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A Method for Probabilistic Stability Analysis of Earth Dams

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ABSTRACT
This paper proposes a new probabilistic methodology for analyzing the stability of Earth Dams, based on the technique of the First Order Reliability Method for Structural Reliability. Differently from others methodologies present in literature, the proposed method interprets the involved variables as random ones. So, three results are provided here: the Structural Reliability Index, the Probability of Rupture and the most probable values of the random variables for the occurrence of a dam break. In order to illustrate it, real data from a cross section of the Left Bank Earthfill Dam of Itaipu Hydroelectric Power Plant (IHPP), located on the city of Foz do Iguaçu, Paraná, Brazil were used. The numerical results achieved by the proposed methodology evidence that IHPP dam has currently good structural conditions, confirming that the safety procedures adopted in Itaipu Dam may be considered as appropriate. The use of the proposed method enables to complement the previously existing knowledge about the structural conditions, improving the process of risk management.
***Keywords* -** Earth Dams, Probability of Rupture, Reliability Index, Stability Analysis, Structural Reliability.

I. Introduction

Regarding the safety analysis of Civil Engineering structures, [1] highlight that the current society is increasingly aware about the inherent risks. The technical community of engineering has been recognizing that no absolute structural safety can be ensured, so that there are residual risks when it is treated using only traditional safety standards. In Brazil, Law number 12,334 from 2010 [2] establishes that national dams must be evaluated regarding their risk according to a methodology that, besides considering technical and conservation techniques, also considers economic, social, environmental and personal impacts. According to the mentioned law, the employer, who is the responsible by the dam, must provide the necessary resources to ensure the structural safety and, therefore, one of the exigencies is the elaboration and presentation of a safety plan for the dam.

Deterministic analysis about the stability of embankment dams is based in calculating the Factor of Safety (*FS*). However, geotechnical variables involved in calculating *FS* are subject to variability due to several reasons, among them: simulating the field conditions of the geotechnical tests, different interpretations during the performance of tests, human failures during tests, spatial variability inherent to the soil properties in distinct places, [3]. Once *FS* value is determined, without considering the randomness of its variables, its value leaves aspects when indicating the safety level of the structure. The consideration about the stochastic fluctuation of the variables involved in the structural safety analysis by researches originated the so-called “Methods of Structural Reliability” [4]. In these methods, the probability of failure and the Reliability Index for the structure are estimated. Due to the several causes for the variability of geotechnical variables, the geotechnical problems were present since the beginning of the development of Structural Reliability techniques. In order to describe the structural safety more precisely, methods of probabilistic Stability Analysis of dams are included in current proposals.

In face of this, this paper proposes an alternative probabilistic methodology based on the First Order Reliability Method (FORM) for the Stability Analysis of Earth Dams. The structural failure to be analyzed is the simulated rupture of the downstream slope. In order to illustrate the method, data from the cross section of Station 122+00 from the Left Bank Earthfill Dam (LBED) of Itaipu Hydroelectric Power Plant (IHPP) are used.

This paper is divided into five Sections. In Section 2 exhibits the Station 122+00 of the LBED, which is used to explain the theory part involved in the proposed methodology. Section 3 brings a review of how the Structural Reliability method is performed - which is the base for the proposed method; in addition, alternatives used for bypassing some difficulties from the FORM algorithm are discussed there. In Section 4, about materials and methods, data from the section of Station 122+00 are presented as well as explanation the iterative actions from the proposed method. Main results from applying the method to the section of Station 122+00 are displayed and commented upon in Section 5. Importantly, the results are estimated are: the Reliability Index, the Probability of Rupture and the most likely values that the random variables must assume for resulting in rupture. Conclusions about the proposed methodology, its results as wells as about its use to complement the risk management of dams, among which LBED stands out, are discussed in Section 6.

II. Cross section of Station 122+00

The Left Bank Earthfill Dam has an extension of 2,294 meters of length. The cross section of Station 122+00 is shown in Fig. 1.

The whole structure of Fig. 1 is over dense basalt, a rock of volcanic origin. The dense basalt, after suffering processes of disintegration and decomposition, (in other words, weathering), gave rise to the yellow layer, known as weathered basalt. Weathered basalt, after suffering more processes of weathering, gave rise to the saprolite layer that is the orange layer. The purple layer, which is the plastic clay of foundation, has originated from the saprolite disintegration processes. The dam body, represented in brown, is composed by clay from the lending area near the place of LBED construction. In the body structure, a gray filter, which is made with sand, can be noted. Green parts are the berms, composed by materials from excavations performed for building other parts of the dam. In pink, upstream, represents a layer of rip rap, rocks with several sizes, gradually organized, in order to avoid erosion that might be caused by the water of the reservoir [5].

