

## Magneto-Thermosolutal Convection in Maxwellian Dusty Viscoelastic Fluid through Porous Medium

Gursharn Jit Singh<sup>1</sup>, Pardeep Kumar<sup>2</sup>

<sup>1</sup>Department of Mathematics, SCD Govt. PG College Ludhiana (Pb), India.

<sup>2</sup>Department of Mathematics, ICDEOL, Himachal Pradesh University, Shimla-171005, India.

### ABSTRACT

Thermosolutal convection in Maxwellian viscoelastic fluid under the simultaneous effects of uniform vertical magnetic field and suspended particles through porous medium is mathematically investigated. A sufficient condition for the invalidity of the 'principle of exchange of stabilities' is derived, in the context, which states that the exchange principle is not valid provided the thermal Rayleigh number  $R$ , solutal Rayleigh number  $R_S$ , the medium permeability  $P_1$  and the suspended particles parameter  $B$  are restricted by the inequality

$$\frac{BP_1}{\pi^2}(R + R_S) < 1.$$

**Keywords:** Maxwellian viscoelastic fluid, porous medium, suspended particles, thermosolutal convection, uniform vertical magnetic field.

### I. INTRODUCTION

The study of onset of convection in a porous medium has attracted considerable interest because of its natural occurrence and of its intrinsic importance in many industrial problems, particularly in petroleum-exploration, chemical and nuclear industries. The derivation of the basic equations of a layer of fluid heated from below in porous medium, using Boussinesq approximation, has been given by Joseph [1]. The study of a layer of fluid heated from below in porous media is motivated both theoretically and by its practical applications in engineering disciplines. Among the applications in engineering disciplines one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in porous medium. Lapwood [2] has studied the stability of convective flow in a porous medium using Rayleigh's procedure. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [3].

A detailed account of the theoretical and experimental results of the onset of thermal instability (Bénard convection) in a fluid layer under varying assumptions of hydrodynamics and hydromagnetics has been given in the celebrated monograph by Chandrasekhar [4]. Veronis [5] has investigated the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. The buoyancy forces can arise not only from density

differences due to variations in temperature but also from those due to variations in solute concentration. Thermosolutal convection problems arise in oceanography, limnology and engineering. The investigation of thermosolutal convection is motivated by its interesting complexities as a double diffusion phenomena as well as its direct relevance to geophysics and astrophysics. Stommel et al. [6] did the pioneering work regarding the investigation of thermosolutal convection. This work was elaborated in different physical situations by Stern [7] and Nield [8]. Examples of particular interest are provided by ponds built to trap solar heat (Tabor and Matz [9]) and some Antarctic lakes (Shirtcliffe [10]). The physics is quite similar in the stellar case in that Helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motion in thermosolutal convection are important (e.g. in lower parts of the Earth's atmosphere, astrophysics and several geophysical situation) are usually far removed from the consideration of a single component fluid and rigid boundaries and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries. A double-diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose has been explained by Brakke [11]. Nason et al. [12] found that this instability, which is deleterious to certain biochemical separations, can be suppressed by rotation in the ultra centrifuge.

The problem of thermosolutal convection in fluids in a porous medium is of importance in

geophysics, soil sciences, ground water hydrology and astrophysics. The development of geothermal power resources has increased general interest in the properties of convection in porous media. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat-transfer mechanism in young oceanic crust (Lister [13]). Generally it is accepted that comets consists of a dusty 'snowball' of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice-versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in the astrophysical context (McDonnell [14]). The effect of a magnetic field on the stability of such a flow is of interest in geophysics, particularly in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The other application of the results of flow through a porous medium in the presence of a magnetic field is in the study of the stability of a convective flow in the geothermal region.

