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Optimization Techniques – A Review

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ABSTRACT
Optimization problem formulation, methods of optimization and solution techniques are presented. Population based methods are also explained. Optimization using constraints in terms of reliability is found to be best option for optimizing structures with discrete parameters.
***Keywords*:** Evolutionary methods, Genetic algorithms, Memetic algorithms, Particle swarm optimization

I. INTRODUCTION

The idea of selecting the best way to accomplish a task that involves fixed requirements is as old as civilization. Optimization can be defined as the art of obtaining best policies to satisfy certain objectives, at the same time satisfying fixed requirements. Recent advances in the field of applied mathematics, operational research, and digital computer technology enable many of the complicated engineering problems to be optimized successfully and thereby optimization technique has attained the status of a science rather than an art.

Design of most engineering systems is a fairly complex process. Engineer would try different trial designs with the quest of arriving at a design that is optimum. It is possible to formulate the engineering design problems as optimization problems with the objective of minimizing the cost or weight subject to satisfaction of all conditions of design.

The success of structural optimization during the past decades motivated the emergence of multidisciplinary design optimization (MDO) recently. The multidisciplinary design optimization is a field of engineering that uses optimization methods to solve structural design problems incorporating a number of disciplines. It is possible to incorporate all the required disciplines simultaneously. The optimum of such problems is superior to that of sequential optimization of each discipline.

An important aspect of the optimal design process is the formulation of the design problem in a mathematical format which is acceptable to an optimization algorithm. It involves the selection of design variables, constraints, objectives, and models of disciplines.

II. OPTIMUM PROBLEM FORMULATION

2.1. Design variables

Parameters chosen to describe the design of a system are called the design variables, which are varied primarily during the optimization process. Formulation of optimization problem begins with identifying the design variables. There is no rigid guideline to choose a priori the parameters which may be important in a problem, because one parameter may be more important with respect to minimizing the cost of the design, while it may be insignificant with respect to maximizing the life of the product. Thus the choice of the important parameters in an optimization problem largely depends on the user. Generally, cross sectional dimensions, thickness etc., are taken as design variables. Design variables are often bounded, that is, they often have maximum and minimum values. Depending upon the solution method, these bounds can be treated as constraints. The following considerations should be given in identifying the design variables.

i. As far as possible the design variables should be independent of each other.
ii. The number of design variables required to formulate the design problem should be minimum.
iii. At the initial formulation stage, it is good to have more number of independent parameters as design variables. Later on some of the design variables can be given a fixed value.

2.2. Constraints

The constraints represent some functional relationships among the design variables and other design parameters satisfying certain physical phenomenon and certain resource limitations. The constraints are mainly three types.

i. Inequality constraints
ii. Equality constraints
iii. Side constraints

2.2.1. Inequality constraints

If the functional relationships among the design variables are either greater than, smaller than, or equal to, a resource value, they are called as inequality constraints. Thus these constraints ensure safety against a failure mode or satisfactory behavior under the given loading conditions.

2.2.2. Equality constraints

The conditions that should exactly match with the resource value for the design to be acceptable are equality constraints.

2.2.3. Side constraints

The constraints such as maximum and minimum value of design variables are side constraints. They impose geometric restrictions on the design variables.

2.3. Objective function

There will be more than one feasible solution for most of the engineering designs and the aim of optimization is to select the best one among the feasible designs. Thus a criterion has to be formulated for selecting the best one out of the feasible solutions. The criterion must be a scalar function of the design variables whose numerical value can be obtained once the design is specified. Such a criterion is called the objective function of the optimization problem.

III. METHODS OF OPTIMIZATION

Various optimization methods are available that can be successfully used to determine the optimum values. These methods can be classified in to two groups: Gradient and Direct search methods (Non gradient methods).

