voltages at predefined instants [7]. For each switch in the output waveform, one can obtain the desired amplitude to the fundamental component, while eliminating a certain number of harmonics of the output voltage and simplifying the filtering stage. Besides, since the inverter operates at low frequency, the overall system efficiency may be improved. The determination of switching instants for SHE-PWM has been extensively studied and several techniques for this task have been proposed [5]–[11]. However, this is only the first stage in the modulation design. In the sequence it is still necessary to specify the voltages to be produced by each H-bridge so thattheSHE-PWMcanbesynthesized.

Grid-tie systems, with sources that may feature different capacities, require attention in specifying voltages of CHB modules. If the output voltage pattern extracts the same energy amounts from modules with different capacities (different current values), the DC-bus voltages of one or more modules may collapse. This requires that the voltage generated by each inverter is carefully defined to proportionally extract

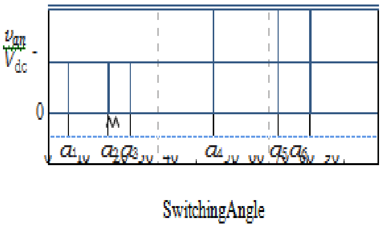

Fig. 2. General waveform for a cascaded multilevel inverter operating with SHE-PWM featuring six transitions per quarter wave.

the available power from each source. This task may easily become too complex without a computational support.

In this context, this main idea of this paper is to propose an automatic method to specify the output voltage waveforms of each module in a cascaded multilevel inverter fed by current sources as in Fig. 1. Based on a genetic algorithm (GA), and for any previously calculated switching pattern, the proposed method provide the optimal waveform to be synthesized by each CHB module such that fundamental components can be balanced, or even that a specific unbalance is achieved. Furthermore, the proposed strategy also keepthe number of switching events to a minimum, avoiding efficiency degradation.

II. SELECTIVE HARMONIC ELIMINATION

In Fig. 2 is presented an example of a SHE-PWM waveform. The six transitions per quarter wave allow six degrees of free- dom to achieve different objectives [5]. The output voltage v_{an} is defined by the Fourier series

$$v_{an}(t) = \sum_{h=1,3,5,\dots}^{\infty} V_h \mathrm{sen}(h\omega t) \qquad (1)$$

where V_h the coefficient of h-th harmonic, defined by

$$V_h = \frac{4V_{dc}}{h\pi}\cos(h\alpha_1) - \cos(h\alpha_2) + \cos(h\alpha_3) + \cos(h\alpha_4) - \cos(h\alpha_5) + \cos(h\alpha_6) \qquad (2)$$

where V_{dc} is the DC bus voltage of H-bridges and α are the switching angles. Assuming that the cos function of a vector returns the cosine of each element in a new vector with the same dimensions, the Fourier coefficients of (2) can be rewritten for any SHE-PWM pattern under the generalized matrix form

$$V_h = \frac{4V_{dc}}{h\pi}\cos(h\boldsymbol{\alpha})\mathbf{1} \qquad (3)$$

ωt

where $\boldsymbol{\alpha} = [\alpha_1, \dots, \alpha_S]$ a row vector containing the transition angles and $\mathbf{T}$ a column vector of S elements that describes the transitions which characterize the PWM pattern on the first quarter wave, where 1 represents a level increase and -1 a level decrease on the output voltage. For the waveform depicted by Fig. 2, $\mathbf{T}$ is defined by

$t(x) =$

$$\mathbf{T}_1^T = [1 \ -1 \ 1 \ 1 \ -1 \ 1] \qquad (4)$$

freedom as observed in (2), one can define the modulation index m and also eliminate five low order harmonics by the solution of the switching angles $\alpha_1, \alpha_2, \dots, \alpha_6$ from the system of nonlinear equations

$$\begin{cases} \frac{4V_{dc}}{\pi}\cos(\boldsymbol{\alpha})\mathbf{T} = m \\ \frac{4V_{dc}}{h\pi}\cos(h\boldsymbol{\alpha})\mathbf{T} = 0, \ h = 5, 7, 11, 13, 17 \end{cases} \qquad (5)$$

subject to the linear inequalities

$$0 < \alpha_1 < \alpha_2 < \alpha_3 < \alpha_4 < \alpha_5 < \alpha_6 < \frac{\pi}{2} \qquad (6)$$

and also to the linear equalities

$$\frac{4V_{dc}}{\mathbf{h}\pi}\cos(\mathbf{h}\boldsymbol{\alpha})\mathbf{T} = 0 \qquad (7)$$

representing the harmonics to be eliminated, where

$$\mathbf{h} = [5 \ 7 \ 11 \ 13 \ 17] \qquad (8)$$

is a vector with the index of the harmonic to be eliminated. This problem can be easily extended to any odd number of levels. It is worth to notice that in this case is assumed that the modulation index is defined as the ratio between the fundamental component of the output voltage by the DC bus voltage ($v_{an,1}/V_{dc}$).