In geophysical situations, more often than not, the fluid is not pure but may instead be permeated with suspended (or dust) particles. The effect of suspended particles on the stability of superposed fluids might be of industrial and chemical engineering importance. Further, motivation for this study is the fact that knowledge concerning fluid-particle mixtures is not commensurate with their industrial and scientific importance. Scanlon and Segel [15] have considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles. Sharma et al. [16] have studied the effect of suspended particles on the onset of Bénard convection in hydromagnetics. The effect of suspended particles was found to destabilize the layer whereas the effect of a magnetic field was stabilizing. Palaniswamy and Purushotham [17] have studied the stability of shear flow of stratified fluids with fine dust and found the effect of fine dust to increase the region of instability. Recently spacecraft observations have confirmed that the dust particle play an important role in the dynamics of atmosphere as well as in the diurnal and surface variations in the temperature of the Martian weather. It is, therefore, of interest to study the presence of suspended particles in astrophysical situations. The fluid has been considered to be Newtonian in all the above studies.

With the growing importance of viscoelastic fluids in geophysical fluid dynamics,

chemical technology and petroleum industry, the investigations on such fluids are desirable. The stability of a horizontal layer of Maxwell's viscoelastic fluid heated from below has been investigated by Vest and Arpaci [18]. The nature of the instability and some factors may have different effects on viscoelastic fluids as compared to the Newtonian fluids. For example, Bhatia and Steiner [19] have considered the effect of a uniform rotation on the thermal instability of a Maxwell fluid and have found that rotation has a destabilizing effect in contrast to the stabilizing effect on Newtonian fluid. Sharma and Kumar [20] have studied the thermosolutal instability in a Maxwellian fluid in porous medium under the effect of Hall current. In another study, Kumar and Singh [21] have studied the superposed Maxwellian viscoelastic fluids through porous medium in hydromagnetics. Linear stability analysis of Maxwell fluid in the Be'nard problem for a double-diffusive mixture in a porous medium based on the Darcy Maxwell model has been studied by Wang and Tan [22]. The problem of double-diffusive convection and cross-diffusion in a Maxwell fluid in a horizontal layer in porous media by using the modified Darcy-Brinkman model has been considered by Awad et al. [23] and analytical expression of the critical Darcy-Rayleigh numbers for the onset of stationary and oscillatory convection are derived. Wang and Tan [24] have studied the double-diffusive convection of viscoelastic fluid with Soret effect in a porous medium by using a modified-Maxwell-Darcy model and have shown that for oscillatory convection the system is destabilizing in the presence of Soret effect. The relaxation time also enhances the instability of the system. The thermal instability of Maxwellian heterogeneous viscoelastic fluid through porous medium has been investigated by Kumar [25] and found that for stationary convection, the medium permeability and density distribution have destabilizing effect.

Keeping in mind the importance and applications in chemical engineering, biomechanics and various applications mentioned above, the effects of magnetic field and suspended particles on thermosolutal convection in Maxwellian viscoelastic fluid through porous medium has been considered in the present paper.

## II. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Let  $T_{ij}$ ,  $\tau_{ij}$ ,  $e_{ij}$ ,  $\mu$ ,  $\lambda$ ,  $p$ ,  $\delta_{ij}$ ,  $v_i$ ,  $x_i$  and  $\frac{d}{dt}$  denote

respectively the total stress tensor, the shear stress tensor, the rate-of-strain tensor, the viscosity, the stress relaxation time, the isotropic pressure, the Kronecker delta, the velocity vector, the position

vector and the mobile operator. Then the constitutive relations  
 Maxwellian viscoelastic fluid is described by the

$$\left. \begin{aligned} T_{ij} &= -p \delta_{ij} + \tau_{ij} , \\ \left( 1 + \lambda \frac{d}{dt} \right) \tau_{ij} &= 2\mu e_{ij} , \\ e_{ij} &= \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) . \end{aligned} \right\} \quad (1)$$

Here we consider an infinite horizontal layer of an electrically conducting incompressible Maxwellian viscoelastic fluid-particle layer of depth  $d$  in porous medium which is acted on by a uniform vertical magnetic field  $\vec{H}(0, 0, H)$  and gravity field  $\vec{g}(0,0, -g)$ .

