

Simulation Model solves exact the Enigma named Generating high Voltages and high Frequencies by Tesla Coil

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ABSTRACT

Simulation model of Tesla coil has been successfully completed, and has been verified the procedure and functioning. The literature and documentation for the model were taken from the rich sources, especially the copies of Tesla patents. The oscillating system's electrical scheme consists of the voltage supply 220/50 Hz, Fe transformer, capacitor and belonging chosen electrical components, the air gap in the primary Tesla coil (air transformer) and spark gap in the exit of the coil. The investigation of the oscillating process Tesla coil's system using the simulation model in MATLAB & SIMULINK have given the exact solution the enigma named the generating high voltage and high frequency the Tesla's coil. The inductance voltage from the spark current in the primary (coil) with its high voltage impulse excites the oscillating series circuit Ce-L3-R3 on the secondary of the air transformer to its own damped oscillations.

Index Terms: Tesla coil system, simulation model of the system, exact solution of the enigma named the generating the high voltage and frequency

I. INTRODUCTION

Simulation model of Tesla coil has been successfully completed, and is largely verified the procedure and its operation. There are available literature related to Tesla coil theory, the theory of generating high voltage and frequency, and subsequent theories of various schools and experts in Tesla's work. Such different conditions and explanations are the great challenge for many researchers and admirers of the Tesla's work.

Tesla could not know in 1881 or thereabouts the complex action Tesla coil. Tesla experimentally adjusted its coil, to produce high voltage and frequency. Theories about Tesla coil, to our knowledge, began appearing around 1895. After successfully completed simulation models of electrical components, electrical oscillating circuits and systems in MATLAB & SIMULINK, of which a few [8] - [16] preceded to the development of simulation model of Tesla Coil. All that imposed the idea for the development of simulation models, of this very complex electrical system, hoping that the mathematical and simulation model will give the exact solution to enigma of generation high voltage and frequency.

II. APPROACH TO PROBLEM SOLVING

Electrical values and parameters of Tesla coil are not always all known and there are no available results of measurement of these quantities, what is a lack of the modeling. In particular, it is specially unknown electrical

behavior of two sparks of the coil, and it was a big obstacle for the development of the model.

Electrical scheme of Tesla coil oscillating system is known and can be easily drawn in the form as a undivided device. System consists of voltage supply 220/50 Hz, Fe transformer, capacitor and belonging chosen electrical components, air gap in the primary Tesla coil (air transformer), than the belonging chosen electrical components and spark gap in the exit of the system.

Intending to write of mathematical relations and formulas for calculation the electrical network as a basis for the simulation model, it should be make the electrical scheme of the device as a substitute scheme. For such plane network, it was possible to write mathematical relations, formulas and logical conditions of the system. Substitute scheme has been successfully made, though at first spark appeared an insurmountable obstacle. Spark gap can not be defined by mathematical relations, but this was done using the simulation tool for generating non-linear and empirically derived forms of electrical values, obtained by analogy with sizes and forms of the sparks testing high voltage and the atmospheric high voltage.

In spite of all the difficulties in the preparations were able to make the complete and exact mathematical and logical model, in terms of differential and algebraic equations, as well as logical and descriptive relationships.

The terms of the mathematical model were adjusted and rearranged in the form of programming to develop a unique simulation model

with the help the powerful simulation tool MATLAB & SIMULINK.

Successfully was made a simulation model for the chosen parameters, which was logically and functionally tested, and using the internal capabilities of the simulation tools to verify in several stages the development's process. It was verified the procedure, functionality and applicability of the simulation model.

Partially was experimented with different parameters, and for some unknown parameters were found values only by multiple tasting numerous of the possible variants. Head result should be the exact solution of the generating high voltage and frequency by Tesla coil. It is expected to adopting this solution of the enigma with help of the appropriate measurement.

For the developed simulation model was made the complete documentation, but in this paper there is no room for it presentation.

III. OVERLOOKED SIMULATION RESULTS TO BETTER UNDERSTAND LATER THE ENIGMA SOLUTION

The simulation model consists of three files and so the control file *teslatd.m* in MATLAB with numerical data on the parameters and electrical values, the certain relationships and commands, while the simulation file *tesla.mdl* in SIMULINK consists of the main system and subsystems created from individual blocks. The third file *frteslae.m* is for the frequency analysis.

