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Solving ONE'S interval linear assignment problem

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ABSTRACT
In this paper to introduce a matrix ones interval linear assignment method or MOILA -method for solving wide range of problem. An example using matrix ones interval linear assignment methods and the existing Hungarian method have been solved and compared. Also some of the variations and some special cases in assignment problem and its applications have been discussed ,the proposed method is a systematic procedure, easy to apply and can be utilized for all types of assignment problem with maximize or minimize objective functions.
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I. INTRODUCTION

We define the assignment matrix, then by using determinant representation we obtain a reduced matrix which has at least one 1 in each row and columns. Then by using the new method, we obtain an optimal solution for interval assignment problem by assigning ones to each row and each column, and then try to find a complete assignment to their ones. In a ground reality the entries of the cost matrix is not always crisp. In many application this parameters are uncertain and this uncertain parameters are represented by interval. In this contribution we propose matrix ones interval assignment methods and consider interval analysis concept for solving matrix ones interval linear assignment methods .This section presents a new method to solve the assignment problem which is different from the preceding method. We call it "Matrix one's- assignment method" ,because of making assignment in terms of ones. The new method is based on creating some ones in the assignment matrix and then try to find a complete assignment in terms of ones.

II. DEFINITION

Each assignment problem has a table or matrix associated with it. Generally the row contains the objects or people we wish to assign and the column comprise the jobs or task we want them assigned to. Consider a problem of assignment of n resources to m activities so as to minimize the overall cost or time in such a way that each resource can associate with one and only one job. The cost matrix (Cij) is given as under:

Activity
Resource

	A_1	A_2	----------------------		An	Available
R_1	C_{11}	C_{12}	-----	-----	C_{1n}	1
R_2	C_{21}	C_{22}			C_{2n}	1
--	---	---	----	----	----	--
--	----	-----	-----	-----	------	---
R_n	C_{n1}	C_{n2}			C_{nm}	1
Required	1	1	----	----	1	

The cost matrix is same as that of a T.P except that availability at each of the resource and the requirement at each of the destinations is unity.

Let x_{ij} denote the assignment of i^{th} resource to j^{th} activity, such that

$1,\wedge\ if resource i is assign j,$

$f(x) = \{0,\wedge\ otherwise|$

Then the mathematical formulation of the assignment problem is

Minimize Z= $\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}x_{ij}$

Subject to the constraints

$\sum_{i=1}^{1} x_{ij}$ = 1 and $\sum_{j=1}^{n} x_{ij}$ =1:x_{ij} = 0 or 1 for all i=1,2----n & j=1,2,-------n

2.1 Definition

The interval form of the parameters may be written as where is the left value [$\underline{x}$] and is the right value [$\overline{x}$]of the interval respectively. We define the centre is m = $\frac{\overline{x}+\underline{x}}{2}$ and w = $\overline{x} - \underline{x}$ is the width of the interval [$\overline{x}, \underline{x}$]

Let [$\overline{x}, \underline{x}$] and [$\overline{y}, \underline{y}$] be two elements then the following arithmetic are well known

(i) [$\overline{x}, \underline{x}$] +[$\overline{y}, \underline{y}$] = [$\overline{x}$ +$\overline{y}$, $\underline{x}$ + $\underline{y}$]

(ii) [$\overline{x}, \underline{x}$] × [$\overline{y}, \underline{y}$] =[min{$\overline{x}\underline{y}\underline{x}\overline{y}$, $\overline{x}\overline{y}$, $\underline{x}\underline{y}$} ,max{$\overline{x}\underline{y}\underline{x}\overline{y}$, $\overline{x}\overline{y}$, $\underline{x}\underline{y}$}]

(iii) [$\overline{x}, \underline{x}$] ÷ [$\overline{y}, \underline{y}$] =[min{$\overline{x} \div \underline{y}\underline{x} \div \overline{y}, \overline{x} \div \overline{y}, \underline{x} \div \underline{y}$} , max{$\overline{x} \div \underline{y}\underline{x} \div \overline{y}, \overline{x} \div \overline{y}, \underline{x} \div \underline{y}$}] provide if $\overline{y}, \underline{y}$] ≠ [0, 0],

Notation:

(i) H M - Hungarian method
(ii) MOAP- Matrix ones Assignment problem
(iii) MOILAP - Matrix ones interval Linear assignment problem
(iv) UBMOAP- Unbalanced Matrix ones Assignment problem
(v) UBMOILAP -Unbalanced Matrix ones Interval linear Assignment problem

III. ALGORITHM FOR HUNGARIAN METHOD:

Step1: Find the opportunity cost table by: (a). subtracting the smallest number in each row of the original cost table or matrix from every number in that row and. (b). Then subtracting the smallest number in each column of the table obtained in part (a) from every number in that column.