Let  $\delta p, \delta p, \theta, \gamma, \bar{q}(u, v, w), \bar{q}_d(l, r, s)$  and  $\vec{h}(h_x, h_y, h_z)$  denote respectively the perturbations in density  $\rho$ , pressure  $p$ , temperature  $T$ , solute concentration  $C$ , fluid velocity (initially zero), particle velocity (initially zero) and magnetic field  $\vec{H}$ . Let  $\kappa, \kappa', \alpha, \alpha', \beta \left( = \left| \frac{dT}{dz} \right| \right), \beta' \left( \left| \frac{dC}{dz} \right| \right)$  stand for thermal diffusivity, solute diffusivity,

thermal coefficient of expansion, an analogous solvent expansion, uniform temperature gradient and uniform solute gradient respectively. The linearized thermosolutal hydromagnetic perturbation equations through porous medium containing suspended particles, following Boussinesq approximation, are

$$\frac{\rho_0}{\varepsilon} \left( 1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial \bar{q}}{\partial t} = \left( 1 + \lambda \frac{\partial}{\partial t} \right) \left[ -\nabla \delta p + \vec{g} \delta \rho + \frac{\mu_e}{4\pi} \left\{ (\nabla \times \vec{h}) \times \vec{H} \right\} + \frac{KN}{\varepsilon} (\bar{q}_d - \bar{q}) \right] - \frac{\mu}{k_1} \bar{q} \quad (2)$$

$$\left( \frac{m}{K} \frac{\partial}{\partial t} + 1 \right) \bar{q}_d = \bar{q} \quad (3)$$

$$\nabla \cdot \bar{q} = 0 \quad (4)$$

$$\nabla \cdot \vec{h} = 0 \quad (5)$$

$$(E + b\varepsilon) \frac{\partial \theta}{\partial t} = \beta(w + bs) + \kappa \nabla^2 \theta \quad (6)$$

$$(E' + b\varepsilon) \frac{\partial \gamma}{\partial t} = \beta'(w + bs) + \kappa' \nabla^2 \gamma \quad (7)$$

$$\varepsilon \frac{\partial \vec{h}}{\partial t} = (\vec{H} \cdot \nabla) \vec{q} + \varepsilon \eta \nabla^2 \vec{h} \quad (8)$$

where  $N(\vec{x}, t)$  denote the number density of the suspended particles,  $\varepsilon$  is the medium porosity and  $K = 6\pi\mu \eta'$  is the Stokes' drag coefficient,  $\eta'$  being the particle radius.

Here  $\vec{x} = (x, y, z)$ ,  $E = \varepsilon + (1 - \varepsilon) \frac{\rho_s C_s}{\rho_o C}$  and  $\rho_o, C_f; \rho_s, C_s$  stand for density and heat capacity of fluid and

solid matrix, respectively.  $E'$  is an analogous solute parameter,  $b = \frac{mNC_p t}{\rho_o C_f}$ ,  $m$  is the mass of particles per unit

volume.  $C, \eta$  and  $e$  stand for speed of light, electrical resistivity and charge of an electron.

The equation of state is

$$\rho = \rho_0 [1 - \alpha(T - T_0) + \alpha'(C - C_0)] \quad (9)$$

where the suffix zero refers to values at the reference level  $z = 0$ , i.e.  $\rho_0, T_0$  and  $C_0$  stand for density, temperature and solute concentration at the lower boundary  $z = 0$ .

The change in density  $\delta \rho$ , caused by the perturbation  $\theta, \gamma$  in temperature and solute concentration, respectively, is given by

$$\delta \rho = -\rho_0 (\alpha \theta - \alpha' \gamma) \quad (10)$$

### III. DISPERSION RELATION

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w, h_z, \zeta, \xi, \theta, \gamma] = [W(z), K(z), Z(z), X(z), \Theta(z), \Gamma(z)] \exp(ik_x x + ik_y y + nt), \quad (11)$$

where  $k_x, k_y$  are horizontal wave numbers,  $k = (k_x^2 + k_y^2)^{1/2}$  is the resultant wave number and  $n$  is, in general, a

complex constant.  $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$  and  $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$  are the z-components of the vorticity and current density,

respectively.