The simulation model examines the behavior of the dynamic system Tesla coil in shorter times to obtain better overlooking of the system's behavior, but also in a very short time (high frequency oscillations) for very fine researches and the shortest changes in the system. Duration of the simulation is limited because of the small integration tstep ($tstep = 5.0 \cdot 10^{-8}$ s). The successful decision has been a time 0.25 s for the observation of the process simulation, which is slightly longer than the period of the supply voltage of 50 Hz. The simulation takes place in 500 000 steps with the duration of the simulation process on my PC about 8 minutes.

Simple commands to draw a qualitative and quantitative diagram are written as standard, while most of them to show results in a very short time contain the definition of short sections, test sections, what will be able to follow later in next chapter. The results obtained by the simulation model are specially selected, in order to finally prove the solution for the enigma of Tesla coil.

For consideration of the entire functioning of the simulation model results are given in the form of qualitative and quantitative diagrams, but

to solve these enigma more results will be analyzed in a very short time.

The input electrical values – vectors – correspond to the data from the public network 220 V and 50 Hz and are marked "a" - the actual value. It can be seen on Fig. 1. These diagrams are *u1a* voltage, current *i1a*, and power *N1a* in the duration little more than one period of 0.20 s.

The measurable power is different from the graph of *N1a* and can be easily calculated as the mean value of vector power *N1a* according to the formula $mean(N1a)$

$$ans = 3.5042e +004 \text{ VA} = 35\ 042 \text{ kVA}$$

Diagram of the *u1a* voltage is correct sinusoidal, while on the diagram currents *i1a* there is the influence of spark like the variable loads.

Diagram power *N1a* as the product of vectors *u1a* and *i1a* is twice of the voltage frequency and shape, which is not known in practice. Used to measure is only its mean value.

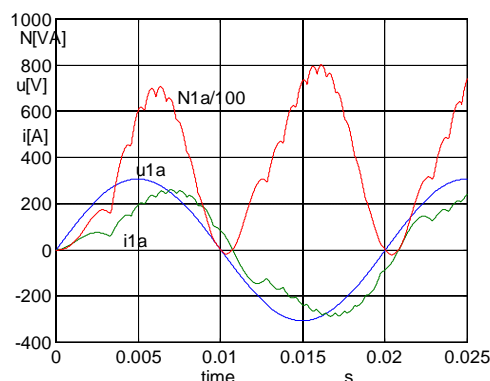


Fig.1. Actual voltage, current and power in first transformer input

In the middle part of the system's scheme with a condenser appear the electrical values with the shapes, which are generated by the condenser discharging, the spark gap and other parameters (Fig. 2)

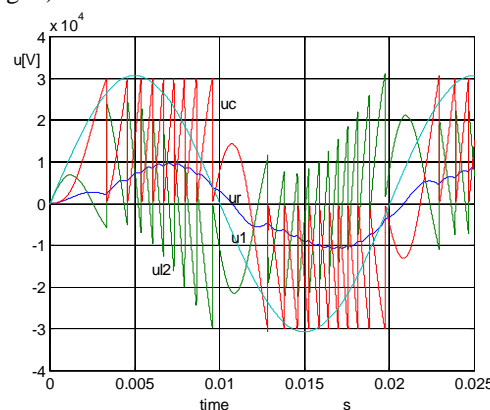


Fig.2. Voltages in the middle loop of the current *i2*

It is easily to follow the behavior of the capacitor voltage (voltage uc), which is repeatedly slow filled and rapidly discharging through a spark. The most important electrical value, the current is is not visible on Fig.2, but there are the diagrams $ul2$, $ur2$, which participate in the process of generating the electric current impulse is . We can see the great influence of voltage $ur2$, but voltage $ul2$ is not negligible too.

The marks of the electrical values are without the index "a", because these values are the same for electrical and substituted scheme.

