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Solutions of Multi Objective Fuzzy Transportation Problems with Non-Linear Membership Functions

Dr. M. S. Annie Christi[1], I.KALPANA[2]
[1]*Associate Professor, Department of Mathematics, Providence College for Women, Coonoor, India.*
[2]*Department of Mathematics, Providence College for Women, Coonoor, India*

ABSTRACT
Multi-objective transportation problem with fuzzy interval numbers are considered. The solution of linear MOTP is obtained by using non-linear membership functions. The optimal compromise solution obtained is compared with the solution got by using a linear membership function. Some numerical examples are presented to illustrate this.
Keywords: Multi-objective decision making, Goal programming, Transportation Problem, Membership function, Fuzzy programming, Interval Numbers.

I. PROBLEM FORMULATION

The transportation problem usually involves multiple, incommensurable and conflicting objective functions in the real-world situations. This kind of problem is called multi-objective transportation problem. Similar to a typical transportation problem in a MOTP a product is to be transported from m sources to n destinations and their capacities are $a_1, a_2, \ldots, a_m$ and $b_1, b_2, \ldots, b_n$ respectively. In addition, there is a penalty c_{ij} associated with transporting a unit of product from source i to destination j. The penalty may be cost or delivery time or safety of delivery or etc. A variable x_{ij} represents the unknown quantity to be shipped from source i to destination j. A mathematical model of MOTP can be written as follows:

Min

$$z_r = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r}\, x_{ij}, \quad r = 1,2,\ldots,k$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,\ldots m,$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,\ldots n,$$

$$x_{ij} \geq 0, \quad \text{for all i, j} \qquad (1)$$

The subscript in Z_r and superscript in c_{ij}^{r} are related to the r^{th} penalty criterion. Without loss of generality, it will be assumed that $a_i \geq 0$ for all i, $b_j \geq 0$ for all j and the equilibrium condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is satisfied.

We denote by S the set of all feasible solutions of the MOTP, i.e.,

$$S = \{x \in \mathbb{R}^{m\times n} \mid \sum_{j=1}^{n} x_{ij} = a_i, \ \sum_{i=1}^{m} x_{ij} = bj, \ xij \geq 0, \ i=1,2,\ldots m, \ j = 1,2,\ldots n\}$$

1.1. Definition:

A feasible solution $X^* = \{x_{ij^*}\} \in S$ is an efficient (non-dominated) solution for MOTP if and only if there do not exist another X = $\{x_{ij}\} \in S$ such that $Z_r(X) \leq Z_r(X^*)$, $r = 1,2,\ldots k$, $and\ Z_l(X) \neq Z_l(X^*)$ for some l, $1 \leq l \leq k$.

1.2. Definition:

A feasible solution $X^* = \{x_{ij^*}\} \in S$ is a weak efficient solution for MOTP if and only if there does not exist another X = $\{x_{ij}\} \in S$ such that $Z_r(X) < Z_r(X^*)$ $\quad r = 1,2,\ldots k$

1.3. Definition: [3]

Optimal compromise solution for MOTP if it is preferred by DM to all other feasible solutions, taking into consideration all criteria contained in the multi-objective functions.

Let E and E^w denote the set of all efficient solutions and all weak efficient solutions for MOTP, respectively, then E $\subseteq E^w$. The best compromise solution of MOTP has to be a weak efficient solution of MOTP.

II. MEMBERSHIP FUNCTIONS

In solving fuzzy mathematical programming problems of the linear membership functions for all fuzzy sets involved in a decision making process. A linear approximation is most commonly used and is defined by fixing two points, the upper and lower levels of acceptability. If fuzzy set theory is to be considered a purely formal theory, such an assumption is acceptable, even though some kind of formal justification of this assumption would be desirable. The fuzzy set theory is used to model real decision making processes, and a fact is made that the resulting models are true models of reality then the practical explanation for this assumption is necessary. Such as concave or convex shaped membership functions are analyzed to determine their impact on the overall design process.

Let L_r and U_r be the aspired level of achievement and the highest acceptable level of achievement for the r^{th} objective function, respectively. Next, we study different membership functions.

