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Application of Fuzzy Algebra in Coding Theory

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ABSTRACT
Fuzziness means different things depending upon the domain of application and the way it is measured. By means of fuzzy sets, vague notions can be described mathematically now a vigorous area of research with manifold applications. It should be mentioned that there are natural ways (not necessarily trivial) to fuzzily various mathematical structures such as topological spaces, algebraic structure etc. The notion of L-fuzzy sets later more generalizations were also made using various membership sets and operations. In this section we let F denote the field of integers module 2, we define a fuzzy code as a fuzzy subset of F^n where $F^n = \{(a_1, ...a_n \mid a_i \in F, i = 1, ...n\}$ and n is a fixed arbitrary positive integers we recall that F^n is a vector space over F. We give an analysis of the Hamming distance between two fuzzy code words and the error – correcting capability of a code in terms of its corresponding fuzzy codes. The results appearing in the first part of this section are from [17].
Key Words: Fuzzy code, Symmetric channel, Hankel matrices, Membership value, Semigroup,

I. INTRODUCTION

Now mention some other way ⇒ fuzzy abstract algebra has been applied. The paper [35] deals with the classification of knowledge when they are endowed with some fuzzy algebraic structure. By using the quotient group of symmetric knowledge as algebraic method is given in [35] to classify them also the anti fuzzy sub groups construction used to classify knowledge. In the paper [20] fuzzy points are regarded as data and fuzzy objects are constructed from the set of given data on an arbitrary group. Using the method of least square, optimal fuzzy subgroups are defined for the set of data and it is shown that one of them is obtained as f fuzzy subgroup by a set of some modified data.In [55], a decomposition of an valued set given a family of characteristic functions which can be considered as a binary block code. Conditions are given under which an arbitrary block code corresponds to L-valued fuzzy set. An explicit description of the Hamming distance, as well as of any code distance is also given all in lattice-theoretic terms. A necessary and sufficient conditions is given for a linear code to correspond to an L-valued fuzzy set.

One of the most important problem in coding theory is to define code whose codeword are "far apart" from each other as possible or whose values EC is maximized. It is also desirable to decode uniquely. For example let n =3 and c = (0 0 0) (1 0 1). Then $d_{min}(C) = 2$ suppose that a codeword is transmitted across the channel and (0, 0, 1) is received. Then (0, 0, 1) is of distance 1 from both the code word (0, 0, 0) and (1, 0, 1). Hence (0, 0, 1) cannot be decoded uniquely. Thus in always order to able be correct a single error we must have $d_{min}(c)$ at least equal to 3. If c = {[0, 0, 0)(1, 1, 1)} then (0, 0, 1) is decoded as (0, 0, 0) since it is closer to (0, 0, 0) then it is to (1, 1, 1).We now examine the fuzzy codes. For any code, $c \subseteq F^n$, we have see that there is a corresponding fuzzy code Φ(c). If $u \in F^n$ is a received word and C is a core word,i.e $c \in C$ then $\tilde{A}_C(u)$ is the probability that c was transmitted. Fuzzy sets appear to be a natural setting for the study of codes in that probability of error in the channel is included in the definition of fuzzy code. In the following we assume that $p \neq q$.

Definition 1.1: $\forall\ u = (u_1, ...u_n) \in F^n$ define the fuzzy subset $\tilde{A}u$ of F^n by $\forall\ u = u_1, ...u_n \in F^n$, $\tilde{A}u(u) = P^{n-d} q^d$, where

$$d = \sum_{i=1}^{n} |x_i - v_i|$$ and p and q are fixed positive real numbers such that p + q = 1

Define $\Phi : F^n \to \tilde{A}^n = \{\tilde{A}_u \mid u \in F^n\}$ by $\Phi(u) = \tilde{A}_u \forall u \in F^n$. Then Φ is a one to one functions of F^n onto $\tilde{A}^n$.