Figure 1: cross section of Station 122+00

It is important to highlight that the cross section of Station 122+00 of the LBED of Itaipu Hydroelectric Power Plant, besides illustrating the proposed analysis method, is also used to explain the theory involved in the present Literature Review, in Section 3.

III. Literature review

This section, addresses the main concepts used as base for the proposed method for evaluating the stability analysis of dams. So, subjects on Structural Reliability are explained in Section 3.1, among them: the Simplified Bishop Factor of Safety used in the analysis (Subsection 3.1.1), and the First Order Reliability Method (FORM) algorithm (Subsection 3.1.2). In face of the difficulties for using the Factor of Safety in analysis, an alternative procedure is indicated, also in that Section.

3.1 Structural Reliability

According [4], among the objectives of Structural Reliability is the estimated calculation of the probability of occurrence of fail in the engineering structures, in any stage of their life. In order to apply the methods of Structural Reliability, first, it is necessary to define the structural failure to be analyzed. This failure is mathematically represented by a function *G*, which, regarding Structural Reliability is called limit state function. Values that *G* assumes are interpreted as performance indicators of a structure being analyzed. In this paper the limit state function considered is the Simplified Bishop Factor of Safety (*FS*). The good performance of the structure happens when $FS > 1$; if $FS = 1$, then the structure is under imminence of rupture; and, finally, if $FS < 1$, it means that the structure breaks [6]. The structural failure, indicated by the simulated rupture of the dam in this paper, is known, in terms of Structural Reliability, as the violation of this limit state. The function of the Factor of Safety is presented in Subsection 3.1.1.

Assuming that $\underline{X}$ is the vector of random variables of resistance and load acting over the structure, $G(\underline{X})$ is the function describing the structural performance for which the violation of the limit happens when $G(\underline{X}) < 0$; and $f_{\underline{X}}(\underline{x})$ is the joint probability density function (p.d.f.) of $\underline{X}$, then the probability of failure, denoted by p_f, is defined in (1):

$$p_f = P[G(\underline{X}) < 0] = \int \ldots \int_{G(\underline{X})<0} f_{\underline{X}}(\underline{x}) d\underline{x} \quad (1)$$

In most of the times, the calculation of the multiple integral (1) is unviable by analytical methods. Besides, the joint p.d.f. of resistance and load variables may be unknown, so this must be estimated or previously known. In face of that, an alternative for obtaining the probability of failure is the use of numerical methods of Structural Reliability [7]. The First Order Reliability Method (FORM) technique of Structural Reliability is presented on Subsection 3.1.2 for solving the problem of stability of dams approached in the current paper.

3.1.1 Limit state function

The desired failure is the simulated rupture of the downstream slope, which may be evaluated by the Factor of Safety, whose values are interpreted: $FS > 1$ indicates safety, if $FS = 1$, the structure is on the verge of rupture; and $FS < 1$ indicates rupture [6]. For calculating the Factor of Safety, a cross section of the dam is considered. The analysis is performed two-dimensionally and the soil over the rupture surface divided into slices. For instance, in Fig. 2 is observed the cross section of Station 122+00, with circular rupture surface and over this surface the soil slices, in green.

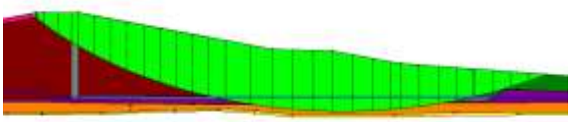

Figure 2: Rupture surface associated with methods subject to limit equilibrium hypotheses

Considering that: the soil breaks abruptly, *FS* is the same along the whole rupture surface and equations of static equilibrium are satisfied until the imminence of rupture. The Simplified Bishop Factor of Safety is defined by the ratio between moment of resistance (M_R) and load moment (M_L):

$$FS_{k+1} = \frac{M_R}{M_L} \quad (2)$$

The Factor of Safety is called simplified when only the equilibrium of strengths or moments is considered [8]. The equilibrium of moments is analyzed in each one of the slices, represented in green in Fig. 2. An arbitrary slice of the rupture surface of Fig. 2, to be used for evaluating the rupture by *FS* of Simplified Bishop is generically presented in Fig. 3.