2.1. Linear Membership Function.

A linear membership function can be defined as follows.

$$\mu_r\big(Z_r(X)\big) = \begin{cases} 1 & \text{if } Z_r \le U_r \\ 1-\frac{Z_r-L_r}{U_r-L_r} & \text{if } L_r < Z_r < U_r, \\ 0 & \text{if } Z_r \ge U_r \end{cases} \tag{2}$$

2.2. Exponential Membership Function.

An exponential membership function is defined by

$$\mu_r^E\big(Z_r(X)\big) = \begin{cases} 1 & \text{if } Z_r \le L_r \\ \frac{e^{-s\psi_r(x)}-e^{-s}}{1-e^{-s}} & \text{if } L_r < Z_r < U_r, \\ 0 & \text{if } Z_r \ge U_r \end{cases} \tag{3}$$

Here $\psi_r(x) = \frac{Z_r-L_r}{U_r-L_r},\ r = 1,2,\ldots,k$ and s is a non-zero parameter prescribed by the decision maker.

2.3. Hyperbolic Membership Function.

The hyperbolic function [27] is convex over a part of the objective function values and is concave over the remaining part. In our problem context is as follows: When the decision maker is worse off with respect to a goal, the decision maker tends to have a higher marginal rate of satisfaction with respect to that goal. A convex shape captures the behavior in the membership function. On the other hand, when one is better with respect to a goal, one tends to have a smaller marginal rate of satisfaction. Such behavior is modeled using the concave portion of the membership function.

$$\mu_r^H(Z_r(x)) = \begin{cases} 1 & \text{if } Z_r \le L_r, \\ \frac{1}{2}\tanh\left(\frac{U_r+L_r}{2} - z_r(x)\alpha_r\right)+\frac{1}{2} & if\ L_r < Z_r < U_r, \\ 0 & \text{if } Z_r \ge U_r \end{cases}$$

Where $\alpha_r = \frac{6}{U_r-L_r}$

This membership function has the following formal properties [32]:

(1) $\mu_r^H(Z_r(x))$ is strictly monotonously decreasing function with respect to $Z_r(x)$;

(2) $\mu_r^H\big(Z_r(x)\big) = \frac{1}{2} \Leftrightarrow Z_r(x) = \frac{1}{2}(U_r + L_r)$;

(3) $\mu_r^H\big(Z_r(x)\big) =$ is strictly convex for $Z_r(x) \ge \frac{1}{2}(U_r + L_r)$ and strictly concave for $Z_r(x) \le \frac{1}{2}(U_r + L_r)$;

(4) $\mu_r^H\big(Z_r(x)\big)$ Satisfies $0 < \mu_r^H\big(Z_r(x)\big) < 1\ for\ L_r < Z_r(x) < U_r$ and approaches asymptotically $\mu_r^H\big(Z_r(x)\big) = 0\ and\ \mu_r^H\big(Z_r(x)\big) = 1\ as\ Z_r(x) \to \infty\ and -\infty$, respectively.

III. FUZZY GOAL PROGRAMMING APPROACH FOR SOLVING MOTP

Mohamed [23] introduced fuzzy goal programming approach for solving multi-objective linear programming problem. In [31], Mohamed's approach was adopted to present a fuzzy goal programming approach for solving multi-objective transportation problems.

Let L_r and U_r be the aspired level of achievement and the highest acceptable level of achievement for the r^{th} objective function, respectively.

To solve MOTP problem based on the fuzzy goal programming technique [31], one can use the following steps:

Step 1:

Solve the multi-objective transportation problem as a single objective transportation problem, taking each time only one objective function and ignoring all others.

Step 2:

Compute the value of each objective function at each solution derived in Step 1.

Step 3:

From Step 2, find for each objective the best (L_r) and the worst (U_r) values corresponding to the set of solutions. Recall that L_r and U_r are the aspired level of achievement and the highest acceptable level of achievement for the r^{th} objective function, respectively.

Step4: Define a membership functions μ_r (linearμ_r^L, hyperbolic μ_r^H, or exponential μ_r^E) for the r^{th} objective function.

If we use the linear membership function as defined in (2) then an equivalent linear model for the model (1) can be formulated as:

Min: φ,
Subject to,

$$\frac{U_r - Z_r}{U_r - L_r} + d_r^- - d_r^+ = 1,$$

$$\varphi \geq d_r^-, \quad r = 1,2, \dots\dots, k,$$

$$d_r^+ \, d_r^- = 0,$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2, \dots\dots m,$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2, \dots\dots n,$$

$$d_r^+, d_r^- \geq 0,$$

$$\varphi \leq 1, \ \varphi \geq 0, \ x_{ij} \geq 0, \ for\ all\ i, j,$$

Here the equilibrium condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is satisfied.