A. Solution of Nonlinear Equations

The system of equations defined in (5) is nonlinear and transcendental, and for its solution there are several methods available in the literature. Patel and Hoft [5] employed the Newton-Raphsonmethod,iterative,fastandwhoseresultis only one of the possible solutions. However, the convergence

of this methods is strongly dependent of the initial guess x_0, which is not always simple to obtain. In an effort to assist the determination of a good x_0, Enjeti and Lindsay [8] have proposed an approximation method of the exact solution for the nonlinear equations exploring the symmetries observed in thetrajectoriesoftheswitchingangles.

Alternatively, the sequential quadratic programming (SQP) is a modern method that is capable to solve nonlinear equations subject to restrictions. SQP performs an inline search, approximating to the Newton method for optimizations with restrictions. In this paper SQP is employed together with an active set optimization by means of a numerical program that implements the algorithm of Fletcher [12].

Finally, the function to be minimized is obtained from the equation that determines the magnitude of the fundamental component

$$\frac{4V_{dc}}{\pi}\cos(\boldsymbol{\alpha})^{\mathbf{T}}-m \qquad (9)$$

Assigning

where the superscript ᵗdenotes the vector or matrixtranspose.

According to Patel and Hoft [5], given S transitions per quarter wave, to control the amplitude of the fundamental component and eliminate S-1 harmonics, S equations are needed. Thus, for the case of Fig. 2, with six degrees of

and solving for the SHE-PWM pattern of Fig. 2 with an initial guess defined by

$$\boldsymbol{\alpha}_{0,1}=[20.1451^\circ \quad 22.0635^\circ \quad 36.2940^\circ \quad 53.7186^\circ \quad 64.0051^\circ \quad 80.8106^\circ] \qquad (10)$$

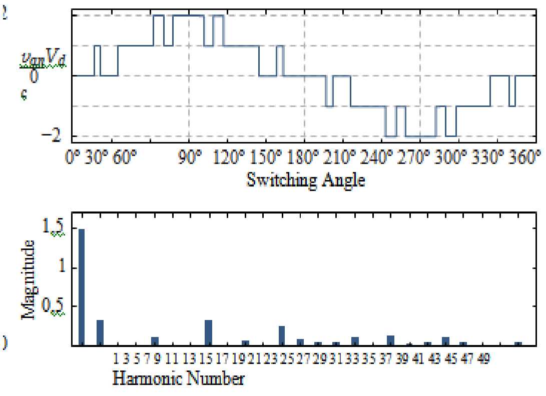

Fig. 3. SHE-PWM pattern and spectrum of the output voltage v_{an} for a modulation index m = 1.5.

leads to the vector of angles

$$\boldsymbol{\alpha}_1=[16.5745^\circ \quad 21.6692^\circ \quad 35.6092^\circ \quad 62.8303^\circ \quad 70.9616^\circ \quad 78.1385^\circ] \qquad (11)$$

whose resultant SHE-PWM pattern, as well as its harmonic components are depicted by Fig. 3.

By the nature of the equation there are multiple possible solutions, which depend on the initial attempt $\boldsymbol{\alpha}_0$. Fig. 4 shows the trajectory of switching angles as a function of the variation of the modulation index for the results obtained from (10) and from other initial guesses defined by

$$\boldsymbol{\alpha}_{0,2}=[21.0734^\circ \quad 37.0704^\circ \quad 42.3072^\circ \quad 54.3622^\circ \quad 69.8550^\circ \quad 79.8245^\circ] \qquad (12)$$

$$\boldsymbol{\alpha}_{0,3}=[3.6956^\circ \quad 23.3308^\circ \quad 35.1796^\circ \quad 44.9371^\circ \quad 65.6209^\circ \quad 81.7783^\circ] \qquad (13)$$

Defined the SHE-PWM pattern and the switching angles, the next step in the design process is to determine the voltage waveform that each individual converter must produce to synthesize the desired pattern. Frequently, these voltages are assigned on the basis of prior knowledge of the designer, seeking to reduce the number of commutations and maintain a balance of DC bus voltages. However, this conventional approachdoesnotguaranteethatthedesigneroptionisoptimal or that voltages produced by the converters are balanced. It is from this problem that this paper proposes an automatic strategy to define the output voltage waveform to be synthesized by each individualconverter.