Expressing the coordinates  $(x, y, z)$  in the new unit of length 'd' and letting

$$a = kd, \sigma = \frac{nd^2}{\nu}, p_1 = \frac{\nu}{\kappa}, p_2 = \frac{\nu}{\eta}, P_1 = \frac{k_1}{d^2}, M = \frac{mN}{\rho_0}, B = b + 1, E_1 = E + b\varepsilon$$

$$E_2 = E' + b\varepsilon, q = \frac{\nu}{\kappa'}, \tau_1 = \frac{\tau\nu}{d^2}, F = \frac{\lambda\nu}{d^2}, D^* = d \cdot \frac{d}{dz} = dD$$

and suppressing the superscript. The physical significance of suspended particles parameter B is that it does not depend on the model under consideration; however, it does depend upon the porosity of the medium.

Equations (2)-(9) with the help of equations (10) and (11), in non-dimensional form become

$$\left[ \frac{\sigma}{\varepsilon} \left( 1 + \frac{M}{1 + \tau_1 \sigma} \right) + \frac{1}{(1 + F\sigma)P_1} \right] (D^2 - a^2)W = \frac{\mu_e Hd}{4\pi\rho_0\nu} (D^2 - a^2)DK - \frac{gd^2 a^2}{\nu} (\alpha\Theta - \alpha'\Gamma), \quad (12)$$

$$(D^2 - a^2 - \sigma p_2)K = -\frac{Hd}{\varepsilon\eta} DW, \quad (13)$$

$$(D^2 - a^2 - \sigma E_1 p_1)\Theta = -\frac{\beta d^2}{\kappa} \left( \frac{B + \tau_1 \sigma}{1 + \tau_1 \sigma} \right) W, \quad (14)$$

$$(D^2 - a^2 - \sigma E_2 q_1)\Gamma = -\frac{\beta' d^2}{\kappa'} \left( \frac{B + \tau_1 \sigma}{1 + \tau_1 \sigma} \right) W. \quad (15)$$

Here we consider the case where both the boundaries are free and the medium adjoining the fluid is non-conducting. The case of two free boundaries is slightly artificial, except in stellar atmospheres (Spiegel [26]) and in certain geophysical situations where it is most appropriate. However, the case of two free boundaries allows us to obtain analytical solution without affecting the essential features of the problem. The appropriate boundary conditions for this case are

$$W = D^2 W = 0, \Theta = 0, \Gamma = 0, DZ = 0, X = 0 \text{ at } z = 0 \text{ and } z = 1. \quad (16)$$

Further  $K = 0$  on both the boundaries if the regions outside the fluid are perfectly conducting or  $DK = \mp aK$  on both the boundaries if the region outside the fluid are insulating. (17)

### IV. MATHEMATICAL ANALYSIS

We first prove the following lemma:

**Lemma:** If  $[\sigma = \sigma_r + i\sigma_i, W, \Theta, \Gamma, K]$  is a non-trivial solution of the double eigen value problem for  $\sigma_r$  and  $\sigma_i$  described by the equations (12)-(15) with the boundary conditions (16) and (17). Then a necessary condition for  $\sigma = 0$  (i.e.  $\sigma_r = \sigma_i = 0$ ) to be an eigen value is that

$$\int_0^1 (|D\Theta|^2 + a^2|\Theta|^2) dz \leq \frac{\beta^2 d^4 B^2}{\kappa^2 \pi^2} \int_0^1 |W|^2 dz.$$

**Proof of Lemma:** Since  $\sigma = 0$  is an eigen value, we have from equation (14)

$$(D^2 - a^2)\Theta = -\frac{\beta d^2 B}{\kappa} W. \quad (18)$$

Multiplying both sides of equation (18) by  $\Theta^*$  (the complex conjugate of  $\Theta$ ), integrating the resulting equation by parts for sufficient number of times over the vertical range of  $z$  by making the use of boundary condition (16) and separating the real parts of both sides of the equation so obtained, we get

$$\int_0^1 \left( |D\Theta|^2 + a^2 |\Theta|^2 \right) dz = \operatorname{Re} \frac{\beta d^2 B}{\kappa} \int_0^1 \Theta^* W dz = \frac{\beta d^2 B}{\kappa} \operatorname{Re} \int_0^1 \Theta^* W dz \quad (19)$$