Step 2: Examine the rows successively until a row with exactly one unmarked zero is found. Enclose this zero in a box(□) as an assignment will be made there and cross (X) all other zeros appearing in the corresponding column as they will not be considered for future assignment. Proceed in this way until all the rows have been examined.

Step 3: After examining all the rows completely, examine the columns successively until a column with exactly one unmarked zero is found. Make an assignment to this single zero by putting square (□) around it and cross out (X) all other zeros appearing in the corresponding row as they will not be used to make any other assignment in that row, Proceed in this manner until all columns have been examined.

Step 4: Repeat the operations (**Step 1**) and (**Step 2**) successively until one of the following situations arises: (i). All the zeros in rows /columns are either marked (□) or corssed (X) and there is exactly one assignment in each row and in each column. In such a case optimal assignment policy for the given problem is obtained. (ii). there may be some row (or column) without assignment, i.e., the total number of marked zeros is less than the order of the matrix. In such a case, proceed to next Step 4.

Step 5: Draw the minimum number of vertical and horizontal lines necessary to cover all the zeros in the reduced cost table obtained from Step 2&3 by adopting the following procedure:

i. Mark (√) all rows that do not have assignments.
ii. Mark (√) all columns (not already marked) which have zeros in the marked rows
iii. Mark (√) all rows (not already marked) that have assignments in marked columns
iv. Repeat steps 4(ii) and (iii) until no more rows or columns can be marked.
v. Draw straight lines through each unmarked row and each marked column. If the number of lines drawn (or total assignments) is equal to the number of rows (or columns) then the current solution is the optimal solution,

Step 6:

Repeat Steps 2 to 4 until an optimal solution is obtained

3.1 Example :

Consider the following linear assignment problem. Assign the four jobs to the three machines so as to minimize the total cost.

	I	II	III	IV
1	10	5	10	15
2	3	9	15	3
3	10	7	3	2
4	2	3	2	4

Solution: Subtracting the smallest number in each row of the original cost table or matrix from every number in that row, as follows

	I	II	III	IV
1	5	0	5	10
2	0	6	12	0
3	8	5	1	0
4	0	1	0	2

Then subtracting the smallest number in each column of the table obtained from every number in that column.

	I	II	III	IV	Min
1	5	**0**	5	10	5
2	**0**	6	12	0	3
3	8	5	1	**0**	2
4	0	1	0	2	2
Min	0	0	0	0	

So the complete assignment is possible and we can assign the Zeros and the Optimum solution is (1→II), (2→I), (3→IV), (4→III), and minimum cost is 12.

IV. ALGORITHM FOR MATRIX ONES ASSIGNMENT PROBLEM

Step1.

In a minimization (maximization) case, find the minimum (maximum) elementof each row in the assignment matrix (say *ai*) and write it on the right handside of the matrix.Then divide each element of i^{th} row of the matrix by a_i. These operations create at least one ones in each rows. In term of ones for each row and column do assignment, otherwise go to step 2.

Step 2.

Find the minimum (maximum) element of each column in assignment matrix (say *bj*), and write it below jth column. Then divide each element of j^{th} column of the matrix by *bj* .These operations create at least one ones in each columns. Make assignment in terms of ones. If no feasible assignment can be achieved from step (1) and (2) then go to step 3.

Step 3.

Draw the minimum number of lines to cover all the ones of the matrix. If the number of drawn lines less than *n*, then the complete assignment is not possible, while if the number of lines is exactly equal to *n*, then the complete assignment is obtained.

Step 4.