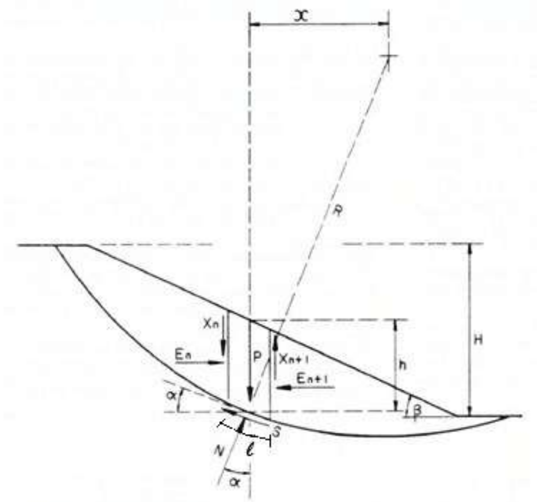

Figure 3: Forces in a generic slice for the Simplified Bishop Factor of Safety [9]

In Figure 3:
P: total weight of the slice;
R: radius of the rupture circle;
X_n,X_{n+1}: vertical shear forces on the sections *n* and *n*+1;
E_n,E_{n+1}: resultants of the horizontal forces on the sections *n* and *n*+1;
l: length of the base of the slice;
N: normal total force acting in the base of the slice;
S: shear force acting on the slice base;
α : inclination angle of the slice base;
x: horizontal distance from the center of the slice until the axis of rotation;
h: height of the slice.

Moment of resistance (M_R) is defined by the sum of moments of resistance from each one of the slices. Shear strength mobilized τ_{mob} is calculated by the Mohr-Coulomb criterions, such that *c*' is the effective cohesion, σ is the normal stress, *u* is the pore pressure (water pressure in pores) and *tgφ*' the tangent of effective friction angle *φ*':

$$\tau_{mob} = \frac{1}{FS_k}[c' + (\sigma - u)\cdot tg\phi'] \quad (3)$$

Thus, the moment of resistance (M_R) is given by:

$$M_R = \sum \frac{\ell \cdot R}{FS_k}\cdot[c' + (\sigma - u)\cdot tg\phi'] \quad (4)$$

And load moment (M_L) is defined as

$$M_L = \sum P \cdot x \quad (5)$$

By replacing (4) and (5) in (2), after some deductions and simplifying assumptions, the Simplified Bishop Factor of Safety [6] is found. Where γ is the specific weight of soil, *b* the slice breadth, secα the secant of angle α and *tg* α the tangent of angle α:

$$FS_{k+1} = \frac{1}{\sum P\cdot sen\alpha}\cdot\sum\left[\left\{c'\cdot b + P\cdot tg\phi'\cdot\left(1-\frac{u}{\gamma}\right)\right\}\cdot\frac{\sec\alpha}{1+\frac{tg\alpha\cdot tg\phi'}{FS_k}}\right] \quad (6)$$

Nowadays there exists some software capable to calculate the Factor of Safety. Here the software SLOPE/W® (http://www.geo-slope.com/support/geostudio2007/examples.aspx) has been adopted to do so.

3.1.2 First Order Reliability Method

FORM algorithm is carried out in the linear space on which the random variables have standard normal distribution (i.e., normal p.d.f. whose average equals zero and standard deviation equals one) and are also stochastically independent. This linear space is usually referred to as a "small space". The limit state function – that is written as a function of variables from the reduced space – provides the rise to the failure surface. Furthermore, this surface describes both the safety region and the failure region. During the development of FORM algorithm, firstly the Reliability Index is obtained – that consists of the smaller distance from the origin of the reduced space to the failure surface. Then the probability of failure is determined. The nearer geometric point on the failure surface of the origin indicates the most probable values that random variables must assume so that violation of the limit state happens. This point is called project point. The iterative actions of FORM for the case when variables are normal and correlated are described next [7]:

Action 1: Transform the random variables in standardized normal and independent ones.
Here the random vector $\underline{X}$ is written as a function of vector $\underline{z}$ of variables belonging to a reduced space which helps to calculate the diagonal matrix of standard deviations estimated for $\underline{X}$ (denoted by $[\sigma_{\underline{X}}]$). The vector of the estimated averages of $\underline{X}$ is represented by $[\mu_{\underline{X}}]$, and T denotes the matrix composed by the normalized eigenvectors of the estimated correlation matrix R. Accordingly, $\underline{X}$ can be decomposed in (7):

$$\underline{X} = [\sigma_{\underline{X}}] \cdot T \cdot \underline{z} + \underline{\mu}_{\underline{X}} \tag{7}$$