These two last examples of overlooked simulation results are from a group of electrical values as the continuous analog function (not impulse) from the first part of Tesla coil system. The mention functions are inherently interesting, but are also very important for generating the electrical spark current is , which in its overview looks as a regular impulse and which will act to change the character of all the following electrical values until the high voltage and high frequency oscillating electrical value at the exit of Tesla coil system.

Generated "impulse" current is as a result of the first part of Tesla coil from the power supply to the spark gap, requires a special show because it represents the most important

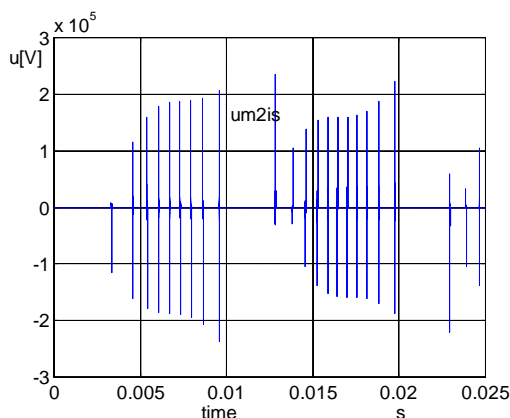


Fig. 3. Spark current is

Electrical value to generate almost all the electrical values which are following as the functions of very short duration, and in their overview look like as impulse values of the transformer. Appearance and duration of the "impulse" values will be seen later in the analysis of their role in creating a high voltage and frequency. .

The "impulse" spark current in overview looks as on the Fig. 3 and has a maximum value of the order 70 A. It appears in every discharging of capacitors in the middle part of the system and every spark in the primary circuit of the air transformer.

Now follows the announced display review of electrical values primary and secondary air transformer (coil), which participate in the mechanism of generating their own oscillations. This impulse current is is the value of the primary air electrical transformer but operates through its mutual inductance on the secondary electrical values.

The inductance voltage $um2is$ generated at the secondary terminals of air transformers is the high voltage impulse, the largest value is about 200 000 V, what can be seen on Fig. 4.

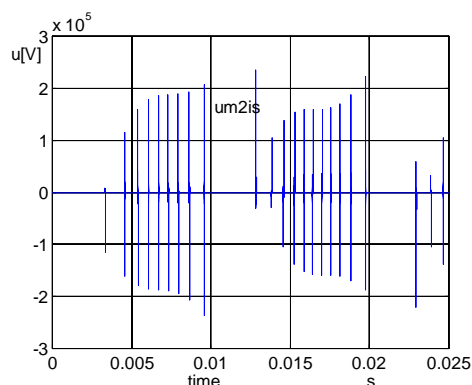


Fig. 4. Inductance voltage $um2is$ in compare to voltage $ur3$

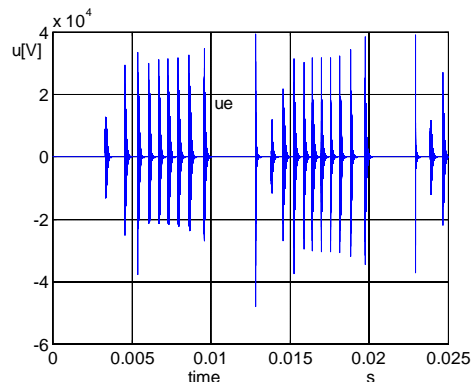


Fig. 5. Output voltage ue

According to our intention to continue providing the most important electrical values obtained from the simulation model on Fig. 5 is the voltage ue at the output, closing the loop current, and than follows diagrams of current $i3$, the inductance voltage $um2i3$ of the current $i3$ to the primary transformer and the current iz of the spark from the final condenser of the transformer.

On the diagram of output voltage ue (Fig. 5) and current $i3$ (Fig. 6) can be seen damped oscillations and it is already the first sign that the simulation model is able to generate a certain voltage and frequency.

It is to note that the form of damping oscillation depends on the binding parameters of inductive transformers and other parameters.

The output voltage ue on its substituted scheme, but the actual voltage uea will be seen later, and will have well-known mark uea .

Output loop current $i3$ (Fig. 6) is the value of its substituted scheme. It is also in the form of damped oscillations, with the largest amounts of the order $\pm 40-50$ A. Current value $i3a$ of the order of a tenth amperes will be seen later.

