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Plasma-Maser Instability of the Electromagnetic Radiation In The Presence Of Electrostatic Drift Wave Turbulence in Inhomogeneous Plasma

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ABSTRACT
The generation mechanism of the electromagnetic radiation in case of inhomogeneous plasma on the basis of plasma-maser interaction in presence of drift wave turbulence is studied. The drift wave turbulence is taken as the low-frequency mode field and is found to be strongly in phase relation with thermal particles and may transfer its wave energy nonlinearly through a modulated field of high-frequency extraordinary mode (X-mode) wave. It has been found that amplification of X-mode wave is possible at the expense of drift wave turbulent energy. This type of high-frequency instability can leads to auroral kilometric radiation (AKR). The growth rate of the X-mode wave, in the form of AKR, has been calculated with the involvement of spatial density gradient parameter. This result may be particularly important for stability of various drift modes in magnetically confined plasma as well as for transport of momentum and energy in such inhomogeneous plasma.

I. INTRODUCTION

Drift waves and drift instabilities occupy a special place in the spectrum of collective plasma processes. This is because under laboratory conditions gradients in temperature, density, magnetic field and even impurity concentration are inevitable. Whenever a gradient exits, a plasma current or particle drift exists; drift waves are supported by these gradients and instabilities can tap the energy in the drifts. In addition drift modes propagating oblique to the magnetic field can tap the thermal energy of particles streaming along field lines. It has been observed that drift wave turbulence is one of the dominating turbulence wave energy available in any magnetically confined plasma [1].

The study on the role of drift wave turbulence associated with plasma inhomogeneity for the generation of various unstable radiation is an interesting issue for plasma physicists. The study of the transfer of wave energy from low-frequency turbulence to ion acoustic wave has been addressed by a number of authors [2,3].

In laboratory and astrophysical plasmas the problem of conversion of wave energy with a large change of frequency is important. Nonlinear interaction of plasma waves and particles can provide possible mechanism for the conversion. One of the attractive possibilities is connected with the plasma maser effect [4,5]. Plasma maser is effective for the energy up-conversion [6,7] from the low-frequency mode to the high-frequency mode. It occurs when nonresonant as well as resonant plasma oscillation are present. The resonant waves are those for which the linear Landau resonance condition $\omega - \vec{k}.\vec{v} = 0$ is satisfied, where ω and $\vec{k}$ are respectively the frequency and wave number of the resonant waves. The nonresonant waves are those for which both the linear and nonlinear (resonance of scattering into resonant waves) Landau conditions are not satisfied, i.e. $\Omega - \vec{K}.\vec{v} \neq 0$ and $(\Omega-\omega)-(\vec{K}-\vec{k}).\vec{v} \neq 0$ here Ω and $\vec{K}$ are respectively the frequency and wave number of the nonresonant wave. Plasma maser process is especially important in strongly magnetized plasma where $\Omega_e > \omega_{pe}$, [8], where Ω_e and ω_{pe} are respectively the electron cyclotron frequency and plasma frequency. However, the energy up conversion be quite effective since the standard Manley-Rowe relations are violated in this type of interactions [9]. Physically, the plasma maser instability is the dissipative type of nonlinear interaction in which the nonresonant wave amplification (or absorption) can be due to perturbation of resonant wave-particle collisionless interaction [6]. The effectiveness of the plasma maser processes is strongly connected with the symmetry properties of the system considered [8] and transition effects [10,11]. Furthermore, the nonlinear evolution of the resonant waves [12] and particle distribution also called inverse plasma maser [9,13], can be observed in the presence of the nonresonant turbulence.

In almost all the studies in plasma maser effect have been carried out considering the the plasma system as homogeneous [14,15], recently attempts have been to investigate the role of density gradient parameter in energy upconversion process through plasma maser effect in inhomogeneous plasma [2,3,16,17,18]. In our investigation, we are considering interaction of the drift wave turbulence present in inhomogeneous plasma with Bernstein mode wave. Considering a Maxwellian model distribution function under standard local approximation for inhomogeneous plasma [19], we have obtained the growth rate for Bernstein mode wave using nonlinear dispersion relation. Here we consider Fourier transform method. The real plasma, particularly in nature, is far from thermal equilibrium state. In such case, the initial value analysis is not valid [20], because the plasma is turbulent at the initial time and accordingly, the steady turbulent state is likely to be maintained by boundary conditions on past history of a plasma. The disribution function is far from a thermal equilibrium Maxwellian and the disrtibution is approximately steady within the time scale considered . Further in steady turbulent state of plasma, the fluctuation is enhanced where the disribution function is far from the thermal equilibrium state.