Action 2: Obtain limit state equation, or equivalently, the failure surface.
The limit state function $G(\underline{X})$ is written as a function of variables from the reduced space ($\underline{z}$) and equated to zero. That is:

$$g(\underline{z}) = 0 \tag{8}$$

Action 3: Write a new project point.
The new project point $\underline{z}^*$ consists of a function of the Reliability Index (β), which must be determined in Action 4, as well as of the vector of direction cosines ($\underline{\alpha}$). That is:

$$\underline{z}^* = -[\sigma_{\underline{z}}] \cdot \underline{\alpha} \cdot \beta \tag{9}$$

The direction cosines consists of a vector function of partial derivatives vector of g in relation to variables of $\underline{z}$ $\left(\frac{\partial g}{\partial \underline{z}}\right)$ as well as of the diagonal matrix of standard deviations estimated of variables $\underline{X}$ from the reduced space ($[\sigma_{\underline{z}}]$) which are defined by the root of eigenvalues of matrix R. So, it follows that:

$$\underline{\alpha} = \frac{[\sigma_{\underline{z}}] \cdot \frac{\partial g}{\partial \underline{z}}}{\left\| [\sigma_{\underline{z}}] \cdot \frac{\partial g}{\partial \underline{z}} \right\|} \tag{10}$$

Action 4: Calculate a new Reliability Index.
The new Reliability Index is obtained from the roots of the limit state equation (9), when it is written as a function of the new project point ($\underline{z}^*$), as in (11):

$$g(\underline{z}^*) = g(-\underline{\alpha} \cdot \beta) = 0 \tag{11}$$

Action 5: Verification of the stopping criterion of the algorithm.
If the difference between Reliability Indexes from the last two successive iterations is an acceptable value, then the execution of the algorithm is stopped and the flow goes to Action 6. Unlike, a new project point is calculated with (9) and the procedure restarts from Action 3.

Action 6: Check the final results of the algorithm execution.

By means of the Reliability Index (β) from the last interaction, the project point of the reduced space is achieved by employing (9), that is then written in the original space with (7). In turn, through the standard normal cumulative distribution function (Φ) the probability of failure (p_f) is estimated. That is:

$$p_f = 1 - \Phi(\beta) \tag{12}$$

The use of recursive functions such as the Factor of Safety of (5) as a function of the limit state may difficult the execution of FORM in Actions which the calculation of partial derivatives and of the roots of the limit state equation is required. In effect, an alternative procedure proposed in this paper is detailed next.

<u>Alternative Procedure</u>
The partial derivatives described in Action 3 must be determined by numerical approximation where t is a value to be incremented:

$$\frac{\partial g}{\partial z_i} = \frac{g(z_1,\dots,z_i+t,\dots,z_n) - g(z_1,\dots,z_i,\dots,z_n)}{t} \tag{13}$$

In order to obtain the roots of the limit state function described in Action 4, [10] suggest the approximation of the equation by a second order Taylor Series centered in a supposed initial Reliability Index β_0:

$$g(\beta) \approx \frac{1}{2} \cdot \left.\left(\frac{\partial^2 g}{\partial \beta^2}\right)\right|_{\beta=\beta_0} \cdot (\beta-\beta_0)^2 + \left.\left(\frac{\partial g}{\partial \beta}\right)\right|_{\beta=\beta_0} \cdot (\beta-\beta_0) + g(\beta_0) = 0 \tag{14}$$

Such as $g(\beta_0)$ corresponds to the value of g in the last new project point of (9) with $\beta = \beta_0$.
The partial derivative of g in relation to β with $\beta = \beta_0$ is given by:

$$\left.\left(\frac{\partial g}{\partial \beta}\right)\right|_{\beta=\beta_0} = \sum_{i=1}^{n} \left(\frac{\partial g}{\partial z_i}\right) \cdot \left.\left(\frac{\partial z_i}{\partial \beta}\right)\right|_{\beta=\beta_0} \tag{15}$$

where $\left(\frac{\partial g}{\partial z_i}\right)$ is obtained from (13) and $\left.\left(\frac{\partial z_i}{\partial \beta}\right)\right|_{\beta=\beta_0}$ from (9) making $\beta = \beta_0$.
The partial second derivative of g in relation to β with $\beta = \beta_0$ is given by:

$$\left.\left(\frac{\partial^2 g}{\partial \beta^2}\right)\right|_{\beta=\beta_0} = \sum_{i=1}^{n} \left(\frac{\partial^2 g}{\partial z_i^2}\right) \cdot \left(\left.\left(\frac{\partial z_i}{\partial \beta}\right)\right|_{\beta=\beta_0}\right)^2 \tag{16}$$

where $\left.\left(\dfrac{\partial z_i}{\partial \beta}\right)\right|_{\beta=\beta_0}$ is obtained from (9) making $\beta = \beta_0$ and $\left(\dfrac{\partial^2 g}{\partial z_i^2}\right)$ is given as follow:

$$\frac{\partial^2 g}{\partial z_i^2} = \frac{g(z_1,..., z_i + 2t,..., z_n)}{t^2} \frac{-2g(z_1,..., z_i + t,..., z_n) + g(z_1,..., z_i,..., z_n)}{} \quad (17)$$

The calculation of the Reliability Index (β) implicates in calculating the roots of (14) approximated. Once a Reliability Index is obtained from those roots, the procedure of executing FORM continues from Action 5.

IV. Material and methods

The purpose of this Section is to treat on questions concerning the practical application of the proposed method. In Subsection 4.1 the data adopted for applying the underlying method in the Stability Analysis by the cross section of Station 122+00 are thoroughly presented. In turn, the explanation of the operational procedures is approached in Subsection 4.2.

4.1 Data for analysis of structural reliability for the section of Station 122+00

Variables interpreted as random ones with their averages and standard deviations for materials of the dam are shown in Table 3 and Table 4. Notice that the normal distribution is supposed here to them.

Table 3: Variables considered as random in the reliability analysis

	Effective cohesion (Kpa)	
Material	Average	Standard Deviation
Dam body clay	55.5181	13.4967
Foundation clay	18.2649	6.0682

Table 4: Variables considered as random in the reliability analysis

	Effective friction angle (degrees)	
Material	Average	Standard Deviation
Dam body clay	27.275	1.3892
Foundation clay	25	1.4142

The estimated correlation matrix (R) named Pearson's correlation [11], between variables $\underline{X}$ is given by:

$$R = \begin{bmatrix} 1 & -0.5401 & 0 & 0 \\ -0.5401 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Is pointed out that the random variables effective cohesion and effective friction angle of the clay in the dam body is -0.5401, and that the other random variables are uncorrelated. On the other hand, the variables exhibited in Table 5 were assumed to be deterministic ones and used for performing the Stability Analysis.

Table 5: Variables considered as deterministic in Reliability Analysis

Material	Specific Weight (KN/m^3)	Effective cohesion (KPa)	Effective friction angle (degrees)
Dam body clay	19.025	------	------
Berms	19.025	58.2517	24
Sand of filters	22.065	0	40
Foundation clay	17.8482	------	------
Saprolite	18.0443	0	25
Weathered basalt	19.1427	19.123	28
Rip rap	21.5747	0	40

4.2 Proposed method for probabilistic analysis of Earth Dams stability

During the development of the FORM algorithm for calculating the partial derivatives by numerical approximation as described in Action 3 (13), an access are made to SLOPE/W®. The value t = 0.001 is incremented in the variable of reduced space ($\underline{z}$), and the values of variables in the original space are obtained from (7). Thus, accesses are made to SLOPE/W® in order to determinate the values of the Factor of Safety (function of the limit state), with the variables in the original space and not in the reduced one. Due to this alternative procedure, it is not necessary to write the equation of limit state as a function of variables of the reduced space ($\underline{z}$), according to (8) of Action 2. For obtaining a new Reliability Index (Action 4), the proposed method suggests the approximation of the limit state (11) by a second-order Taylor Series (14) as a function of the Reliability Index (β). Thus, Action 4 is performed without the need of writing (8) (limit state equation) due to the new project point, according to (11). In this procedure, the limit state (8) of Action 2 is treated indirectly, according to approximations given by (13) and (14) indicated in the alternative procedure.

The rupture surface was considered as fixed, passing through the saprolite layer. This is one because the material layer, is highly permeable.

The method presented in this Section allows performing stability analysis of Earthfill dams account for the randomness of some variables, unlikely traditional methods. Importantly, even if a dam is considered to be safe according to traditional stability analysis, there exists residual risk that should be taken into account. Hence the proposed method aims to mitigate the risk management and to give more support to final decision of the responsible technical sector.