III. A GENETIC ALGORITHM TO SPECIFYOPTIMAL OUTPUT VOLTAGE WAVEFORMS

The strategy for the optimization of voltages of the converters investigated in this paper is conducted by means of a genetic algorithm whose solution sequence is detailed in this

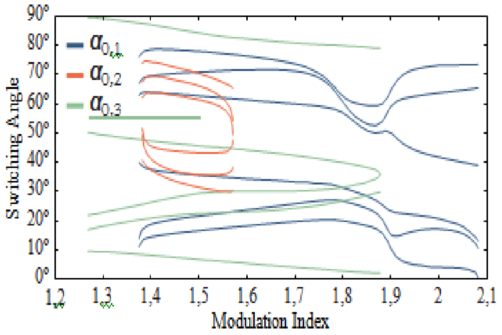

Fig. 4. Trajectories of switching angles for the transition vector defined on (4) and from the different initial guesses α_0.

2) Group genes by type according to voltage they are able toproduce;

section, which can be summarized in the following steps:

1) Maps the synthesizable voltages by the CHB and determines the possible genes to be employed in the algorithm;

3) Randomly creates the initialpopulation;
4) Generate voltage waveforms for each H-bridge ($v_{a,N}$), evaluatestheFFTandsolvetheobjectivefunction;
5) Sortschromosomesbasedonitscosts;
6) Evaluate the stop condition. If it is met, terminates the algorithm;
7) Apply naturalselection;
8) Creates the next generation from crossover or from modifications on random genes(mutation);
9) Return to the step4.

Instead of the quarter-wave symmetry assumed for determi-nationofswitchingangles,thisstepassumestheoutputvoltage of the converters has half-wave symmetry. This assumption increases the degree of freedom in the search for optimal solution.

To avoid the GA indefinitely continue searching for an optimalsolution,atleastonestopcriteriashouldbeestablished by the designer [13]. One of the criteria used in this paper sets the maximum number of generations in three hundred, while the other ends the evolutionary process if the cost of the best chromosome does not significantly evolve in fifteen generations.

A. Initial PopulationCreation

The first step for the proposed genetic algorithm is the creation of genome, in the identification of genes and in the generation of the initial population, which is created randomly from the available genes. Each gene is associated with the phase voltage level produced by the inverter in the time interval between two switches. Observing again the pattern depicted by Fig. 2, one can observes that for a half cycle of the output voltage v_{an}, there are thirteen time slots and, in turn,thirteengeneswillcomposeeachchromosome.

Not all voltage combinations that can be generated by the converter modules may be employed to reproduce the SHE PWMpattern. Thus,duringtheinitializationofGAallpossible voltage combinations that can be produced by each phase leg are determined. After the combinations that can be used to synthesize the output voltage required at each interval are identified. For the arrangement of two converters in thepattern illustrated by Fig. 2, and knowing that the normalized voltages each converter is capable to produce are 1, 0, 1, one canmap thesynthesizableoutputvoltages v_{an}asdefinedbyTableI.

TABLE I
SYNTHESIZABLE OUTPUT VOLTAGES FOR A PHASE LEG OF THE HREE-LEVEL INVERTER AND ITSTYPES OF ASSOCIATED GENES.

V_{dc}	$\frac{V_{a,2}}{V_{dc}}$	$\frac{V_{an}}{V_{dc}}$	Gene
−1	−1	−2	—
−1	0	−1	—
−1	1	0	Type0
0	−1	−1	—
0	0	0	Type0
0	1	1	Type1
1	−1	0	Type0
1		1	Type1
1	1	2	Type 2

TABLE II
GENOME EMPLOYED TO SOLVE THE SHE-PWM PATTERN OF FIG. 3.