Now

$$\begin{aligned} \operatorname{Re} \int_0^1 \Theta^* W dz &\leq \left| \int_0^1 \Theta^* W dz \right| \\ &\leq \int_0^1 |\Theta^* W| dz \\ &\leq \int_0^1 |\Theta| |W| dz \\ &\leq \sqrt{\int_0^1 |\Theta|^2 dz} \sqrt{\int_0^1 |W|^2 dz} \end{aligned}$$

(by Schwartz inequality)

Equation (18) and inequality (19) implies that

$$\int_0^1 \left( |D\Theta|^2 + a^2 |\Theta|^2 \right) dz \leq \frac{\beta d^2 B}{\kappa} \sqrt{\int_0^1 |\Theta|^2 dz} \sqrt{\int_0^1 |W|^2 dz} \quad (20)$$

which in turn implies that

$$\int_0^1 |D\Theta|^2 dz \leq \frac{\beta d^2 B}{\kappa} \sqrt{\int_0^1 |\Theta|^2 dz} \sqrt{\int_0^1 |W|^2 dz} \quad (21)$$

whence we derive from inequality (21) using Rayleigh-Ritz inequality

$$\int_0^1 |D\Theta|^2 dz \geq \pi^2 \int_0^1 |\Theta|^2 dz \quad , \quad (\text{since } \Theta = 0 \text{ at } z = 0 \text{ and } z = 1) \quad (22)$$

$$\sqrt{\int_0^1 |\Theta|^2 dz} \leq \frac{\beta d^2 B}{\pi^2 \kappa} \sqrt{\int_0^1 |W|^2 dz} \quad (23)$$

Inequalities (20) and (23) lead to

$$\int_0^1 \left( |D\Theta|^2 + a^2 |\Theta|^2 \right) dz \leq \left( \frac{\beta d^2 B}{\pi \kappa} \right)^2 \int_0^1 |W|^2 dz \quad (24)$$

and hence the lemma.

The contents of the above lemma when presented otherwise from the point of view of theoretical hydrodynamics imply that

**Lemma:** A necessary condition for the validity of the principle of exchange of stabilities in thermosolutal convection configuration of Maxwellian viscoelastic fluid in porous medium in the presence of magnetic field and suspended particles is that

$$\int_0^1 \left( |D\Theta|^2 + a^2 |\Theta|^2 \right) dz \leq \left( \frac{\beta d^2 B}{\pi \kappa} \right)^2 \int_0^1 |W|^2 dz \quad .$$

$$\text{Similarly, } \int_0^1 \left( |D\Gamma|^2 + a^2 |\Gamma|^2 \right) dz \leq \left( \frac{\beta' d'^2 B}{\pi \kappa'} \right)^2 \int_0^1 |W|^2 dz \quad (25)$$

The essential contents of lemma are true for this case also.

**We now prove the following theorem:**

**Theorem:** If  $[\sigma = \sigma_r + i\sigma_i, W, \Theta, \Gamma, K]$  is a non-trivial solution of the double eigen value problem for  $\sigma_r$  and  $\sigma_i$  described by the equations (12)-(15) with the boundary conditions (16) and (17) for given values of other parameters, then a sufficiency condition for the invalidity of  $\sigma = 0$  (i.e.  $\sigma_r = \sigma_i = 0$ ) to be an eigen value

is that  $\frac{BP_1}{\pi^2}(R + R_s) < 1$ .