If a complete assignment program is not possible in step 3, then select the smallest (largest) element (say *dij*) out of those which do not lie on any of the lines in the above matrix. Then divide by *dij* each element of the uncovered rows or columns, which *dij* lies on it. This operation create some new ones to this row or column. If still a complete optimal assignment is not achieved in this new matrix, then use step 4 and 3 iteratively. By repeating the same procedure the optimal assignment will be obtained.

4.1 Example :

Consider the following linear assignment problem. Assign the four jobs to the three machines so as to minimize the total cost.

	I	II	III	IV
1	10	5	10	15
2	3	9	15	3
3	10	7	3	2
4	2	3	2	4

Solution:

Find the minimum element of each row in the assignment matrix and write it on the hand side of the matrix, then divide each element of i^{th} row of the matrix ,These operations create ones to each rows, and the matrix reduces to following matrix

	I	II	III	IV	Min
1	2	**1**	2	3	5
2	**1**	3	5	1	3
3	5	$\frac{7}{2}$	$\frac{3}{2}$	**1**	2
4	1	$\frac{3}{2}$	**1**	2	2

Find the Minimum element of each column in interval assignment matrix and write it below j^{th} column. Then divide each element of j^{th} column of the matrix

	I	II	III	IV
1	2	**1**	2	3
2	**1**	3	5	1
3	5	$\frac{7}{2}$	$\frac{3}{2}$	**1**
4	1	$\frac{3}{2}$	**1**	2
Min	1	1	1	1

So the complete assignment is possible and we can assign the ones and the Optimum solution is (1→II), (2→I), (3→IV), (4→III), and minimum cost is 12.

V. ALGORITHM FOR BALANCED MATRIX ONES INTERVAL LINEAR ASSIGNMENT PROBLEM

Step 1:

The interval of cost matrix is balanced; find out the mid values of each interval in the cost matrix.

Step 2: Find the Minimum element of each row in the interval assignment matrix and write it on the right hand side of the matrix. Then divide each element of i^{th} row of the matrix . These operations create at least one ones in each rows. In term of ones for each row and column do assignment, otherwise go to step 4.

Step 3: Find the Minimum element of each column in interval assignment matrix and write it below j^{th} column. Then divide each element of j^{th} column of the matrix .These operations create at least one ones in each columns. Make interval assignment in terms of ones.

Step 4: Draw lines through appropriate rows and columns so that all the intervals contain one of the cost matrix are covered and the minimum number of such lines is used.

Step5: Test for optimality (i) If the minimum number of covering lines is equal to the order of the cost matrix, then optimality is reached. (ii) If the minimum number of covering lines is less than the order of the matrix, then go to step 6.

Step 6: Determine the smallest mid value of the intervals which are not covered by any lines divide this entry from all un-crossed element and add it to the crossing having an interval contain one's. Then go to step3.

5.1 Example:

Consider the following linear assignment problem. Assign the five jobs to the five machines so as to minimize the total cost.

	I	II	III	IV
1	10	5	10	15
2	3	9	15	3
3	10	7	3	2
4	2	3	2	4

Solution;

Now change the entry of the cost matrix by some interval form. Then we get a new cost matrix as follow. Cost matrix with interval entries

	I	II	III	IV
1	[9,11]	[4,6]	[9,11]	[14,16]
2	[2,4]	[8,10]	[14,16]	[2,4]
3	[9,11]	[6,8]	[2,4]	[1,3]
4	[1,3]	[2,4]	[1,3]	[3,5]

Find the minimum element of each row in the assignment matrix and write it on the hand side of the matrix, then divide each element of i^{th} row of the matrix .

	I	II	III	IV	Min
1	$[\frac{9}{4},\frac{11}{6}]$	[1,1]	$[\frac{9}{4},\frac{11}{6}]$	$[\frac{14}{4},\frac{16}{6}]$	[4,6]
2	[1,1]	$[\frac{8}{2},\frac{10}{4}]$	$[\frac{14}{2},\frac{16}{4}]$	[1,1]	[2,4]
3	$[\frac{9}{1},\frac{11}{3}]$	$[\frac{6}{1},\frac{8}{3}]$	$[\frac{2}{1},\frac{4}{3}]$	[1,1]	[1,3]
4	[1,1]	$[\frac{2}{1},\frac{4}{3}]$	[1,1]	$[\frac{3}{1},\frac{5}{3}]$	[1,3]