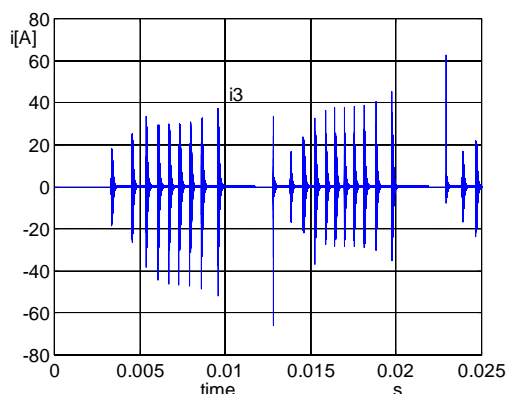


Fig. 6. Output loop current $i3$

The simulation model allows the observation the sparks current iz (Fig. 7) at the exit.

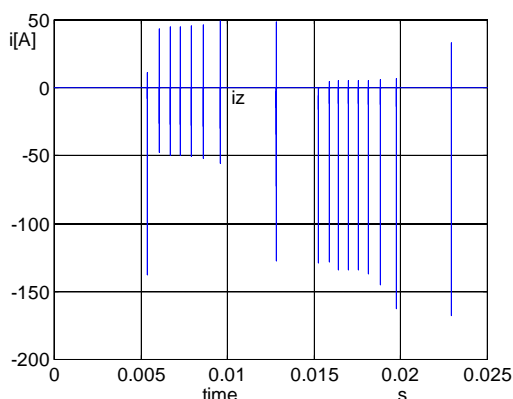


Fig. 7. Spark current iz

The current iz is the "impulse" without oscillation and in some areas is quite large, the order of 150 A. This value is studied in detail, but to solve enigma Tesla coil is not a priority.

Inductance voltage $um2i3$ from the current $i3$ to the air transformer primary side can be seen on Fig. 8. And this electrical value is of interest and detail has been studied, but in this paper is not necessary to specify.

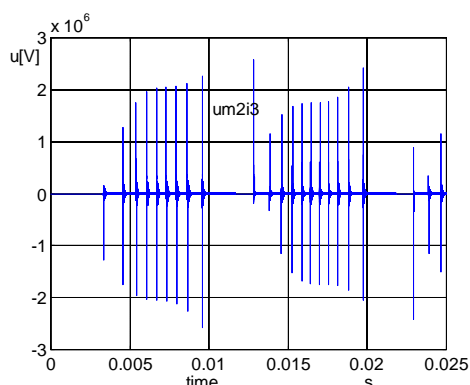


Fig. 8. Inductance voltage $um2i3$

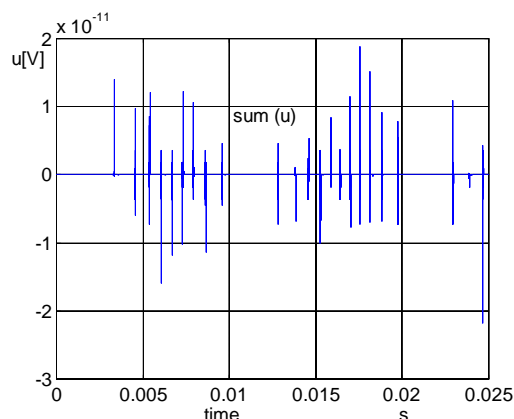


Fig. 9. The sum of all the voltages in the $i3$ loop - model's verification

The approach and functionality of the simulation model, in order to prove his credibility, was examined on several occasions and in several parts. The common way of checking is to look for the sum of all the voltages in the main current loops. The example on Fig. 9 is the sum of all the voltages output loop. It can be seen that the sum of all voltages in the output $i3$ current loop and during observation time of 0-25 ms is about 10^{-11} , which can be consider almost zero. Credibility and functionality of the approach and the simulation models is on one way proved!