In our study it is found that amplification of Bernstein mode wave is possible at the expense of drift wave turbulence energy. This result is mainly important in stabilizing various drift modes in fusion plasma as well as explaining transport of momentum and energy in such inhomogeneous system of finite extent. Moreover, it is also important as nonlinear interaction of low frequecy resonant mode with thermal particles in long spatial lengths in magnetosphere can trasfer wave energy burst in space [21].

II. FORMULATION

We consider an inhomogeneous plasma in the presence of an enhanced electrostatic drift wave turbulence and an external magnetic field $B_0(\vec{z})$. The interaction of the turbulent fields with electrons which leads to the X-mode radiation test wave is governed by the Vlasov-Maxwell equations

$$\left[\frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial \vec{r}} - \frac{e}{m}\left(\vec{E} + \frac{\vec{v}\times\vec{B}}{c}\right)\cdot\frac{\partial}{\partial \vec{v}}\right] f_e(\vec{r},\vec{v},t) = 0. \tag{1}$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{2}$$

$$\vec{\nabla} \times \vec{B} = -\frac{1}{c}\frac{\partial \vec{E}}{\partial t} - \frac{4\pi}{c}\int \vec{v} f_e(\vec{r}, \vec{v}, t) d\vec{v}. \tag{3}$$

The inhomogeneous plasma considered for our study has spatial gradient in the Y-direction. The confining magnetic field with negligibly small gradient is taken along the $\vec{z}$ direction. For such an inhomogeneous system [22] the particle distribution function is considered as

$$\begin{aligned} f_{0e}(v_\perp, v_\parallel, y + \frac{v_x}{\Omega_e}) &\simeq f_{0e}(\vec{v}, y) + \frac{v_x}{\Omega_e}\frac{\partial f_{0e}(\vec{v}, y)}{\partial y} \\ &\simeq f_{0e}(\vec{v}, y)\left\{1 + \frac{v_x}{\Omega_e}\epsilon'\right\}. \end{aligned} \tag{4}$$

.

where

$$\epsilon' = \left[\frac{1}{f_{0e}}\frac{\partial f_{0e}}{\partial y}\right]_{y=0}$$

is the density gradient ,$v_x = v_\perp \cos\phi$, $v_\perp$ and $v_\parallel$ are components of velocity along and perpendicular direction of the external magnetic field, ϕ is the phase angle of the particle in the orbit, $\Omega_e = eB_0/mc$ is the electron cyclotron frequency and at $y = 0$, we have

$$f_{0e}(\vec{v}, y) = \left(\frac{m}{2\pi T_e}\right)^{3/2} \exp\left[-\frac{m}{2T_e}\left(v_\parallel^2 + v_\perp^2\right)\right] \tag{5}$$

.

Since the drift wave turbulence which is assumed to propagate almost perpendicular to the magnetic field with propagation vector $\vec{k} = (k_\perp, 0, k_\parallel)$ is present in our system, the various physical quantities can be written as

$$f_e = f_{0e} + \epsilon f_{1e} + \epsilon^2 f_{2e} + \delta f,$$

$$\vec{B} = \vec{B}_0 + \delta\vec{B}_l \tag{6}$$

and

$$\vec{E} = \epsilon\vec{E}_l + \delta\vec{E}.$$

where f_{0e} is the space-and time-averaged part, f_{1e} and f_{2e} are the low-frequency fluctuating parts of the electron distribution functions, $\vec{E}_l$ is the electrostatic ion-cyclotron wave field which is assumed to be in the Z and X- directions, ϵ is a small parameter, δf is the perturbed distribution function, $\delta\vec{E}(\vec{r}, t)$ and $\delta\vec{B}(\vec{r}, t)$ is the perturbed electric and magnetic fields of the high-frequency X-mode test wave.