Then divide each element of i^{th} row of the matrix .These operations create ones to each rows, and the matrix reduces to following matrix

Now find the minimum element of each column in assignment matrix and write it below column. Then divide each element of j^{th} column of the matrix

	I	II	III	IV
1	$[\frac{9}{4},\frac{11}{6}]$	[1,1]	$[\frac{9}{4},\frac{11}{6}]$	$[\frac{14}{4},\frac{16}{6}]$
2	[1,1]	$[\frac{8}{2},\frac{10}{4}]$	$[\frac{14}{2},\frac{16}{4}]$	[1,1]
3	$[\frac{9}{1},\frac{11}{3}]$	$[\frac{6}{1},\frac{8}{3}]$	$[\frac{2}{1},\frac{4}{3}]$	[1,1]
4	[1,1]	$[\frac{2}{1},\frac{4}{3}]$	[1,1]	$[\frac{3}{1},\frac{5}{3}]$
Min	[1,1]	[1,1]	[1,1]	[1,1]

	I	II	III	IV
1	$[\frac{9}{4},\frac{11}{6}]$	**[1,1]**	$[\frac{9}{4},\frac{11}{6}]$	$[\frac{14}{4},\frac{16}{6}]$
2	**[1,1]**	$[\frac{8}{2},\frac{10}{4}]$	$[\frac{14}{2},\frac{16}{4}]$	[1,1]
3	$[\frac{9}{1},\frac{11}{3}]$	$[\frac{6}{1},\frac{8}{3}]$	$[\frac{2}{1},\frac{4}{3}]$	**[1,1]**
4	[1,1]	$[\frac{2}{1},\frac{4}{3}]$	**[1,1]**	$[\frac{3}{1},\frac{5}{3}]$

So the complete assignment is possible and we can assign the ones and the solution is (1,→II), (2→I), (3→IV), (4→III), and minimum cost is [8,16]. Now we are applying ones Assignment method, then we get a minimum assignment cost is 12

Result:
Balanced optimum solution for three methods is same solution

Problem	HM-Method	MOA method	MOILAP	Optimum
01	12	12	12	12

VI. UNBALANCED ASSIGNMENT PROBLEM

The method of assignment discussed above requires that the number of columns and rows in the assignment matrix be equal. However, when the given cost matrix is not a square matrix, the assignment problem is called an unbalanced problem. In such cases a dummy row(s) or column(s) are added in the matrix (with zeros as the cost elements) to make it a square matrix. For example, when the given cost matrix is of order 4×3, a dummy column would be added with zero cost element in that column. After making the given cost matrix a square matrix, Once the unbalanced assignment problem is converted into balanced assignment problem then we can follow usual algorithm to solve the assignment problem.

6.1Example:

Consider the following linear assignment problem. Assign the four jobs to the three machines so as to minimize the total cost.

	I	II	III	IV
1	10	5	13	15
2	3	9	18	3
3	10	7	3	2

Solution:
The unbalanced assignment problem is converted into balanced assignment problem

	I	II	III	IV
1	10	5	13	15
2	3	9	18	3
3	10	7	3	2
4	0	0	0	0

	I	II	III	IV	Min
1	5	**0**	8	10	5
2	**0**	6	15	0	3
3	8	5	1	**0**	2
4	0	0	**0**	0	0
Min	0	0	0	0	

So the complete assignment is possible and we can assign the Zeros and the Optimum solution is (1→,II), (2→I), (3→IV), (4→III), and minimum cost is 11.

VII. 7UNBALANCED MATRIX ONES ASSIGNMENT PROBLEM

The method of one's Assignment problem discussed above requires that the number of columns and rows in the assignment matrix be equal. However, when the given cost matrix is not a square matrix, the assignment problem is called an unbalanced problem. In such cases a dummy row(s) or column(s) are added in the matrix (with ones as the cost elements) to make it a square matrix. For example, when the given cost matrix is of order 4×3, a dummy column would be added with ones element in that column. After making the given cost matrix a square matrix, once the unbalanced ones assignment problem is converted into balanced ones assignment problem then we can follow usual algorithm to solve the assignment problem.