IV. EXACT SOLUTION ENIGMA NAMED THE GENERATION HIGH VOLTAGES AND FREQUENCIES BY TESLA COIL

Neither one image in the overview diagrams of the electrical values, except the diagrams of output voltage ue and output current $i3$ as oscillating values, are at first sight helpful in searching the solution enigma of the generating high voltage and frequency Tesla coil. This main evidence, the task of this work, will be performed using the powerful simulation tools MATLAB & SIMULINK, by using the features of the simulation tools to stretch the time of observation, and become

visible very short parts of the process and very short changes in the system.

How are looking the electrical values in the i_s current loop on primary of the air transformer (coil), when it is observing a mechanism of working process on the simulation model and so in the separate narrow test section, which is defined with a interval of integration defined as time steps segment $t(133300:136000)$. In this test section the output voltage ue is greater than 30,000 V and we have output sparking between capacitor electrodes. The simulation model will respond to this request and further requirements, which will lead to exact solution of the enigma Tesla coil.

Without the mentioned possibility of simulation tools to consider short part of the process in a very short time, it could never reach a solution.

From the plenty researches and the results obtained, which exactly solve the enigma of Tesla coil, was elected the test section and its short duration.

The starting point of the test section on the time axis is

$t1(133\ 300)$
 $t2(136\ 000)$
 $ans = 0.0067\ s$
 Duration of observation
 $t2(136\ 000)-t1(133\ 300)$
 $ans = 1.3500e-004 = 0.000135\ s = 135\ \mu s$

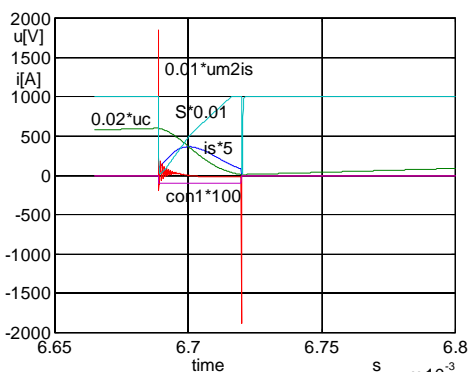


Fig. 10. The characteristic values of the primary air transformer

The characteristic values of the primary air transformer for the short time of the test section according to the command

`plot(t(133300:136000),is(133300:136000)*5,t(133300:136000),uc(133300:136000)*0.02,t(133300:136000),um2is(133300:136000)*0.01,t(133300:136000),S(133300:136000)*0.01,t(133300:136000),con1(133300:136000)*100),grid` can be seen on Fig. 10.

The electrical values and parameters on the Fig. 10 are: current i_s , capacitor voltage uc , inductance voltage $um2is$ on the secondary side of transformer but from the primary current i_s , developed electrical resistance S and the conductivity $CON1$ during the spark.

The diagrams represent the qualitative and quantitative values. Clear are the flow of each value, or vectors, and their amounts.

The current i_s is not a pure impulse but is quiet function except the parts at the beginning and end, when there are jumping. It must be warned immediately to the form of inductance voltage $um2is$, which will play later the most important role to solve the Tesla coil enigma. The voltage $um2is$ has at the beginning the very large positive impulse and small damped oscillation, than reduced value during the way to the final point of the test section. In the final point the inductance voltage has the large negative impulse and then the voltage fast disappears. Once again, this will be seen later on the Fig. 13.

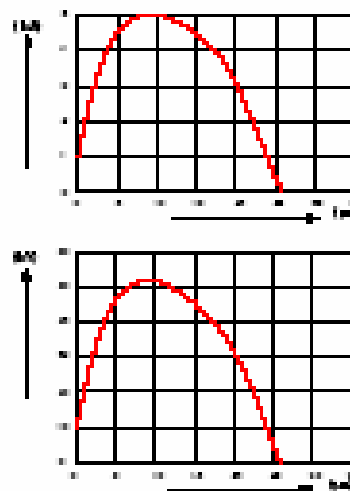


Fig. 11. Current and voltage by lightning strikes

Looking for the confirmation form of electricity sparks in the literature there was found approximately similar to the loading sparks with lightning strikes, as it is seen on Fig. 11.

On the test section we are going to seek behaviors of the all main electrical values and parameters relevant to our intention and our task of seeking solutions Tesla coil's enigma.