To investigate the problem of plasma-maser from a quasisteady turbulent plasma with enhanced electrostatic drift waves we perturb the steady state by introducing high-frequency electromagnetic X-mode which is assumed to propagate perpendicular to the magnetic field with propagation vector $\vec{K} = (K_\perp, 0, 0)$. According to the linear response theory [20] of a turbulent plasma, we have

$$\begin{aligned} \delta\vec{E} &= \mu\delta\vec{E}_h + \mu\epsilon\delta\vec{E}_{lh} + \mu\epsilon^2\Delta\vec{E} \\ \delta\vec{B} &= \mu\delta\epsilon\delta\vec{B}_h + \mu\epsilon\delta\vec{B}_{lh} + \mu\epsilon^2\Delta\vec{B} \qquad (7) \\ \delta f &= \mu\delta f_h + \mu\epsilon\delta f_{lh} + \mu\epsilon^2\Delta f \end{aligned}$$

where μ is another small parameter ($\mu \ll \epsilon$) and $\delta\vec{E}_{lh}$, $\Delta\vec{E}$, $\delta\vec{B}_{lh}$, $\Delta\vec{B}$, δf_{lh} and Δf come from the mixed mode perturbation. Linearising the Vlasov-equation (1) to the order ϵ, μ, $\mu\epsilon$ and $\mu\epsilon^2$, we obtain respectively

$$Pf_{1e} = \frac{e}{m}\left(\vec{E}_l \cdot \frac{\partial}{\partial\vec{v}}\right) f_{oe}. \qquad (8)$$

$$P\delta f_h = \frac{e}{m}\left(\delta\vec{E}_h + \frac{\vec{v}\times\delta\vec{B}_h}{c}\right)\cdot\frac{\partial}{\partial\vec{v}} f_{oe}. \qquad (9)$$

$$\begin{aligned} P\delta f_{lh} = \frac{e}{m}\left(\vec{E}_l \cdot \frac{\partial}{\partial\vec{v}}\right)\delta f_h + \frac{e}{m}\left(\delta\vec{E}_h + \frac{\vec{v}\times\vec{\delta}B_h}{c}\right)\cdot\frac{\partial}{\partial\vec{v}} f_{1e} + \\ \frac{e}{m}\left(\delta\vec{E}_{lh} + \frac{\vec{v}\times\delta\vec{B}_{lh}}{c}\right)\cdot\frac{\partial}{\partial\vec{v}} f_{0e}. \end{aligned} \qquad (10)$$

.

$$P\Delta f = \frac{e}{m}\left(\vec{E}_l \cdot \frac{\partial}{\partial\vec{v}}\right)\delta f_{lh} + \frac{e}{m}\left(\delta\vec{E}_{lh} + \frac{\vec{v}\times\delta\vec{B}_{lh}}{c}\right)\cdot\frac{\partial}{\partial\vec{v}} f_{1e} \qquad (11)$$

$$P = \frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial\vec{r}} - \frac{e}{m}\left(\frac{\vec{v}\times\vec{B}_0}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}.$$

Eqs.(9) to (11) are the basic equations for the induced Bremsstrahlung interaction which comes from electron acceleration due to nonlinear forces.

For low-frequency electrostatic waves, the electron motion along the magnetic field is important. The Fourier component of the corresponding distribution function f_{1e}, from Eq.(8), is given by

$$f_{1e}(\vec{k},\omega) = \left(\frac{ie}{m}\right)\frac{E_{l\|}(\vec{k},\omega)\frac{\partial}{\partial v_\|} f_{oe}}{\omega - k_\| v_\| + i0}. \qquad (12)$$

where $i0$ is the small imaginary part associated with ω and $\vec{k}$ is the wave number of ion-cyclotron wave and $\|$ means parallel to the magnetic field.

Now, we obtain dielectric response function for high-frequency X-mode wave by using Poisson's equation

$$\nabla\cdot\delta\vec{E}_h = -4\pi e n_e \int \left[\delta f_h(\vec{K},\Omega) + \Delta f(\vec{K},\Omega)\right] d\vec{v}.$$

This takes the form

$$\delta E_h(\vec{K},\Omega)\epsilon_h(\vec{K},\Omega) = 0$$

The dispersion relation thus obtain can be written as

$$\epsilon_h(\vec{K},\Omega) = \epsilon_0(\vec{K},\Omega) + \epsilon_d(\vec{K},\Omega) + \epsilon_p(\vec{K},\Omega)$$

where $\epsilon_0(\vec{K},\Omega)$ is the linear part, $\epsilon_d(\vec{K},\Omega)$ is the direct coupling part and $\epsilon_p(\vec{K},\Omega)$ is the polarization coupling part, The expressions of these parts are given by

$$\epsilon_0(\vec{K},\Omega) = 1 + \frac{\omega_{pe}^2}{K_\perp^2}\left(\frac{m}{T_e}\right)\int\left\{1 + \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right)\right. \\ \left. \times \sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp\{i(b-a)\theta\}}{a\Omega_e - \Omega}\right\} f_{0e} d\vec{v}. \tag{13}$$