Gene	1	2	3	4	5	6	7	8	9	10	11	12	13
Type	0	1	0	1	2	1	2	1	2	1	0	1	0

TABLE III
INDEXERS AND AVAILABLE GROUPS OF GENES

[V_{a1}; V_{a2}]				
	Index	Type 0	Type1	Type2
[0; 0]	1 2	[−1; 1]	[0; 1] [1; 0]	[1;1] —
3	[1;−1]	—	—	

In addition to mapping the possible voltage combinations, one need to identify which ones may be used for each gene. Given the half wave symmetry of the output voltage, combinations that results in a negative voltage are not considered and may be discarded, while positive combinations are grouped according the voltage produced by each phase of the inverter. For the group of combinations that produce zero level in the output voltage the genes are named as Type 0, and so on for the other groups of combinations that generate positive voltage levels, as defined by Table I.

The genome employed in the solution of the SHE-PWM pattern of Fig. 3 is presented by Table II and can be easily redefined for other patterns by simple inspection of the v_{an} waveform.

During the creation of the initial population, the algorithm generates a random value for each gene. This value corresponds to an index of the available genes for each group, as presented by Table III. This ensures that the algorithm uses only the appropriate type of gene, producing feasible combinations whose sum of voltages of the H-bridges correspond exactly to the analyzed SHE-PWM pattern.

B. Chromosome Cost Evaluation

The objective of each chromosome is to determine the output voltage waveform of each CHB module so the fundamental components can be equalized, or even that a specific balance between them is imposed. To provide this feature a positive-defined diagonal weighting matrix is defined as

$$\mathbf{W} = \begin{bmatrix} w_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & w_N \end{bmatrix} \qquad (14)$$

allowing one to establish a specific ratio between each fundamental component of the output voltages, where w_N a positive weight to be given to the components of the N -th converter. Also, it is of practical interest that the total number of switching in the converters c_s by period of the output voltage be minimal so that efficiency is not adversely affected by switching losses. To simultaneously achieve both targets, the GA employs an objective function defined by

where c_s the number of switching events per period of the SHE-PWM pattern and $\mathbf{H}$ a row vector containing the magnitude of the fundamental components of each converter.

C. Natural Selection and Crossover

After evaluating the objective function, chromosomes are sorted according to their costs in ascending order. A x_{nat} rate defines how many chromosomes will survive the natural selection and generate offspring to replace chromosomes with worst cost, previously discarded. For the solution of the proposed problem a rate of $x_{nat} = 0.2$ is defined. This value has been chosen after a series of preliminary studies in which it was found that this rate allows a better compromise between convergence time and accuracy of the results. The pair of chromosomes that will generate a pair of offspring is selected by the roulette wheel method, in which the area of the corresponding section to an individual at roulette is proportional to its expectation of selection [13]. In this way, individuals who have a lower cost have higher probability of selection.

In the GA developed for this paper, offspring generation is carried out by means of scattered crossover. In this method, for each pair of chromosomes selected to generate offspring, a random binary mask with the same number of genes is generated. For the first offspring, all elements that in the mask has a value 0 the offspring receives the genes of the father, and for the other elements that the mask has a value 1, the offspring receive the genes of the mother. For the second offspring the same mask is employed, however in a complimentary way.

D. Mutation

Mutation is responsible to include new genes in the population, increasing its diversity. This is carried out by means of random modifications in some genes. A mutation rate x_{mut} establishes the portion of genes that will suffer mutation.

$$f_{obj} = - \frac{\sum_{i=1} \mathbf{H}_i \mathbf{W}_{i,i}}{c_s \quad 1 + N|\max(\mathbf{HW}) - \min(\mathbf{HW})|} \qquad (15)$$

Here a rate of $x_{mut} = 0.1$ is employed by the same reason of x_{nat}. To perform mutation is first necessary to identify the type of gene to be modified through its position in the genome (as in Table II, for example). After, a random integer value is generated, which needs to belong to the range of one to the number of available genes belonging to the same type of the gene.