**Proof:** Multiplying equation (12) by  $w^*$  (the complex conjugate of  $w$ ) and integrating the resulting equation over the vertical range of  $z$ , we obtain

$$\left[ \frac{\sigma}{\varepsilon} \left( 1 + \frac{M}{1 + \tau_1 \sigma} \right) + \frac{1 + F_0 \sigma}{(1 + F \sigma) P_1} \right] \int_0^1 W^* (D^2 - a^2) W dz - \frac{\mu_e H d}{4 \pi \rho_0 \nu} \int_0^1 W^* (D^2 - a^2) D K dz + \frac{g d^2 a^2}{\nu} \int_0^1 W^* (\alpha \Theta - \alpha' \Gamma) dz = 0, \quad (26)$$

Integrating equation (26) by parts for sufficient times by making use of boundary conditions (16) and (17) and equations (13)-(15), it follows that

$$\left[ \frac{\sigma}{\varepsilon} \left( 1 + \frac{M}{1 + \tau_1 \sigma} \right) + \frac{1 + F_0 \sigma}{(1 + F \sigma) P_1} \right] I_1 + \frac{\mu_e \varepsilon \eta}{4 \pi \rho_0 \nu} [I_2 + \sigma^* p_2 \{ I_3 + a (|K|_1^2 + |K|_0^2) \}] - \frac{g a^2}{\nu} \left( \frac{1 + \tau_1 \sigma^*}{B + \tau_1 \sigma^*} \right) \left[ \frac{\alpha \kappa}{\beta} (I_4 + \sigma^* E_1 p_1 I_5) - \frac{\alpha' \kappa'}{\beta'} (I_6 + \sigma^* E_2 q_1 I_7) \right] = 0, \quad (27)$$

where  $\sigma^*$  is the complex conjugate of  $\sigma$  and

$$I_1 = \int_0^1 (|DW|^2 + a^2 |W|^2) dz, \quad I_2 = \int_0^1 (D^2 - a^2) |K|^2 dz, \quad I_3 = \int_0^1 (|DK|^2 + a^2 |K|^2) dz, \\ I_4 = \int_0^1 (|D\Theta|^2 + a^2 |\Theta|^2) dz, \quad I_5 = \int_0^1 |\Theta|^2 dz, \quad I_6 = \int_0^1 (|D\Gamma|^2 + a^2 |\Gamma|^2) dz, \\ I_7 = \int_0^1 |\Gamma|^2 dz,$$

where the integrals  $I_1 - I_7$  are all positive definite.

Putting  $\sigma_r = 0$  in equation (27) and separating the real and imaginary parts of the resulting equation, we derive

$$\left[ \frac{M \tau_1 \sigma_i^2}{\varepsilon (1 + \sigma_i^2 \tau_1^2)} + \frac{(1 + F F_0 \sigma_i^2)}{P_1 (1 + F^2 \sigma_i^2)} \right] I_1 + \frac{\mu_e \varepsilon \eta}{4 \pi \rho_0 \nu} I_2 - \frac{g a^2}{\nu} \left( \frac{B + \sigma_i^2 \tau_1^2}{B^2 + \sigma_i^2 \tau_1^2} \right) \left[ \frac{\alpha \kappa}{\beta} I_4 - \frac{\alpha' \kappa'}{\beta'} I_6 \right] + \frac{g a^2}{\nu} \frac{(B - 1) \tau_1 \sigma_i^2}{B^2 + \sigma_i^2 \tau_1^2} \left[ \frac{\alpha \kappa}{\beta} E_1 p_1 I_5 - \frac{\alpha' \kappa'}{\beta'} E_2 q_1 I_7 \right] = 0, \quad (28)$$

and

$$\sigma_i \left\langle \left[ \frac{1}{\varepsilon} \left( 1 + \frac{M}{1 + \sigma_i^2 \tau_1^2} \right) - \frac{(F - F_0)}{1 + F^2 \sigma_i^2} \right] I_1 - \frac{\mu_e \varepsilon \eta p_2}{4 \pi \rho_0 \nu} [I_3 + a (|K|_1^2 + |K|_0^2)] - \frac{g a^2}{\nu} \left( \frac{\tau_1 (1 - B)}{B^2 + \sigma_i^2 \tau_1^2} \right) \left[ \frac{\alpha \kappa}{\beta} I_4 - \frac{\alpha' \kappa'}{\beta'} I_6 \right] - \frac{B + \tau_1^2 \sigma_i^2}{B^2 + \sigma_i^2 \tau_1^2} \left[ \frac{\alpha \kappa}{\beta} E_1 p_1 I_5 - \frac{\alpha' \kappa'}{\beta'} E_2 q_1 I_7 \right] \right\rangle = 0. \quad (29)$$