Definition 1.2: If $c \subseteq F^n$ then $\Phi(c)$ is called a fuzzy code corresponding to the code C.If $c \in C$ then $\tilde{A}_c$ is called a fuzzy code word. We consider an example let n = z and c = {(0, 0, 0) (1, 1, 1). If (0, 0, 0) is transmitted and (0, 1, 0) is received, then assuming $q < ½$ there is a greater likelihood that (0, 0, 0,) was transmitted then (1, 1, 1) since we are assuming burst errors do not occur.

Let $u \in F^n$ and $u^n \in F^n$ then $\sum_{i=1}^{n} |u_i - u_i|$ is the number of coordinates position in which u and u differ. The number of errors required to transform u into U equal this number. We let d(u, v) denote $\sum_{i=1}^{n} |u_i - u_i| d(u,v)$ is called the Hamming distance of uv.

Definition 1.3: Let $C \subseteq F^n$ be a code the minimum distance c is defined to be $d_{min}(c) = \Lambda\{d(a,b) | (a,b) \in c, a \neq b\}$. If C is a subspace of F^n then $d_{min}(c)$ $= \Lambda\{d(a,0) | a \in c, a \neq 0\}$
0 = (0, ..., 0) now $d(a_0, 0)$ is the number of non zero entries in a and is called the weight of a and often is denoted by | a | when a code C is a subspace of F^n, we called it a linear code.
Let [[ ]] denote the greater integer function on the real numbers for any code $c \subseteq F^n$
$E_c = [[d_{min}(C) - 1)/2]]$ in the maximum number of errors allowed in the channel for each n bits transmitted for which received signals may be correctly decoded **Definition 1.4:** Let $\subseteq F^n$ be a code. Define
$Q: F^n \to \{\tilde{A} / \tilde{A}$ is a fuzzy subset of $F^n\}$ by $\forall u \in F^n$,
$Q(u) = \{\tilde{A}_c \mid c \in C, \tilde{A}_c(u)(u) \geq \tilde{A}_b(u) \quad \forall b \in C\}$
A code for which $|\theta(u)| = 1 \ \forall u \in F^n$ is uniquely decodable. In such a case u is decoded as $\Phi^{-1}(\theta(u))$. An important criteria for designing good code is s spasing the code words as far apart from each other as possible, the Hamming distance is the metric used in F^n to measure distance. Analogously the generalized Hamming distance between fuzzy subsets may be used as a metric in $\tilde{A}^n$. It is defined by $\forall \tilde{A}_u . \tilde{A}_u \in \tilde{A}^n$,

$$d(\tilde{A}_u, \tilde{A}_u) \sum_{w \in F^n} |\tilde{A}_u(w) - \tilde{A}_u(w)|.$$

The theorem which follows show that $d(\tilde{A}_u, \tilde{A}_u)$ is independent of n.

Let $u, v \in F^n$ be such that d(u, x) = d), if $P \neq q$ and $p \neq 0$, then $d(\tilde{A}_u, \tilde{A}_u) = \sum_d$, where

$$\sum_d = \sum_{i=0}^{d} \binom{d}{i} | p^i q^{d-i} - P^{d-i} q^i,$$

Theorem 2.1: Let C_1 and C_2 be two codes used in the same channel. If $p \neq q$ and $p \neq 0, 1$ then $E_{C_1} = E_{C_2}$ if and only if $d_{min}(\Phi(c_1)) = d_{min}(\Phi(c_2))$
$d_{min} = (\Phi(C_1)) = d_{min}(\Phi(C_2))$.

Proof: Let $d_i = d_{min}(\Phi(c_1))$ for i = 1, 2. If $E_{C_1} = E_{C_2}$, then either $d_1 = d_2$ in which case the desired result holds or $d_2 = d_1 + 1$, where d_1 is a an odd positive integer. From Lemma we have if $p \neq q$ and $P \neq 0, 1$ then $\sum_1 < \sum_2 = \sum_2 < \sum_3 = \sum_4 < \sum_5 = ...$
$d_{min}(\Phi(c_1)) = d_{min}(\Phi(c_2))$, then it follows from lemma.