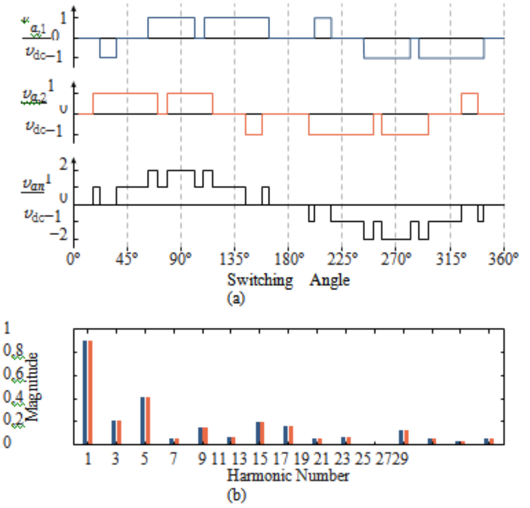

Fig. 5. Results obtained by the proposed GA for the switching pattern of Fig.3withbalancedfundamentalcomponents(a)voltagewaveformsproduced by each H-bridge; (b) harmonic components of the output voltage for each converter.

IV. CASESTUDIES

The GA-based optimization strategy proposed in the previous sectionisvalidatedbymeansofcasestudiesbasedon simulations. Results for cascaded multilevel inverters with five andnine-levels arepresented.Inthefirstcasetheobjective is to achieve the desired modulation index and match thefundamentalcomponentsofafivelevelinverter.Thisisthecase,forexample,ofsourceswithequalcurrentsIdc,1andIdc,2onFig.1.Forthesecondcase,theobjective istoalsoreachthemodulationindex,butinthiscasetoforceaspecificunbalancebetweenfundamentalcomponentsoftheoutputvoltagesforaninelevelinverter.Anexamplewheredifferentpowerlevelsneedtobeextractedfromeachconvertersothatvoltagesofdcbusesaremaintainedbalanced.Resultsandcommentsfor each case are presentedbelow.

A. Five-levels: balanced fundamentals

For this first case the objective is to find the optimal output voltage for the individual CHB converters so that the SHE- PWM pattern described by the transition vector (4) provide equalfundamentalcomponents.Thismaybeachievedddefining a weighting matrixas

$$W1=\text{diag} \quad 1 \;\; 1 \qquad (16)$$

that will be employed by the objective function (15), where diaga function for diagonal matrices creation. The optimal waveforms obtained by the proposed GA is depicted by Fig. 5(a). Fig. 5(b) detail the harmonic components, where one can verifies that both fundamentals of each converterareequalized.Moreover,asitcanbeseenonFig.6,thisbalanceis maintained over the full range of modulation indexes that the SHE-PWM pattern is able to synthesize. The solution found by the GA features 24 switching, which for this case is equal tothenumberofswitchingpatternitself.

B. Nine-levels: unbalanced fundamentals

To evaluate the proposed algorithm in a more complex scenario, it is also presented a case study for a nine-level inverter where it is desirable to impose an unbalance for the fundamental voltages of the converters. This would be the case, for example, of DC sources with different capacities, as batteries with distinct charge levels. Other application of this feature is for CHB fed by capacitors, where a specific unbalance may desirable to provide resources for the system toreestablishtheequilibriumofDCbusvoltages.

In this case, assuming that the harmonics 5, 7, 11, 13, 17, 19 and 23 must be eliminated, it is necessary 8 switching events per quarter wave so that the magnitude of fundamental com- ponent for the voltage v_{an}can be controlled. Thus, defining aSHE-PWMswitchingpatternbythetransitionvector

$$\mathbf{T} = \quad 1 \quad 1 \;\; -1 \quad 1 \quad 1 \quad 1 \;\; -1 \quad 1^T \qquad (17)$$

with a modulation index of $m = 3, 8$ and using as an initial guess

$$\boldsymbol{\alpha}_{0,4}=[7.524^\circ \quad 19.400^\circ \quad 30.682^\circ \quad 40.750^\circ \quad 55.047^\circ \quad 69.999^\circ \quad 75.063^\circ \quad 84.203^\circ] \qquad (18)$$

it is obtained vector of angles

$$\boldsymbol{\alpha}_2=[7.700^\circ \quad 25.332^\circ \quad 28.447^\circ \quad 30.255^\circ \quad 43.160^\circ \quad 62.242^\circ \quad 67.978^\circ \quad 73.445^\circ] \qquad (19)$$

For this case, the resulting SHE-PWM pattern and as its harmonic components are illustrated by Fig. 7. It is worth noting that the triplet harmonics has not been eliminated since a final three phase system is assumed.