If we use the exponential membership function as defined in (3), then an equivalent nonlinear model for the model (1) can be formulated as:

Min: φ,
Subject to,

$$\frac{e^{-sZ\psi_r(x)} - e^{-s}}{1 - e^{-s}} + d_r^- - d_r^+ = 1,$$

$$\varphi \geq d_r^-, \quad r = 1,2, \dots\dots, k,$$

$$d_r^+ \, d_r^- = 0,$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2, \dots\dots m,$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2, \dots\dots n,$$

$$d_r^+, d_r^- \geq 0,$$

$$\varphi \leq 1, \ \varphi \geq 0, \ x_{ij} \geq 0, \ for\ all\ i, j,$$

Here the equilibrium condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is satisfied.

If we use the hyperbolic membership function as defined in (4) then an equivalent nonlinear model for the model (1) can be formulated as:

Min: φ,

Subject to,

$$\frac{1}{2} + \frac{1}{2} \frac{e^{\{\frac{(U_r+L_r)}{2} - Z_r\}\alpha^r} - e^{-\{\frac{(U_r+L_r)}{2} - Z_r\}\alpha^r}}{e^{\{\frac{(U_r+L_r)}{2} - Z_r\}\alpha^r} + e^{-\{\frac{(U_r+L_r)}{2} - Z_r\}\alpha^r}} + d_r^- - d_r^+ = 1,$$

$$\varphi \geq d_r^-, \quad r = 1,2, \dots\dots, k,$$

$$d_r^+ \, d_r^- = 0,$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2, \dots\dots m,$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2, \dots\dots n,$$

$$d_r^+, d_r^- \geq 0,$$

$$\varphi \leq 1, \ \varphi \geq 0, \ x_{ij} \geq 0, \ for\ all\ i, j,$$

Here the equilibrium condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is satisfied.

Step 5: Solve the equivalent crisp model obtained in Step 4. The solution obtained in Step 5 will be the optimal compromise solution of MOTP model [29].

IV. EXAMPLES

To illustrate the efficiency of the proposed method, using interval numbers we consider the following numerical example.

$$\text{Min} \quad Z_1 = [16,18]\,x_{11} + [19,21]x_{12} + [12,14]x_{13} + [20,22]x_{14} + [22,24]x_{21} \ [13,15]x_{22} + [19,21]x_{23} + [8,10]x_{24} + [14,16]\,x_{31} + [28,30]x_{32} + [8,10]x_{33} + [5,7]x_{34}$$

$$\text{Min} \quad Z_2 = [9,11]x_{11} + [14,16]x_{12} + [12,14]x_{13} + [4,8]x_{14} + [16,18]x_{21} + [10,12]x_{22} + [14,18]x_{23} + [3,5]x_{24} + [8,10]x_{31} + [20,22]x_{32} + [6,8]x_{33} + [5,7]x_{34}$$

Subject to,

$$x_{11} + x_{12} + x_{13} + x_{14} = [7,9]$$

$$x_{21} + x_{22} + x_{23} + x_{24} = [17,21]$$

$$x_{31} + x_{32} + x_{33} + x_{24} = [16,18]$$

$$x_{11} + x_{21} + x_{31} + x_{41} = [10,12]$$

$$x_{12} + x_{22} + x_{32} + x_{42} = [2,4]$$

$$x_{13} + x_{23} + x_{33} + x_{34} = [13,15]$$

$$x_{14} + x_{24} + x_{34} + x_{44} = [15,17]$$

$x_{ij} \geq 0$ i = 1,2,3 , j = 1,2,3,4

In the following the proposed steps of the previous section are presented.