Conversely if $d_{min}(\Phi(c_1)) = d_{min}(\Phi(c_2))$, either $d_1 = d_2$ and so $E_{C_1} = E_{C_2}$ or $d_1 = d_2 \pm 1$ where $d_1 \cap d_2$ is odd and so $E_{C_1} = E_{C_2}$ .Let M be a subset of . Let $C \subseteq F^n$ be a code suppose c ∈ C is transmitted across the channel and that u ∈ F^n is received. The signal that are transmitted are usually distorted by varying degrees.

Let M be a subset of R. Let $C \subseteq F^n$ be a code.

Suppose c∈C is transmitted across the channel and that u∈F^n is received. The signal that are transmitted are usually destroyed by varying degrees. The electrical receiver may record the signals is one of the two ways. The electrical waves received representing the word 4 Rs. measured bit by bit as real numbers. The signals then are either recorded as n types over M or each bit of u is transformed into an element of F. u is then coded in F^m for some positive integer's m. For example suppose that (1, 0, 1) is encoded as C = (1, 0, 1, 0) and transmitted electronically across the channel. Supposed the received waves are measured as u = (1, 02, 0.52, 0.98, 0.02) and recorded as U = (1, 1, 1, 0), Then some information is lost in recording u as u – Some possible directories for further study to overcome this loss and suggested in [15]. For example let x∈ F^n and define the fuzzy subset $\tilde{A}_x$ of M^n by

$\forall$ y∈ M^n $\tilde{A}_x(y) = p(x, y)$

where P(x, y) is the probability that if x is coded and transmitted across the channel, y is received. Then soft decoding may be studied via { $\tilde{A}_x$ /x∈F^m}We have assumed that error in the transmitted of words across a noisy channel were symmetric in nature i.e., the probability of $1 \Rightarrow 0$ and $0 \Rightarrow 1$ cross over failures were equally likely. However error in VSLI circuit and many computer memories are on a undirectional nature [8] A undirectional error model assumes that both $1 \Rightarrow 0 \Rightarrow 1$ cross overs can occur, but only are type of error occurs in a particular data word. This has provided the basis for a new direction in coding theory and fault tolerance computing. Also the failure of the memory cells of some of the LSI transistor cell memories and NMOS memories are most likely caused by leakage of charge. If we represent the presence of charge in a cell by 1 and the absence of charge by 0, then the errors in those type of memories can be modeled as $1 \Rightarrow 0$ type symmetric errors, [8].The result in the remainder of this section are from [15]. Once again F denotes the field of integers module 2 and F^n the vector space of n-tuples over F, we let p denotes the transmitted 1 will be received as a1 and a transmitted 0 will be received as a 0, Let q = 1−p. Then q is the probability that there is an error in transmission in an arbitrary bit

Definition ⇒ (1.5): Let u = $u_1, \ldots, u_n \in F^n$. Define the fuzzy subset $\tilde{A}(u)$ of F^n as follows: ∀ u

∀ u = $(u_1, \ldots, u_n) \in F^n$

$$\tilde{A}_u(U) = \begin{cases} 0 & \text{if } K_1 \Lambda K_2 \neq 0 \\ P^{m-d} q^d & \text{otherwise} \end{cases}$$

Where, $K_1 = \sum_{i=1}^{n} OV(u_i - v_i), \quad K_2 = \sum_{i=1}^{n} OV(u_i - v_i)$

$$d = \begin{cases} K_1 & \text{if } K_2 = 0 \\ K_2 & \text{if } K_1 = 0 \end{cases} \quad m = \begin{cases} \sum_{i=1}^{n} u_i & \text{if } K_2 = 0 \\ n - \sum_{i=1}^{n} u_i & \text{if } K_1 = 0 \\ \left(\sum_{i=1}^{n} u_i\right) V \left(n - \sum_{i=1}^{n} u_i\right) & \text{if } K_1 = K_2 = 0 \end{cases}$$

By this definition $\tilde{A}_u(u)$ is zero if both one and zero transitions have occurred. It allows either by themselves, to occur in a given received word. In the case that a received word is the same as that transmitted, one may choose either the one's or the zero's as possible toggling. In our definitions, we choose which ever there are more of for example: if there are more one's, the definition would expect $1 \rightarrow 0$ transitions only.