$$\epsilon_d(\vec{K},\Omega) = -\left(\frac{\omega_{pe}}{K_\perp}\right)^2\left(\frac{e}{m}\right)^2\int\left[\frac{m}{T_e}\left(1 + \left\{\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\}\times\right.\right. \\ \left.\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp\{i(b-a)\theta\}}{a\Omega_e - \Omega}\right)\times \\ \left\{\frac{E_{l\perp}}{K_\perp}\frac{m}{T_e}\left(1 + \left\{\Omega - \omega + k_\parallel v_\parallel - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\}\times\right.\right. \\ \left.\sum_{p,q}\frac{J_p(\alpha')J_q(\alpha')\exp\{i(q-p)\theta\}}{p\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\right) \\ \left.-E_{l\parallel}\frac{\partial}{\partial v_\parallel}\sum_{p,q}\frac{J_p(\alpha')J_q(\alpha')\exp\{i(q-p)\theta\}}{p\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\right\} f_{0e} + \\ \frac{m}{T_e}\left(\frac{E_{l\parallel}\frac{\partial}{\partial v_\parallel}f_{0e}}{-\omega + k_\parallel v_\parallel + i0}\right)\left(1 + \{\Omega - \omega + k_\parallel v_\parallel - \right. \\ \left.\left.\frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\}\sum_{a,b}\frac{J_a(\alpha')J_b(\alpha')\exp\{i(b-a)\theta\}}{a\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\right)\right]\times \\ \left[\frac{E_{l\perp}}{K_\perp}\frac{m}{T_e}\left(1 + \left\{\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\}\times\right.\right. \\ \left.\sum_{s,t}\frac{J_s(\alpha)J_t(\alpha)\exp[i(t-s)\theta]}{s\Omega_e - \Omega}\right) - \\ \left.E_{l\parallel}\frac{\partial}{\partial v_\parallel}\sum_{s,t}\frac{J_s(\alpha)J_t(\alpha)\exp[i(t-s)\theta]}{s\Omega_e - \Omega}\right] .d\vec{v}. \tag{14}$$

and the expression for $\epsilon_p(\vec{K},\Omega)$ can be put in the form taking dominant contribution into consideration

$$\epsilon_p(\vec{K},\Omega) = \frac{\omega_{pe}^2}{K_\perp^2}\left(\frac{e}{m}\right)^2 \frac{\omega_{pe}^2(\Omega-\omega)}{(\Omega-\omega)^2 - c^2K'^2}|E_{l\|}(\vec{k},\omega)|^2 \times (A+B) \tag{15}$$

where

$$\begin{aligned} A = \int v_\| \frac{m}{T_e}\left(\frac{\frac{\partial}{\partial v_\|}f_{0e}}{-\omega + k_\| v_\| + i0}\right)\{1 + (\Omega - \omega + k_\| v_\| - \\ \frac{\epsilon' T_e K_\perp}{m\Omega_e}\Big) \sum_{a,b}\frac{J_a(\alpha')J_b(\alpha')\exp[i(b-a)\theta]}{a\Omega_e - \Omega - k_\| v_\| + \omega}\Big\} \times \\ \left[\left(1 - \frac{k_\| v_\|}{\Omega - \omega}\right)\frac{m}{T_e}\left\{1 + \left(\Omega - \omega + k_\| v_\| - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right)\right.\right. \\ \left.\sum_{a,b}\frac{J_a(\alpha')J_b(\alpha')\exp[i(b-a)\theta]}{a\Omega_e - \Omega - k_\| v_\| + \omega}\right\} + \frac{k_\| v_\|}{\Omega - \omega}\times \\ \left.\frac{\partial}{\partial v_\|}\sum_{a,b}\frac{J_a(\alpha')J_b(\alpha')\exp[i(b-a)\theta]}{a\Omega_e - \Omega - k_\| v_\| + \omega}\right] \times \\ \frac{\partial}{\partial v_\|}\sum_{s,t}\frac{J_s(\alpha)J_t(\alpha)\exp[i(t-s)\theta]}{s\Omega_e - \Omega} d\vec{v}. \end{aligned} \tag{16}$$