With the order

`plot(t(133300:136000),i3(133300:136000)*50,t(133300:136000),ue(133300:136000)*0.05,t(133300:136000),um2is(133300:136000)*0.01,t(133300:136000),v(133300:136000)*100000,t(133300:136000),con1(133300:136000)*600,t(133300:136000),iz(133300:136000)*10),grid` are received the qualitative and quantitative values of

the current i_3 , voltage ue , inductive voltage $um2is$, conductivity v during spark on the output capacitor, conductivity $CON1$ for spark in the primary of Tesla coil and the last the arc current iz during discharging the output capacitor (Fig.7) in test section. We are able to analyze every value from Fig. 12. During the test section the voltage ue is.

$e = ue(133300:136000)$;

$\max(e) \text{ ans} = 3.1345e+004 \text{ V}$

The spark appears and the iz spark current in test section is

$i = iz(133300:136000)$; $\max(i)$

$\text{ans} = 44.7918 \text{ A}$

this can be checked on Fig. 7.

The voltage ue and the current i_3 in the first point of the test section briefly oscillate and seemingly disappear until the second point of the test section. In the second point they begin very oscillate and with less attenuation decrease. What is going on with all the electrical values in a short time segment of the process of passing the test section?

The simulation model will give the answer to it, but also the answer to the enigma of the Tesla coil.

We return back to the inductance voltage $um2is$, especially given on Fig. 13. The first point of the test section inductance voltage $um2is$ has, as already stated, positive impulse, and the briefly damped oscillation.

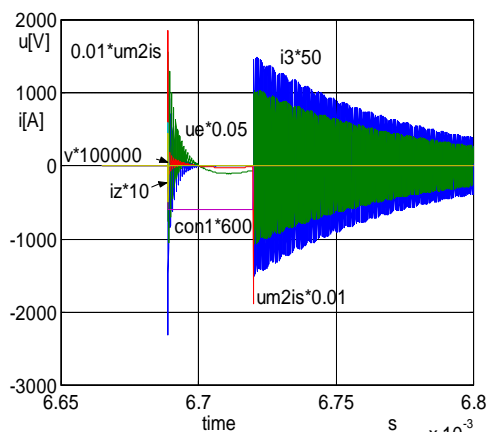


Fig. 12. Electrical values in the current i_3 loop.during test section

What is going to happen in the first point of the test section? Why there are not here oscillation of the voltage ue and current i_3 ?

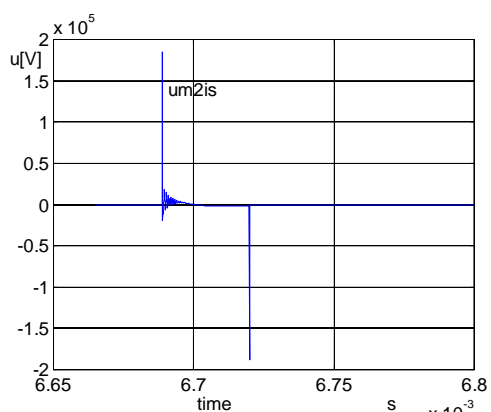


Fig. 13. Inductance voltage $um2is$

With the order

```
plot(t(133300:136000),ue(133300:136000),t(133300:136000),ul3(133300:136000),t(133300:136000),um2is(133300:136000)),grid
```

we can see on Fig. 14 qualitatively and quantitatively the voltage on the capacitor ue , the voltage on the inductor L3 and the inductance voltage $um2is$ of the test section.

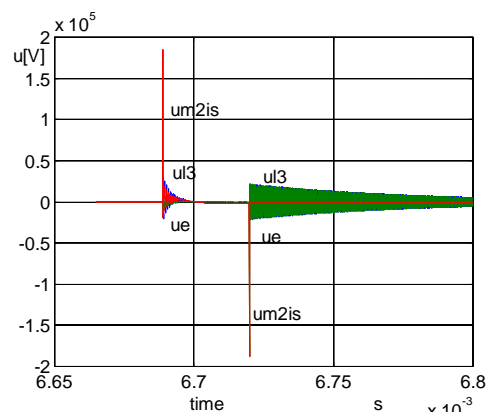


Fig. 14. Voltages ue , $ul3$ and $um2is$ on the test section

Using MATLAB & SIMULAB we are able to go in deeply in a process.