Equations (28) and (29) must be satisfied when  $\sigma_r = 0$ . Further since  $\sigma_i$  is also zero as a necessary condition of the theorem, equation (29) is identically satisfied while equation (28) reduces to

$$\frac{1}{P_1} I_1 + \frac{\mu_e \varepsilon \eta}{4 \pi \rho_0 \nu} I_2 - \frac{g a^2 \alpha \kappa}{\nu \beta B} I_4 + \frac{g a^2 \alpha' \kappa'}{\nu \beta' B} I_6 = 0. \quad (30)$$

Now making use of inequalities (24), (25) and the inequality

$$\int_0^1 \left( |DW|^2 + a^2 |W|^2 \right) dz \geq a^2 \int_0^1 |W|^2 dz \quad , \text{ (which is always valid),}$$

we derive from the equation (30)

$$\left\{ \frac{1}{P_1} I_1 + \frac{\mu_e \varepsilon \eta}{4 \pi \rho_0 \nu} I_2 + \frac{g a^2 \alpha' \kappa'}{\nu \beta' B} I_6 - \frac{g a^2 \alpha \kappa}{\nu \beta B} I_4 \right\} > \left\{ \frac{a^2}{P_1} \left[ 1 - \frac{B P_1}{\pi^2} (R + R_s) \right] \int_0^1 |W|^2 dz + \frac{\mu_e \varepsilon \eta}{4 \pi \rho_0 \nu} I_2 + \frac{2 g a^2 \alpha' \kappa'}{\nu \beta' B} I_6 \right\} \quad (31)$$

where  $R = \frac{g \alpha \beta d^4}{\nu \kappa}$  and  $R_s = \frac{g \alpha' \beta' d^4}{\nu \kappa'}$  are the thermal Rayleigh number and solutal Rayleigh number, respectively..

Now if  $\frac{B P_1}{\pi^2} (R + R_s) < 1$ , then the right hand side of inequality (31) is a positive definite which in turn implies that the left hand side of the inequality (31) must also be positive definite and therefore (30) cannot be satisfied.

Thus a sufficiency condition for the invalidity of zero being an eigen-value for  $\sigma$  is that  $\frac{B P_1}{\pi^2} (R + R_s) < 1$ .

It is clear from above that when regions outside the fluid are perfectly conducting

$$a \left\{ \left( |K|^2 \right)_1 + \left( |K|^2 \right)_0 \right\} = 0, \quad (32)$$

and hence the above analysis holds good for this case.

**Presented otherwise from the point of view of theoretical hydrodynamics, we have the following theorem:**

**Theorem:** A sufficiency condition for the invalidity of principle of exchange of stabilities in a thermosolutal convection configuration of Maxwellian viscoelastic fluid in porous medium in the presence of suspended particles and magnetic field is that the thermal Rayleigh number  $R$ , solutal Rayleigh number  $R_s$ , the medium permeability  $P_1$  and suspended particles parameter

$B$  are restricted by the inequality  $\frac{B P_1}{\pi^2} (R + R_s) < 1$ ,

**or in the context of overstability, we can state the above theorem as:**

**Theorem:** A sufficiency condition for the existence of overstability in a thermosolutal convection configuration of Maxwellian viscoelastic fluid in porous medium in the presence of suspended particles is that the thermal Rayleigh number  $R$ , solutal Rayleigh number  $R_s$ , medium permeability  $P_1$  and suspended particles parameter

$B$  are restricted by the inequality  $\frac{B P_1}{\pi^2} (R + R_s) < 1$ .

## REFERENCES

[1]. D.D. Joseph, *Stability of fluid motions* vol. 1 and 2 (Springer-Verleg, Berlin, 1976).