3.1 Gradient based methods

Gradient based methods make use of calculus and derivatives of the objective function and constraints to search for the optimum. In general, the methods of optimization which require gradient information are considered more efficient. (Krishnamoorthy [1]). During the process of optimization of the design variables, the constraints placed on the solution must also be considered. There are various methods that can be employed to determine the optimum set of design variables that can provide the minimum or maximum value for a specific function. Of these methods, there are two basic ways of determining the optimum value, using a differential method or search method in the design field. These two methods can be further divided in to two sub groups as, problems with constraints and without constraints. For problems without constraints, the differential calculus method is considered as the best means of achieving an optimum solution. For problems with constraints there is a choice between differential calculus methods (Lagrangean and Kuhn-Tucker) or search methods (Linear Programming and Integer-Linear Programming) (Osvaldo Querin[2])

3.2 Non - gradient based methods

In these methods, only function values at different points are used to perform a search and do not use the partial derivatives of the function and hence are called non - gradient methods. These methods are most suitable for simple problems involving a relatively small number of variables. (Rao [3])

IV. MULTIDISCIPLINARY DESIGN OPTIMIZATION METHODS

4.1 Response surface methodology

Response surface methodology (RSM) is an approach for constructing approximation models based on either physical experiments or computer simulations.(S. Raissi et.al.,[4])The objective is to optimize a response(output variable) which is influenced by several independent input variables. Response Surface Methodology (RSM) is a collection of statistical and mathematical techniques useful for developing, improving, and optimizing processes. (Kathleen.et.al., [5]).

4.2 Evolutionary methods

In 1992, a new method of structural optimization was developed by Xie and Steven (Osvaldo Querin[2]) called the Evolutionary Structural Optimization (ESO) method. This method was developed from observations of natural process and evolution. ESO can locate both local or global minima. It does not stop when an apparent minimum is located, instead continue to search for a better one because of its evolutionary characteristics. The principles of this method were to gradually remove all under stressed material from the structural domain in an evolutionary fashion. The resulting structure will have a lower weight and a modified shape. The main idea of ESO is to determine the optimal shape and topology of the structure.

4.3 Reliability- based design optimization (RBO)

This is a growing area of interest in multi disciplinary design optimization techniques. Objective of this method is to design structures, which are reliable and economical. Reliability based methods can be grouped in to three categories. They are minimization of cost or weight for a allowable failure probability; minimization of probability of failure for a fixed cost; and minimization of the total overall cost. All the said methods use the concept of system reliability, ie, all possible failure modes and the corresponding failure probabilities of the structural system are considered for the computation of the overall failure probability. The overall failure probability or the structural reliability is the only behavior constraint in reliability based optimization.

The reliability based design optimization can be defined as minimization of an objective function subject to the probabilistic constraints. Simulation methods or moment methods can be used for the evaluation of the probabilistic constraints.

V. PROBLEM SOLUTION

Appropriate technique from the field of optimization is to be selected for solving the problem. These include gradient based algorithms, population based algorithms and others.

5.1 Gradient-based methods

5.1.1. Steepest descent method

Begins with an initial estimate for minimum design. Then the direction of the steepest descent at that point is calculated. If the direction is non zero, a line search along the negative of the derivative direction is performed to find the minimum point along that direction. The minimum point becomes the current point and continue search from this point. Repeat till convergence is reached.

5.1.2. Newton's method

This method uses second order derivatives to create search directions. The function value about the current point is determined by using second order Taylor's series expansion. Any design iteration which makes the Hessian matrix, a positive semi definite one, gives a global minimum.

5.1.3. Conjugate gradient method.

It is a modified version of the steepest descent method. The search directions can be obtained by using the recursive equation given by Fletcher and Reeves,[6]. The search direction thus obtained becomes linearly dependent after a few iterations. The extent of linear dependence is obtained by calculating the included angle between the two consecutive search directions. If the included angle is very close to zero the algorithm is to be restarted.

5.1.4. Variable-metric method (DFP method).

Davidon, [7] proposed this method and was modified by Fletcher and Powell,[8]. This method is considered as the most powerful method for the minimization of a function. In this method an estimate of the inverse of the Hessian matrix is created using first order derivatives. The expensive computation of the Hessian matrix and its inverse is thus eliminated.