V. Results

Results from the analysis of Structural Reliability of the LBED through the section of Station 122+00, using the proposed method (Section 3), with the Simplified Bishop Factor of Safety as a function of the limit state, are in Table 6. The Reliability Indexes had close values in both analysis, as well as the respective Rupture Probabilities. Those values, once determined, are not unique, but they depend on the Reliability Index chosen to begin the iterative process. Two results are presented, one for a initial Reliability Index of 2.0 and another for 2.4. It is important to observe that the dam is in good safety conditions, because the values of the Reliability Index and consequently the ones of Probability of Rupture, are inside the interval considered as safe, established by [12] for Earth Dams.

Table 6: Reliability and Probability of Rupture Indexes

Initial Reliability Index	Final Reliability Index	Probability of Rupture
2.0	4.682	0.000142%
2.4	4.512	0.000321%

The values of effective cohesion and of effective friction angle (project point) resulting from the analysis are presented in Table 7. Similar values are observed for all variables in both analysis. For clay from dam body, for instance, the values of effective cohesion were 12.271^{o} and 11.769^{o}. It is observed that a small change in the initial Reliability Index may change the project point at the end of the analysis. Close values of the initial Reliability Indexes generate values close to the project point. This procedure indicates intervals for the project point variables.

Table 7: Project point resulting from analysis with distinct initial Reliability Indexes

	Dam body clay		Foundation clay	
Initial Reliability Index	c' (KPa)	φ' (degrees)	c' (KPa)	φ' (degrees)
2.0	12.217	25.839	15.034	23.945
2.4	11.769	26.405	15.489	23.094

The rupture surface with its slices at the end of both analysis is similar, since the rupture surface was considered as fixed. The surface is shown in Fig. 5.

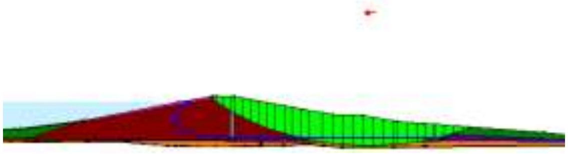

Figure 5: Rupture surface resulting from analysis with initial Reliability Index 2.0 and 2.4

It was verified that the convergence of the proposed model is associated with the selection of the initial Reliability Index and of the increment value *t*. Accordingly, the beginning of the analysis have taken more time. For analysis executed with both initial Reliability Index 2.0 and 2.4, the selected value of increment *t* was the same – namely, $t = 0.001$.

VI. Conclusion

In this paper a method for probabilistic analysis of Earth Dam stability is put forward. Given a cross section of the dam, the proposal, based on the First Order Reliability Method aims to obtain the Probability of Rupture of the Earthfill, the Reliability Index for the structure and the more probable values that the random variables must assume for the rupture to happen. Two analysis were executed according to the initially chosen Reliability Indexes: 2.0 and 2.4. In both analysis the function describing the performance of the structure (limit state function) was the Simplified Bishop Factor of Safety. The probabilistic methods of Stability Analysis differ from traditional deterministic methods by considering the randomness of the variables involved in the problem at hand. In the proposed methodology, the variables considered as random ones were the effective cohesion and the effective friction angle of materials: clay from dam body and clay from foundation.

By applying the underlying method, information from the cross section of Station 122+00 on the Left Bank Earthfill Dam of Itaipu Hydroelectric Power Plant was used. The results obtained from the application evidence the excellent performance of the proposed method, as well as the good security conditions in which the dam is, according to comments about results in Section 4.

Furthermore, it was verified that both analysis have values close to the Index of Reliability and Probability of Rupture. The numerical analysis performed by traditional methods of Structural Reliability was also indicated. The application of the Monte Carlo Simulation Method, with the same limit state functions considered in this paper, will probably point to values of Index of Reliability and Probability of Rupture close to the obtained ones. The greater the amount of information is, more precision there is to infer about structural security and for indicating procedures of risk management.

The probabilistic view of the question, by considering the randomness of variables involved in the stability problem, favors the execution of analysis taking into account the risks, and that, by providing a vision closer to reality, may complement the previously existing knowledge about the structural conditions. The technical complexity, presented by the mathematical resources structuring the proposed methodology, is no bar for applying the method in Earthfill dams, since they may be operated with relative simplicity, using the software used for the analysis development of this paper.

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