and

$$\begin{aligned} B = \int v_\| \left[\frac{m}{T_e}\left\{1 + \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right)\times\right.\right. \\ \left.\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta]}{a\Omega_e - \Omega}\right\} \times \\ \left.\frac{\partial}{\partial v_\|}\sum_{p,q}\frac{J_p(\alpha')J_q(\alpha')\exp[i(q-p)\theta]}{p\Omega_e - \Omega - k_\| v_\| + \omega}\right] \times \\ \left[\left(1 - \frac{k_\| v_\|}{\Omega - \omega}\right)\left(\frac{\frac{\partial}{\partial v_\|}f_{0e}}{-\omega + k_\| v_\| + i0}\right)\times\right. \\ \left\{1 + \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right)\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta]}{a\Omega_e - \Omega}\right\} - \\ \frac{k_\| v_\|}{\Omega - \omega}\frac{\partial}{\partial v_\|}\left(\frac{\frac{\partial}{\partial v_\|}f_{0e}}{-\omega + k_\| v_\| + i0}\right)\times \\ \left.\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i(b-a)\theta]}{a\Omega_e - \Omega}\right] d\vec{v}. \end{aligned} \tag{17}$$

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III. GROWTH RATE OF THE X-MODE RADIATION

The growth rate of X-mode wave is calculated by using

$$\frac{\gamma_h}{\Omega} = -\left[\frac{I_m\epsilon_h}{\Omega\partial\epsilon_0/\partial\Omega}\right]_{\Omega=\Omega_r} . \tag{18}$$

By considering the fact that Plasma-maser effect arises from the condition $\omega = k_{\|}v_{\|}$. The most dominant contribution comes from $a = s = 1$ and $p = 0$ terms in summation. After simple calculation, by taking dominating terms only, we get from Eq.(13)

$$\begin{aligned}\epsilon_0(\vec{K},\Omega) =& 1 + \left(\frac{\omega_{pe}}{K_\perp}\right)^2\left(\frac{m}{T_e}\right) + \frac{1}{2}\left(\frac{\omega_{pe}}{\Omega_e^2}\right)^2 \times \\ & \left(\frac{1}{\Omega_e - \Omega}\right)\left\{\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e\Omega}\right\}.\end{aligned} \tag{19}$$

.

From Eq.(19),we obtain,

$$\frac{\partial\epsilon_0}{\partial\Omega} = \left(\frac{\omega_{pe}^2}{K_\perp^2}\right) \times \frac{1}{(\Omega_e - \Omega)^2} \times \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right). \tag{20}$$

Since, the plasma-maser comes from the imaginary parts of the direct coupling term and polarization coupling term, so we calculate the contributions of direct and polarization coupling terms separately.

3.1 Growth Rate of the X-Mode Radiation Due To Direct Coupling Term:

First we calculate the imaginary part of direct coupling term and applying plasma-maser conditions, we have, after lengthy but straightforward calculations (by partial integrations), from Eq.(14)

$$I_m\epsilon_d(\vec{K},\Omega) = 0$$

Thus, we find that contribution from direct coupling term is zero.

3.2 Growth Rate of the X-Mode Radiation Due To Polarization Coupling Term:

First we calculate the imaginary part of direct coupling term and applying plasma-maser conditions, we have, after lengthy but straightforward calculations from Eq.(15) as

$$I_m\epsilon_p(\vec{K},\Omega) = \frac{\omega_{pe}^2}{K_\perp^2}\left(\frac{e}{m}\right)^2 \frac{\omega_{pe}^2(\Omega-\omega)}{(\Omega-\omega)^2 - c^2K'^2}|E_{l\|}(\vec{k},\omega)|^2 \times I_m(A+B) \tag{21}$$

Now we calculate I_mA and I_mB. Integrating partially and after simplification, applying plasma-maser condition, the dominating terms are written as from Eq.(16), we have

$$I_mA = \left[1 + \frac{1}{2}\left(\frac{\Omega - \epsilon' T_e K_\perp / m\Omega_e}{\Omega_e - \Omega}\right)\left(\frac{K_\perp - k_\perp}{\Omega_e - \Omega}\right)^2 \frac{T_e}{m} + \left(\frac{\Omega - \epsilon' T_e K_\perp / m\Omega_e}{\Omega_e - \Omega}\right)^2 \Lambda_1\right] \times \frac{2\sqrt{\pi}}{v_e^3}\frac{\omega}{k_\parallel |k_\parallel|}\exp\left\{-\left(\frac{\omega}{k_\parallel v_e}\right)^2\right\}. \tag{22}$$

.