In the first point above voltages disappear very quickly but in and after the second point are the large oscillations of the same voltages, but only the voltage $um2is = 0$.

The diagrams of the voltages ue , $ul3$ and $um2is$ (Fig. 14) are giving us the solution of the enigma, but we are going further to the completely clean solution.

In particular attention is paid to changes in the electrical values of the initial and the final point of the test section.

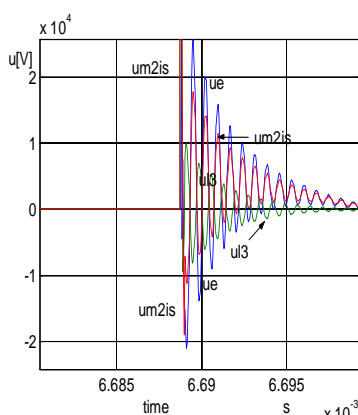


Fig. 15. Dampd voltages in the first point of the test section

The behavior of voltages *ue*, *ul3* and *um2is* in the first point is better seen if we make zoom the voltages particularly in this first point, so it is better to see their diagrams (Fig. 15).

Voltage *um2is* (red diagram) has phase opposition to the voltage *ul3* and as damper reduces direct the voltage *ul3*, but indirect too voltage *ue*. This is the solution for the first point of the test section. No remarkable oscillations!

The solution of the enigma and very high own oscillation of *ue* and *i3* are expected now in the analysis the voltages relations in the final point of the test section (Fig. 16).

Voltages are the same as in the previous analysis (*ue*, *ul3* and *um2is*) too in zoom state.

Solution of the enigma now gives inductive voltage *um2is*. This voltage appears at the final point of the test section just as the large negative impulse and momentarily disappears, because there is no more current *is* in the primary of the transformer (Fig. 13).

The single negative impulse voltage *um2is* excites the oscillation of the series resonant circuit with inductor L3 capacitor Ce, resistance R3 and this series resonant circuit oscillates slowly damped with the voltages *ue* and *ul3* and the current *i3*. All of this is shown on Fig. 16 and this is the solution of the enigma Tesla coil.

Oscillator Ce-L3-R3 will oscillate with damping without additional energy from the outside, depending on its parameters and the size of the circuit excitation voltage impulse *um2is*. In this case, parameters, and according to Fig. 5 own oscillations disappear quite quickly, but with new negative impulse voltage *um2is* oscillator oscillates again.

Energy in the resonant circuit delivers impulse inductance voltage *um2is* and there is no classical energy transfer across the air transformer.

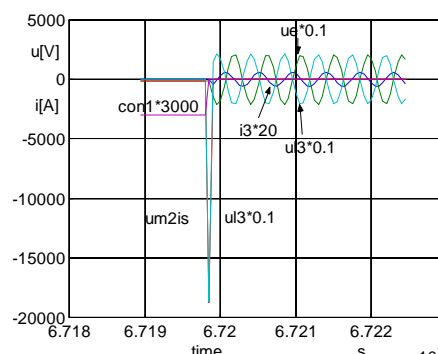


Fig. 16. Very high selfoscillation in the final point of the test section – solution of enigma

The observed diagrams of the own damped oscillations electrical values in the output circuit of transformers are on its substituted scheme. We need the actual electric values, which means the values on real Tesla coil.

The diagrams of own oscillations of the output voltage *uea* and *i3a* (the "a" means the actual value of the actual electric scheme, and not according to its substituted scheme) are given on Fig. 17 and Fig. 18..