Hence in equal to the numbers of is and d = 0. In any event we only allow m one to take the value of the number of bits which can transition in calculating our membership functions. The bits that cannot change are not considered in the function. In the following definition .We define a fuzzy word for an asymmetric error model in which only errors may occour. **Example 3.1:** Let n = 2 let u = (0, 0) and d $(\tilde{A}_u, \tilde{A}_u) = | p^2q^0 - p^0q^2 | + | p^1q^1 - p^1q^1 | + | p^1q^1 - p^1q^1| + | p^0q^2 - p^2q^0 | = 2 (p - q)$ and $d_H(u, u) = 2$.

Now let n = 3, w = (1, 1, 1) and x = (0, 0, 1) in F^3 then

$$d(\tilde{A}_u, \tilde{A}_u) = | p^0q^3 - p^0q^1| + |p^1q^2 - p^2q^0| + |p^1q^2 - 0| + |p^2q^1 - p^1q^1| + | p^1q^2 - 0 | + | p^2q^1 - p^1q^1 | + | p^3q^0 - p^0q^2 | = q - q^2 - q^3 + p^3 + p^2 + 2pq + pq^2 - p^2q \neq 2(p - q)$$ and

$D_H(u, v) = 2$.

We some Hamming distance between u, U, w and x respectively but different distance between the corresponding fuzzy code word. We now state a result which says that the distance between fuzzy code word is independent of n for ideal symmetric error. The result holds when the Hamming distance or the as symmetric distance is used as the distance metric between two code words.

Lemma 4.1: $\Phi_1 < \Phi_2 < .....\Phi_n < 1$. As the asymmetric between code words on which fuzzy codes will be based become large, there is only a small increase in the measurable distance between codewords. For unidirectional errors, the case is that the space of the code will effect the distance between the fuzzy code words. These issues must be taken into account in designing fuzzy codes.

Lemma4.2:$\Gamma_1 < \Gamma_2 < ... \Gamma_n > 2$.

Proof: If instead of using the Hamming distance between two fuzzy codes, we used the asymmetric distance, so that

$$D_a(\tilde{A}_u, \tilde{A}_u) = \left(\sum_{w \in F^n} \left(\tilde{A}_u(w) - \tilde{A}_u(w)\right) V \sum_{w \in F^n} \left(\tilde{A}_v(w) - \tilde{A}_u(w)\right) V \right)$$

Then the following theorem holds.

Corollary 5.1: A fuzzy column vectors $h^1 \in A^n$ is independent of a set of fuzzy column vectors $\{\tilde{h}_1, ..., \tilde{h}_n\}$.

If S(i) = φ for any i ∈ {1, ...,m}.

Proof: We give the algorithm for checking if a non-null column x_k in the Sub semimodule F is linearly dependent on a set of fuzzy vectors at the end of this section. In a set of the column vectors $\tilde{g}_i$, i = 1, ...n, is given a complete set of independent fuzzy vectors $\tilde{f}_i$, i = 1, ...i, can be selected such that subsemimodule generated by $\{\tilde{f}_i, ..., \tilde{f}_m\}$ contains the $\tilde{g}_i$ s the procedure is shown in the form of the flow chart.

We now consider a positive sample set R^+ = 0.8ab, 0.8aa, bb, 0.3ab, 0.2bc, 0.99bbc. The finite submatrix of the fuzzy Hankel matrix H(r) is shown.

Using the algorithm DEPENDENCE the independent columns of the fuzzy Hankel matrix have been indicated as F_1, F_2, F_5, F_6, F_7.