In order to force an unbalance of fundamentals, the weight- ing matrix is defined as

$$\mathbf{W}_2=\text{diag} \quad 1,0 \;\; 0,9 \;\; 0,8 \;\; 0,7 \qquad (20)$$

and the GA is run for the SHE-PWM pattern of Fig. 7.The results obtained are depicted by Fig. 8, where one can observe 40 switching events, while the output voltage vanfeatures 32 switchingevents.

The development of the fundamental components of the converter due to the variation of the modulation index for the resulting configuration is illustrated by Fig. 9, where expected unbalance of the fundamental components can be observed. The forced unbalance occurs over the entire range of modulation indexes that the SHE-PWM patter is able to synthesize, although it need to be highlighted that a fixed ratio cannotbemaintainedthroughoutthisrange

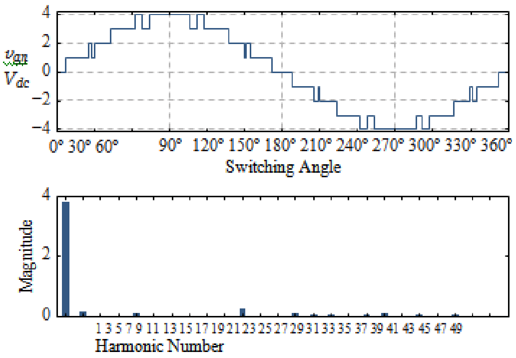

Fig. 7. SHE-PWM pattern defined in (17) and its harmonic components for $m = 3.8$.

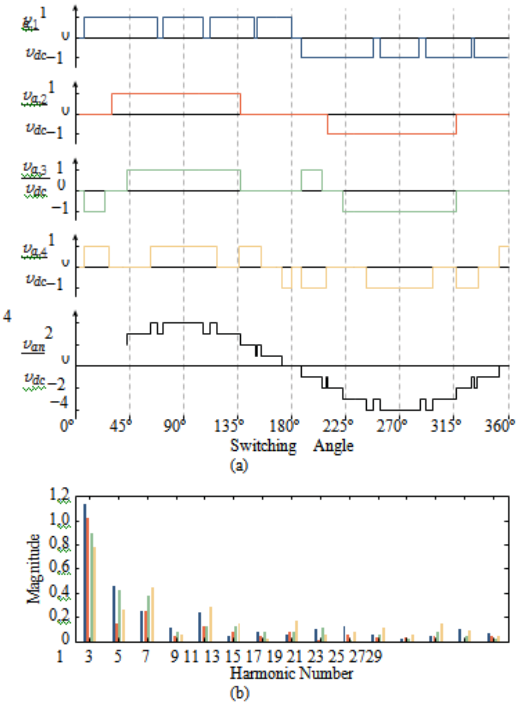

Fig. 8. Optimized converter waveforms for the SHE-PWM pattern of Fig. 7 with forced fundamental unbalance (a) voltages produced by individual converters; (b) harmonic components of output voltages of the converters

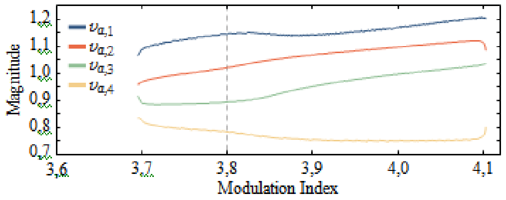

Fig. 9. Trajectories of fundamental components of the individual converters subject to variation of modulation index. (unbalanced case).

V. CONCLUSION

This paper presented an automatic method for specification of voltage waveforms to be synthesized by cascaded H- bridges multilevel converters employing SHE-PWM. Basedon a GA, and for any given SHE-PWM pattern with precomputedswitchingangles,theproposedmethodenablesabetterbalanceofpowerprocessedbyeachconverter.Besides,italsoallows to obtain waveforms with unbalanced fundamentals that can be used by control strategies as an additional resource for balancing and equalizing DC buses directly on the modulation stage. This makes the proposed strategy a majorcandidate for grid-tie systems fed by renewable sources. Case studies forfive and nine-level converters requiring balanced or unbal- anced fundamental components was presented, and the results corroborates with the applicability of the proposed strategy. It is worth noticing that although it was reported results only for those configurations, the proposed strategy can be easily applied to inverters with higher number of levels, with the elimination of as much harmonics as necessary, and even for asymmetric multilevelconverters.

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