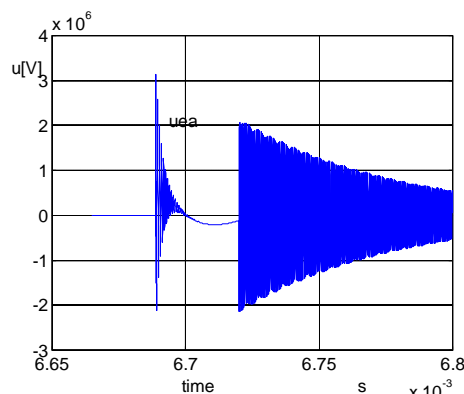


Fig. 17. Actual output voltage *uea*

On the Fig. 17 can be seen that the highest voltage of the *uea* in the first point of the test section, just when the spark occurs and could be the short-lived value order of the 3 million. In the final point of the test section occurs a little bit lower amplitude of the voltage *uea* (about 2 million V), but it oscillates with the visual attenuation. Simulation result of Tesla coil is very high voltage of several MV as by the real Tesla coil.

The actual current *i3a* in the output circuit (Fig. 18) is very similar with his shape to the output voltage *uea*. Its greatest value is small and amounts to about 0.3 A

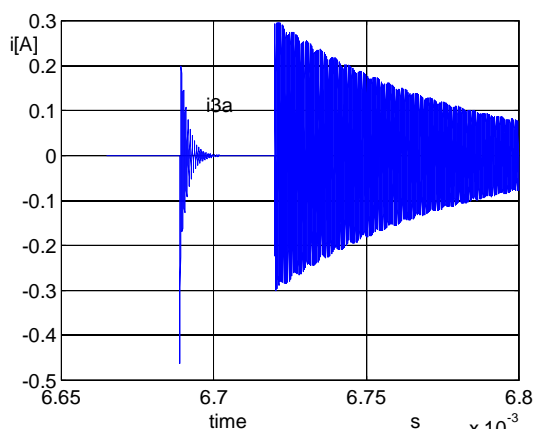


Fig. 18 Actual current i_{3a} in the output circuit

For the final verification of the process and simulation model competently for the solution of Tesla coil enigma is missing just check the frequency of the output actual voltage of the uea in the test section.

Commands for the frequency analysis the output voltage during the test section

```
e = uea (133300:136000);
y = e;
frteslae
```

The result shows Fig.19.

The frequency of the actual voltage uea at the test section on the Fig. 19 is about 2.2 MHz and is corresponding to the frequency of the real Tesla coil.

V. CONCLUSION

On the selected results from the researches the oscillating process Tesla coil's system on the simulation model was solved exact the enigma named the generating high voltage and frequency Tesla coil.

Inductance voltage from the spark current in the primary air transformer (coil) with its high voltage impulse and in a certain point and certain mutual relations excites on the secondary air transformer to oscillate series circuit Ce-L3-R3 on its own damped oscillations.

In addition to the many installations performed at present in laboratories, Tesla coil simulation model could become irreplaceable research and educational tool.

Successfully completed a very complex simulation model of Tesla coil system encourages us to approach the development of the simulation models of an individual and groups of the wind power-plants completed with the part to the control system for the optimization of the system in

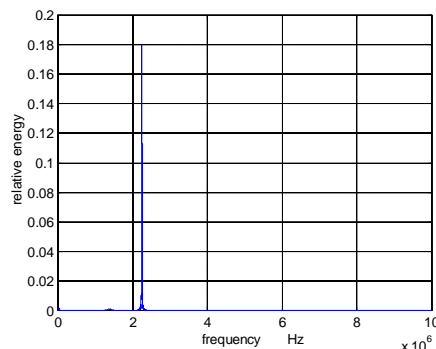


Fig. 19. Frequency characteristic of the uea in test section

Tesla coil frequency can be finding precisely with help of MATLAB Parth of the procedure according the Fig. 19 is

```
find(Pyy>0.18)
ans =56171
    56172
    56173
    56174
    443828
    443829
    443830
    443831
```

```
f(56174)
ans = 2246920 = 2.24 MHz
```

It could be enough arguments to make conclusion.

normal, in transition and in the conflict working situations.

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VII. BIOGRAPHIES



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Scientific Field:

Simulation of train movement and optimal
 control using Pntryagin's principle

Mathematical modelling and simulation of
 electrical components and systems

Mathematical modelling and simulation of
 mechanical components and systems

Retired in 1998 from the University of Zagreb

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Emeritus member of Croatian Academy of
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