The finite submatrix of the fuzzy Hankel matrix $H(r)^2$.

$$\mu(a) = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .03 & 0 & 1 & 0 & .08 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .08 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\mu(b) = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\mu(c) = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\mu(c) = \begin{array}{c} \\ \lambda \\ a \\ ab \\ abc \\ bc \\ abb \\ abbc \\ aa \\ aab \\ aabb \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccccccccc} \lambda & ab & a & abc & bc & c & abbc & bbc & aabb & abb & bb \end{array} \\ \begin{bmatrix} 0 & .8 & 0 & .3 & .2 & 0 & .9 & 0 & .8 & 0 & 0 \\ 0 & 0 & .8 & 0 & .3 & 0 & 0 & .9 & 0 & .8 & 0 \\ .8 & 0 & 0 & 0 & .9 & .3 & 0 & 0 & 0 & 0 & 0 \\ .3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ .2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .9 & 0 & 0 & 0 & 0 & 0 \\ .9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ .8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{ccccccccccc} F_1 & & F_7 & & F_3 & F_4 & F_2 & F_5 & & & F_6 \end{array} \end{array}$$

The algorithm depends also identifies column if any column vector $\tilde{h}(j)$ is dependent on the set of generator H ∪ (m) = $H \cup (m) = \{\hat{f}_1, ..., \hat{f}_m\}$ of the Hankel matrix as constructed intable. It also identifies the coefficient S_i, using the procedure ARRANGE CS(i), N, CARD(i) and the procedure COMPARESO(k), SO(K – 1).

Definition 1.6: A code C over the alphabet A is called a prefix (suffix) code if it satisfies $CA^+ \cap C = \Theta$ ($A^+ C \cap C = \Theta$). C is called a biprefix code of it is a prefix and a suffix code. A submonoid M of any monoid N. Satisfying of proposition $CA^+ \cap C = \Theta$ is called the left unitary in N. M is called right unitary in N if it is satisfies the dual of $\forall x \in A^{\leftrightarrow}$, $Mw \cap M \neq \theta$ implies $w \in M$ namely $A^+ C \cap C = \Theta$.
Let M be a submonoid of a free monoid $A^{\leftrightarrow}$ and c its base then the following conditions are equivalent:
(i) $\forall w \in A^{\leftrightarrow}$, $Mw \cap M \neq \Theta$ implies $w \in M$
(ii) $CA^+ \cap C = \Theta$

Proposition 6.1: Let $A^{\leftrightarrow}$ be a free monoid and C be a subset of $A^{\leftrightarrow}$. Define the subset D_i of $A^{\leftrightarrow}$ recursively by $D_0 = C$ and $D_i = \{w \in A^{\leftrightarrow} \mid D_{i-1}w \cap c \neq 0 \text{ or } Cw \cap D_{i-1} \neq \Theta\}$, i = 1, 2, ... Then c is a code over A if and only if $C \cap D_i = \Theta$ for i = 1,2, ...
Suppose e is a finite then the length of the word in C. Hence there is only a finite number of distinct D_i and this proposition gives an algorithm for deciding whether or c is a code.

II. CONCLUSION

The search for suitable codes for communication theory is known. It was proposed by Garla-that-L-semi-group theory be used. To this end free, pure, very pure, left unitary, right unitary, unitary such L-submsemigroup there is a family of codes associated with it. An L-subsemigroup of a free semigroup if free,one are, pure, very pure, left unitary right unitary respectively. Thus any method used to construct an L-=subsemigroup of a free semigroup of one of these types yields a family of semigroups of the same types. Namely the level sets of the L-subsemigroup. The basic idea is that the class of non-fuzzy system, that are approximately equivalent to a given type of system from the point of view of their behaviours is a fuzzy class of systems.For instances the class of system that are approximately linear. This idea of fuzzy classification of system was first hinted at by Zadeh 1965. Saridis 1975 applied it to the classification of nonlinear systems according to their nonlinearies, pattern recognition methods are first used to build crisp classes. Generally, this approach does not answer the question of complete identification of the nonlinearlies involved within one class. To distinguish between the nonlinearlies belonging to a single class, membership value in this class are defined for each nonlinearly one of thee considered as a references with a membership value 1.The membership value of each nonlinearity is calculated by comparing the coefficients of its polynomial series expansion to that of the reference nonlinearlity.

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