and from Eq.(17), we have

$$I_mB = \left[\frac{k_\parallel}{\Omega - \omega}\frac{m}{T_e}\frac{1}{\Omega_e - \Omega}\left\{\frac{1}{2}\left(\frac{K_\perp - k_\perp}{\Omega_e - \Omega}\right)^2 + \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right)\Lambda_1\right\}\right] \frac{2\sqrt{\pi}}{v_e^3}\frac{\omega}{k_\parallel |k_\parallel|}\exp\left\{-\left(\frac{\omega}{k_\parallel v_e}\right)^2\right\}. \tag{23}$$

.

where

$$\Lambda_1 = \int_0^\infty 2\pi v_\perp^2 J_1^2(\alpha) J_1^2(\alpha') f_{0e}(v_\perp) dv_\perp.$$

By substituting Eqs.(22)and (23)in Eq.(21)we can write the expression for $I_m\epsilon_p(\vec{K}, \Omega)$

Next we estimate $R(\vec{K} - \vec{k}, \Omega - \omega)$ by expanding from Eq.(A.10), about the small argument $\vec{k}$ and ω in R and we have used the relation $\epsilon(\vec{K}, \Omega) = 0$. To the lowest order approximation for $\Omega > \omega$,

$$\frac{1}{R(\vec{K} - \vec{k})[(\Omega - \omega)^2 - c^2 k_\perp^2]} \simeq \frac{1}{c^2 k_\perp^2}$$

Hence the growth rate due to polarization coupling term by using formula (18), we have (after simplification and taking dominating contribution only)

$$\frac{\gamma_h}{\Omega} = 2\sqrt{\pi}\frac{\omega_{pe}^2}{\Omega_e^2}|E_{l\parallel}(\vec{k}, \omega)|^2 \left\{1 - \frac{\epsilon' T_e K_\perp}{m\Omega_e \Omega}\right\}\frac{K_\perp^2}{k_\perp^2}\exp\left\{-\left(\frac{\omega}{k_\parallel v_e}\right)^2\right\}. \tag{24}$$

When there is no density gradient ($\epsilon' = 0$) i.e., for homogeneous plasma, the growth rate of X-mode wave is obtained, from Eq.(24),as

$$\frac{\gamma_h}{\Omega} = 2\sqrt{\pi}\frac{\omega_{pe}^2}{\Omega_e^2}|E_{l\parallel}(\vec{k}, \omega)|^2 \frac{K_\perp^2}{k_\perp^2}. \tag{25}$$

When high density gradient is present in the system i.e., for inhomogeneous plasma system, the growth rate of the X-mode wave is estimated, from Eq.(24), as

$$\frac{\gamma_h}{\Omega} = 2\sqrt{\pi}\frac{\omega_{pe}^2}{\Omega_e^2}\frac{|E_{l\|}(\vec{k},\omega)|^2}{\Omega}\frac{1}{v_e^2}\frac{K_\perp^2}{k_\perp^2} \times \epsilon'. \quad (26)$$

However, for small-order gradient there is no effect in the growth rate and Eq.(25) will give the estimate for such a small-gradient situation.

IV. APPLICATION AND DISCUSSION

As an illustration, we apply the result of our investigation to AKR. Accordingly, we take the typical plasma parameters [23]

$E_{l\|} \sim 10\mu vm^{-1}$, $\Omega \sim 200KHz$, $k_\perp \sim 2\pi \times 10^{-5}cm^{-1}$, $K_\perp \sim 8.3 \times 10^{-6}cm^{-1}$, $\Omega_e \sim 10\omega_{pe}$. We have from Eq.(25)

$$\frac{\gamma_h}{\Omega} \simeq 10^{-2} \quad (27)$$

and from Eq. (26), we have

$$\frac{\gamma_h}{\Omega} \simeq 10^{-2} \times \epsilon' \quad (28)$$

and from Eq.(28), by taking $\epsilon' = 10$, we have

$$\frac{\gamma_h}{\Omega} \simeq 10^{-1} \quad (29)$$

The growth rate is much enhanced because all the accelerated background electrons take part in the emission.