[2]. E.R. Lapwood, Convection of a fluid in a porous medium, *Proc. Camb. Phil. Soc.*, 44,1948, 508-554.  
 [3]. R.A. Wooding, Rayleigh instability of a thermal boundary layer in flow through a porous medium, *J. Fluid Mech.*, 9, 1960, 183-192.  
 [4]. S. Chandrasekhar, *Hydrodynamic and hydromagnetic stability* (Dover Publication, New York, 1981).  
 [5]. G. Veronis, On the finite amplitude instability in thermohaline convection, *J. Marine Res.*, 23, 1965, 1-17.  
 [6]. H. Stommel, A.B. Arons, and D. Blanchard, An oceanographical curiosity: The perpetual salt fountain, *Deep Sea Res.*, 3, 1956, 152-153  
 [7]. M.E. Stern, The salt fountain and thermohaline convection, *Tellus*, 12, 1960, 172-175.  
 [8]. D.A. Nield, The thermohaline convection with linear gradients, *J. Fluid Mech.*, 29, 1967, 545-558.  
 [9]. H. Tabor, and R. Matz, Solar pond project, *Solar Energy*, 9, 1965, 177-182.  
 [10]. T.G.L. Shirtcliffe, *J. Geophys. Res.*, 69, 1964, 5257-5268.  
 [11]. M.K. Brakke, Zone electrophoresis of dyes, proteins and viruses in density-gradient columns of sucrose solutions, *Arch. Biochem. Biophys.*, 55, 1955, 175-190.

- [12]. P. Nason, V. Schumaker, B. Halsall, and J. Schwedes, Formation of a streaming convective disturbance which may occur at one gravity during preparation of samples for zone centrifugation, *Biopolymers*, 7, 1969, 241-249.
- [13]. P.F. Lister, On the thermal balance of a mid-ocean ridge, *Geophys. J. Roy. Astr. Soc.*, 26, 1972, 515-535.
- [14]. J.A.M. McDonnell, *Cosmic dust* (John Wiley and Sons, Toronto, pp. 330, 1978).
- [15]. J.W. Scanlon, and L.A. Segel, Some effects of suspended particles on the onset of Be'nard convection, *Phys. Fluids*, 16, 1973, 1573-1578.
- [16]. R.C. Sharma, K. Prakash, and S.N. Dube, Effect of suspended particles on the onset of Be'nard convection in hydromagnetics, *Acta Phys. Hung.*, 40, 1976, 3-10.
- [17]. V.I. Palaniswamy, and C.M. Purushotham, Stability of shear flow of stratified fluids with fine dust, *Phys. Fluids*, 24, 1981, 1224-1229.
- [18]. C.M. Vest, and V.S. Arpaci, Overstability of a viscoelastic fluid layer heated from below, *J. Fluid Mech.*, 36, 1969, 613-619.
- [19]. P.K. Bhatia, and J.M. Steiner, Convective instability in a rotating viscoelastic fluid layer, *Z. Angew. Math. Mech.*, 52, 1972, 321-324.
- [20]. R.C. Sharma, and P. Kumar, Hall effect on thermosolutal instability in a Maxwellian viscoelastic fluid in porous medium, *Arch. Mech.*, 48, 1996, 199-209.
- [21]. P. Kumar, and M. Singh, On superposed Maxwellian viscoelastic fluids through porous medium in hydromagnetics, *JeJEMTA*, 3, 2008, 146-158.
- [22]. S. Wang, and W. Tan, Stability of double-diffusive convection of Maxwell fluid in a porous medium heated from below, *Phys. Lett. A*, 372, 2008, 3046-3050.
- [23]. F.G. Awad, P. Sibanda, and S.S. Motsa, On the linear stability analysis of a Maxwell fluid with double-diffusive convection, *Appl. Math. Modelling*, 34, 2010, 3509-3517.
- [24]. S. Wang, and W. Tan, Stability analysis of soret-driven double-diffusive convection of Maxwell fluid in a porous medium, *Int. J. Heat Fluid Flow*, 37, 2011, 88-94.
- [25]. P. Kumar, Stability analysis of viscoelastic fluid in porous medium, *Int. J. Fluid Mech. Research*, 40(5), 2013, 382-390.
- [26]. E.A. Spiegel, Convective instability in a compressible atmosphere, *Astrophys. J.*, 141, 1965, 1068-1070.