7.1Example:

Consider the following linear assignment problem. Assign the four jobs to the three machines so as to minimize the total cost.

	I	II	III	IV
1	10	5	13	15
2	3	9	18	3
3	10	7	3	2

Solution:
The unbalanced ones assignment problem is converted into balanced ones assignment problem

	I	II	III	IV
1	10	5	13	15
2	3	9	18	3
3	10	7	3	2
4	1	1	1	1

	I	II	III	IV	Min
1	$\frac{10}{5}$	**1**	$\frac{13}{5}$	3	5
2	1	3	6	1	3
3	5	$\frac{7}{2}$	$\frac{3}{2}$	**1**	2
4	1	1	**1**	1	1
Min	1	1	**1**	1	

So the complete assignment is possible and we can assign the ones and the optimum solution is (1→,II), (2→I), (3→IV), (4→III), and minimum cost is 11.

VIII. UNBALANCED MATRIX ONES INTERVAL LINEAR ASSIGNMENT PROBLEM

The method of one's interval Assignment problem discussed above requires that the number of columns and rows in the assignment matrix be equal. However, when the given cost matrix is not a square matrix, the ones interval assignment problem is called an unbalanced problem. In such cases a dummy row(s) or column(s) are added in the matrix (with interval ones as the cost elements) to make it a square matrix. For example, when the given cost matrix is of order 4×3, a dummy column would be added with interval ones element in that column. After making the given cost matrix a square matrix, once the unbalanced ones interval assignment problem is converted into balanced ones interval assignment problem then we can follow usual algorithm to solve the interval assignment problem.

8.1Example:
Consider the following linear assignment problem. Assign the four jobs to the three machines so as to minimize the total cost.

	I	II	III	IV
1	10	5	13	15
2	3	9	18	3
3	10	7	3	2

Solution:
Now change the entry of the cost matrix by some interval form. Then we get a new cost matrix as follow.& the unbalanced ones assignment problem is converted into balanced ones assignment problem

	I	II	III	IV
1	[9,11]	[4,6]	[12,14]	[14,16]
2	[2,4]	[8,10]	[17,19]	[2,4]
3	[9,11]	[6,8]	[2,4]	[1,3]
4	[1,1]	[1,1]	[1,1]	[1,1]

	I	II	III	IV	Min
1	$[\frac{9}{4},\frac{11}{6}]$	**[1,1]**	$[\frac{12}{4},\frac{14}{6}]$	$[\frac{14}{4},\frac{16}{6}]$	[4,6]
2	**[1,1]**	$[\frac{8}{2},\frac{10}{4}]$	$[\frac{17}{2},\frac{19}{4}]$	[1,1]	[2,4]
3	$[\frac{9}{1},\frac{11}{3}]$	$[\frac{6}{1},\frac{8}{3}]$	$[\frac{2}{1},\frac{4}{3}]$	**[1,1]**	[1,3]
4	[1,1]	[1,1]	**[1,1]**	[1,1]	[1,1]
Min	[1,1]	[1,1]	[1,1]	[1,1]	

The Hungarian condition satisfied, we are applying the proposed interval Hungarian method and solve this problem. We get an optimal assignment as 1,2,3,4 machines are assign to II,I,IV,III operators respectively and the minimum assignment cost is [8,14] Now we are applying ones Assignment method, then we get an minimum assignment cost is 11

IX. CONCLUSION

We conclude that the balanced optimum solution and unbalanced optimum solution are

(i) Balanced problem for three methods is same solution

Problem	HM-Method	MOA method	MOILAP	Optimum
01	12	12	12	12

(ii)Unbalanced problem for three methods is same solution

Problem	HM-Method	MOA method	MOILAP	Optimum
02	11	11	11	11

The MOILA-method can be used for all kinds of assignment problems, whether maximize or minimize objective. The new method is based on creating some ones in the assignment matrix and finds an assignment in terms of the ones. As considerable number of methods has been so for presented for assignment problem in which the Hungarian Method is more convenient method among them. Also the comparisons between both the methods have been shown in the paper. Therefore this paper attempts to propose a method for solving assignment problem which is different from thepreceding methods.

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