Step 1: The solution of each single objective transportation problem is

$$X^1 = (x_{11} = 0, x_{12} = 0, x_{13} = 8, x_{14} = 0, x_{21} = 11, x_{22} = 3, x_{23} = 5, x_{24} = 0, x_{31} = 0, x_{32} = 0, x_{33} = 1, x_{34} = 0)$$

$$X^2 = (x_{11} = 8, x_{12} = 0, x_{13} = 8, x_{14} = 0, x_{21} = 11, x_{22} = 3, x_{23} = 0, x_{24} = 16, x_{31} = 8, x_{32} = 0, x_{33} = 14, x_{34} = 0)$$

Step 2: The objective function values are:

$$Z_1(X^1) = 604, \; Z_1(X^2) = 568, \; Z_2(X^1) = 406, Z_2(X^2) = 302$$

Step 3: The upper and lower bounds of each objective function can be written as follows:

$$568 \leq Z_1 \leq 604 \;, 302 \leq Z_2 \leq 406 \;,$$
$$L_1 = 568, U_1 = 604, L_2 = 302, U_2 = 406$$

Step 4: If we use the linear membership function as defined in (2), an equivalent crisp model can be formulated as:

Min: φ,

Subject to,

$$\frac{604 - Z_1}{36} + d_1^- - d_1^+ = 1,$$

$$\frac{406 - Z_2}{104} + d_2^- - d_2^+ = 1,$$

$$\varphi \geq d_r^- , \quad r = 1,2, \ldots \ldots, k,$$

$$d_r^+ \, d_r^- = 0,$$

The problem is solved and the results are:

$$x_{11}^* = 4, x_{14}^* = 4, x_{22}^* = 3 \,, x_{23}^* = 5 \,, x_{24}^* = 11 \,, x_{31}^* = 7, x_{33}^* = 9 \,, x_{34}^* = 1$$

$$d_1^- = 0.5 \,, d_1^* = 0, d_2^- = 0.5, d_2^+ = 0, \varphi = 0.5$$

$$Z_1^* = 586 \,, Z_2^* = 354$$

The other variables that are not in the above have a zero value.

If we use the exponential membership function as defined in (3) with the parameter s = 1, an equivalent crisp model can be formulated as:

Min: φ,

Subject to,

$$\frac{e^{-(z_1 - 568)/36} - e^{-1}}{1 - e^{-1}} + d_1^- - d_1^+ = 1,$$

$$\frac{e^{-(z_2 - 302)/104} - e^{-1}}{1 - e^{-1}} + d_2^- - d_2^+ = 1,$$

$$\varphi \geq d_r^- , \quad r = 1,2, \ldots \ldots, k,$$

$$d_r^+ \, d_r^- = 0$$

The problem is solved and the results are:

$$x_{11}^* = 4, x_{14}^* = 4, x_{22}^* = 3 \,, x_{23}^* = 5 \,, x_{24}^* = 11 \,, x_{31}^* = 7, x_{33}^* = 9 \,, x_{34}^* = 1$$

$$d_1^- = 0.38 \,, d_1^* = 0, d_2^- = 0.38, d_2^+ = 0, \varphi = 0.38$$

$$Z_1^* = 586 \,, Z_2^* = 354$$

The other variables that are not in the above have a zero value.

If we use the hyperbolic membership function as defined in (4) then an equivalent crisp model can be formulated as:

Min: φ,

Subject to,

$$\frac{1}{2} + \frac{1}{2} \frac{e^{6(568 - z_1)/36} - e^{-6(568 - z_1)/36}}{e^{6(568 - z_1)/36} + e^{-6(568 - z_1)/36}} + d_1^- - d_1^+ = 1,$$

$$\frac{1}{2} + \frac{1}{2} \frac{e^{6(406 - z_2)/104} - e^{-6(406 - z_2)/104}}{e^{6(568 - z_2)/104} + e^{-6(568 - z_2)/36}} + d_2^- - d_2^+ = 1,$$

$$\varphi \geq d_r^- , \quad r = 1,2, \ldots \ldots, k,$$

$$d_r^+ \, d_r^- = 0,$$

The problem is solved and the results are:

$$x_{11}^* = 4, x_{14}^* = 4, x_{22}^* = 3 \,, x_{23}^* = 5 \,, x_{24}^* = 11 \,, x_{31}^* = 7, x_{33}^* = 9 \,, x_{34}^* = 1$$

$$d_1^- = 0.5 \,, d_1^* = 0, d_2^- = 0.5, d_2^+ = 0, \varphi = 0.5$$

$$Z_1^* = 586 \,, Z_2^* = 354$$

The other variables that are not in the above have a zero value

V. CONCLUSION

Multi-objective fuzzy transportation problem can be solved by the non-linear membership functions. To solve the non-linear type of problem these nonlinear membership functions can be used. Apart from the transportation problems, for the multi-objective nonlinear programming problems, nonlinear membership functions are useful.

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