It should be mentioned that the electromagnetic emission without inverted electron population is most important characteristic of the plasma-maser. Similar high-frequency radiation is also observed in laboratory experiments [24]. The plasma maser contribution $I_m\epsilon_h(\vec{K},\Omega)$is composed of two parts:$I_m\epsilon_d(\vec{K},\Omega)$ and $I_m\epsilon_p(\vec{K},\Omega)$, which are the Plasma maser contribution due to direct coupling term and polarization coupling term respectively. It was pointed out in Ref.[6] and [9] that $I_m\epsilon_d(\vec{K},\Omega) \neq 0$ and $I_m\epsilon_p(\vec{K},\Omega) = 0$ for un-magnetized plasma. Furthermore, only for closed system, without external magnetic field, the plasma maser contribution from the direct coupling term exactly cancels out with the reverse absorption effect if the slow time change of medium due to quasilinear interaction is considered [6]. This is because, for closed system, both the low-frequency turbulence and the background electron distribution function are not fixed by external agents but are free to evolve self- consistently to form a quasilinear plateau. Then, both the plasma maser and the reverse process due to quasilinear effect coexist and there is no net growth of the nonresonant high-frequency test wave. On the other hand, for an open system (a plasma in the external magnetic field considered here)with a particle supply from outside, the electron distribution function is fixed by external agents and the reverse absorption effect vanishes. Then the stationary state without quasilinear plateau is possible. Accordingly the energy transferred from low-frequency wave by resonant interaction must go to an unstable high-frequency mode.

Appendix

The Vlasov equation (9)-(11) are now solved by integrating along the orbits of the particles in the unperturbed fields. In cylindrical coordinates $v_x = v_\perp \cos\phi$, $v_y = v_\perp \sin\phi$, $v_z = v_\parallel$, the particle orbits $r'(\tau)$ are given $v'_x = v_\perp \cos(\theta - \Omega_e \tau)$, $v'_y = v_\perp \sin(\theta - \Omega_e \tau)$, $v'_z = v_z$ and

$$x' = x - \frac{v_\perp}{\Omega_e}\sin(\theta - \Omega_e\tau) + \frac{v_\perp}{\Omega_e}\sin\theta$$

$$y' = y + \frac{v_\perp}{\Omega_e}\cos(\theta - \Omega_e\tau) - \frac{v_\perp}{\Omega_e}\cos\theta$$

$$z' = z + v_\parallel \tau, \quad \tau = t' - t$$

From Eq. (9), after using Maxwell's equation and integrating partially, we have

$$\delta f_h(\vec{K},\Omega) = \frac{ie}{m}\frac{\delta E_h}{K_\perp}\frac{m}{T_e}\left[1 + \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right) \times \sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp i(b-a)\theta}{a\Omega_e - \Omega}\right] f_{0e}.$$

From Eq.(10), we have

$$\delta f_{lh} = \int \frac{e}{m}\left[\left(\vec{E}_l \cdot \frac{\partial}{\partial \vec{v}}\right)\delta f_h + \left(\delta\vec{E}_h + \frac{\vec{v}\times\vec{\delta}B_h}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}f_{1e} + \left(\delta\vec{E}_{lh} + \frac{\vec{v}\times\delta\vec{B}_{lh}}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}f_{0e}.\right] d\vec{v}.$$
$$= I_1^{lh} + I_2^{lh} + I_3^{lh}.$$

Now

$$I_1^{lh} = \frac{e}{m}\int\left(\vec{E}_l\cdot\frac{\partial}{\partial\vec{v}}\right)\delta f_h d\vec{v}.$$
$$= \frac{ie}{m}\left[\frac{ie}{m}\frac{\delta E_h}{K_\perp}\frac{m}{T_e}\left\{1 + \left(\Omega - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right) \times \sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp\{i(b-a)\theta\}}{a\Omega_e - \Omega}\right\}\right] \times$$
$$\left[\frac{E_{l\perp}}{K_\perp}\frac{m}{T_e}\left(1 + \left\{\Omega - \omega + k_\parallel v_\parallel - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\} \times \sum_{p,q}\frac{J_p(\alpha')J_q(\alpha')\exp i(q-p)\theta}{p\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\right) - E_{l\parallel}\frac{\partial}{\partial v_\parallel}\sum_{p,q}\frac{J_p(\alpha')J_q(\alpha')\exp i(q-p)\theta}{p\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\right] f_{0e}.$$

$$I_2^{lh} = \frac{e}{m}\int\left(\delta\vec{E}_h + \frac{\vec{v}\times\vec{\delta}B_h}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}f_{1e}d\vec{v}.$$

$$= \frac{ie}{m}\frac{\delta E_h}{K_\perp}\frac{m}{T_e}\left(\frac{ie}{m}\times\frac{E_{l\|}(\vec{k},\omega)\frac{\partial}{\partial v_\|}f_{0e}}{\omega - k_\| v_\| + i0}\right)\times$$

$$\left\{1 + \left(\Omega - \omega + k_\| v_\| - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right)\times\right.$$

$$\left.\sum_{a,b}\frac{J_a(\alpha')J_b(\alpha')\exp\{i(b-a)\theta\}}{a\Omega_e - \Omega - k_\| v_\| + \omega}\right\}$$