5.2. Direct search methods

5.2.1. Hooke- Jeeves method.

It is sequential technique, each step consists of two kinds of moves, one called the exploratory move and other called the pattern move. The exploratory move is performed in the vicinity of the current point to find the best point around the current point. Then two such points are used to make the pattern search.

5.2.2. Powell's conjugate direction method

Powell's method is the most widely accepted direct search method. It uses the history of the previous solutions to create new search directions. A set of linearly independent directions are created and unidirectional searches are performed along each of these directions, beginning from the previous best point.

5.3. Population based methods

5.3.1. Genetic algorithm

Genetic algorithms have been developed by John Holland at the University of Michigan.(Goldberg, [9])Genetic algorithms are computerised search and optimization algorithms based on the mechanics of natural selection and natural genetics. They combine survival of the fittest among string structures and a systematic information exchange guided by random operators to form a sensible search procedure. Genetic algorithms begin with a randomly created population of string structures. Each string is then evaluated. To create better population, the random population is operated by three operators – reproduction, cross over and mutation. The reproduction operator selects good strings in a population and forms the mating pool. The cross over operator selects two strings from the mating pool at random and exchange some portion of the strings. The third operator mutation changes the string locally to create a better string with a small probability. Then the population is evaluated and tested for termination. The population is repeatedly operated by these operators till the termination criteria are attained.

5.3.2. Memetic algorithm

Memetic algorithms start with a population, which can be created in a random way or according to an initialization procedure. Each of these individuals is improved using a local search method. The individual members of the population now interact between each other. Operators of genetic algorithms, such as selection, crossover, mutation and elitism are used to combine the information present in the individuals of the population

5.3.3. Particle swarm optimization

PSO is a stochastic global optimization method based on simulation of social behavior of birds flocking or fish schooling developed by Eberhart and Kennedy in 1995. (Dian Palupi Rini et.al.,[10]). The operators inspired by natural evolution are not used for creating new population. PSO depends on the exchange of information between the individuals, called particles, in the population, called swarm. Each particle of the swarm is treated as a point in a hyper space, which fly through it with two essential capabilities: their memory of their own best position -local best , and knowledge of their global or their neighborhood's best – global best. The basic concept lies in accelerating each particle toward its local best and global best. The performance of each particle is measured by fitness function. The convergence is influenced by the inertia weight (Konstantinos,E.P.et.al.,[11]). The inertia weight is used to control the impact of previous velocity on current velocity. A suitable value of the inertia weight is to be used for creating a balance between the local and global exploration ability of the swarm there by providing the best solution.

5.4. Other methods

5.4.1. Random search method

A number of points are created at random. The function value is calculated in each of these points and the smallest value is chosen as the minimum. The search interval can be reduced by a constant factor depending on the function value.

5.4.2. Grid search method

This methodology involves setting up of grids in the decision space and evaluating the values of the objective function at each grid point. The point which gives the best value of the function is considered to be the optimum solution.

5.4.3. Simulated annealing

Simulated annealing is a point by point method. This procedure simulates the process of slow cooling of molten metal to get the minimum function value. The cooling phenomenon is simulated by controlling a temperature like parameter introduced with the concept of the Boltzman probability distribution. The algorithm begins with an initial point and a high temperature. A second point is created at random in the vicinity of the initial point and the difference in the function values at these points is calculated. If the second point has a smaller function value, that is accepted. Otherwise the point is accepted with a small probability. This completes one iteration .
(Kalyanmoy Deb, [12])

VI. CONCLUSION

Various optimization and solution methods are discussed in this chapter. Most of these traditional algorithms terminate when the gradient of the objective function is very close to zero. They do not guarantee to find global optimal solutions. In most of the real engineering problems, some simulations are required to evaluate the objective function. Thus exact evaluation of the gradient may not be possible in some problems. This suggests that to solve discrete or discontinuous problems, some robust search algorithms such as genetic algorithm is required. They work with a coding of variables which discretises the search space even though the function is continuous. They process more than one string simultaneously and the solution will be a global optimum. Genetic algorithms permit the use of probabilistic rules also to guide their search.

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