.

$$I_3^{lh} = \frac{e}{m}\int\left(\delta\vec{E}_{lh} + \frac{\vec{v}\times\delta\vec{B}_{lh}}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}f_{0e}d\vec{v}.$$

$$= \left(\frac{ie}{m}\right)\frac{\delta E_{lh}(\vec{K}-\vec{k})}{|\vec{K}-\vec{k}|}\left[\left(1 - \frac{k_\| v_\|}{\Omega - \omega}\right)\frac{m}{T_e}\{1 + (\Omega - \omega\right.$$

$$\left. + k_\| v_\| - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right)\sum_{a,b}\frac{J_a(\alpha')J_b(\alpha')\exp[i(b-a)\theta]}{a\Omega_e - \Omega - k_\| v_\| + \omega}\Bigg\} f_{0e} +$$

$$\left.\frac{k_\| v_\perp}{\Omega - \omega}\frac{\partial f_{0e}}{\partial\vec{v}}\sum_{a,b}\frac{(2a/\alpha')J_a(\alpha')J_b(\alpha')\exp[i(b-a)\theta]}{a\Omega_e - \Omega - k_\| v_\| + \omega}\right]$$

.

Now we obtain mixed modulation electric field $\delta E_{lh}(\vec{K}-\vec{k}, \Omega - \omega)$, as follows

$$\delta E_{lh} = \frac{4\pi e n_e(\Omega - \omega)}{mR(\Omega - \omega)^2 - c^2 K'^2}\frac{\delta E_h}{K_\perp}\int v_\| \left[\left\{\left[\frac{ie}{m}\frac{m}{T_e}\{1 + (\Omega - \right.\right.\right.$$

$$\left.\left.\frac{\epsilon' T_e K_\perp}{m\Omega_e}\right)\sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp\{i(b-a)\theta\}}{a\Omega_e - \Omega}\right\}\right]\times$$

$$\left[\frac{E_{l\perp}}{K_\perp}\frac{m}{T_e}\left(1 + \left\{\Omega - \omega + k_\| v_\| - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right\}\times\right.\right.$$

$$\left.\sum_{p,q}\frac{J_p(\alpha')J_q(\alpha')\exp i(q-p)\theta}{p\Omega_e - \Omega - k_\| v_\| + \omega}\right) -$$

$$\left. E_{l\|}\frac{\partial}{\partial v_\|}\sum_{p,q}\frac{J_p(\alpha')J_q(\alpha')\exp i(q-p)\theta}{p\Omega_e - \Omega - k_\| v_\| + \omega}\right]\Bigg\} f_{0e} + \qquad (A.9)$$

$$\frac{ie}{m}\frac{m}{T_e}\left(\frac{ie}{m}\times\frac{E_{l\|}(\vec{k},\omega)\frac{\partial}{\partial v_\|}f_{0e}}{\omega - k_\| v_\| + i0}\right)\times$$

$$\left\{1 + \left(\Omega - \omega + k_\| v_\| - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right)\times\right.$$

$$\left.\left.\sum_{a,b}\frac{J_a(\alpha')J_b(\alpha')\exp\{i(b-a)\theta\}}{a\Omega_e - \Omega - k_\| v_\| + \omega}\right\}\right] d\vec{v}.$$

.

where $K' = K_\perp - k_\perp$ and

$$R = 1 + \frac{\omega_{pe}^2(\Omega-\omega)}{(\Omega-\omega)^2 - c^2K'^2}\int v_\parallel \left[\left(1-\frac{k_\parallel v_\parallel}{\Omega-\omega}\right)\frac{m}{T_e}\left\{1+(\Omega - \omega + k_\parallel v_\parallel - \frac{\epsilon' T_e K_\perp}{m\Omega_e}\right)\sum_{a,b}\frac{J_a(\alpha')J_b(\alpha')\exp[i(b-a)\theta]}{a\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\right\} f_{0e} + \frac{k_\parallel v_\parallel}{\Omega-\omega}\frac{\partial f_{0e}}{\partial \vec{v}}\sum_{a,b}\frac{(2a/\alpha')J_a(\alpha')J_b(\alpha')\exp[i(b-a)\theta]}{a\Omega_e - \Omega - k_\parallel v_\parallel + \omega}\Bigg] d\vec{v}. \quad (A.10)$$

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