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Effects of Thermal Radiation and Chemical Reaction on MHD Free Convection Flow past a Flat Plate with Heat Source and Convective Surface Boundary Condition

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ABSTRACT
This paper analyzes the radiation and chemical reaction effects on MHD steady two-dimensional laminar viscous incompressible radiating boundary layer flow over a flat plate in the presence of internal heat generation and convective boundary condition. It is assumed that lower surface of the plate is in contact with a hot fluid while a stream of cold fluid flows steadily over the upper surface with a heat source that decays exponentially. The Rosseland approximation is used to describe radiative heat transfer as we consider optically thick fluids. The governing boundary layer equations are transformed into a system of ordinary differential equations using similarity transformations, which are then solved numerically by employing fourth order Runge-Kutta method along with shooting technique. The effects of various material parameters on the velocity, temperature and concentration as well as the skin friction coefficient, the Nusselt number, the Sherwood number and the plate surface temperature are illustrated and interpreted in physical terms. A comparison of present results with previously published results shows an excellent agreement.

Key words: Chemical reaction, Heat and Mass Transfer, Heat source, MHD, Radiation.

I. INTRODUCTION

Steady free convection boundary layer flow past a horizontal plate is a very practical and basical issue that is worthy enough to be discussed perfectly. So researchers in the heat transfer field paid tangible attention toward this problem. Laminar boundary layer about a flat-plate in a uniform stream of fluid continues to receive considerable attention because of its importance in many practical applications in a broad spectrum of engineering systems like geothermal reservoirs, cooling of nuclear reactors, thermal insulation, combustion chamber, rocket engine, etc. Stewartson [1] concentrated on an isothermal horizontal semi-infinite plate and his results, which were later discussed by Gill *et al*.[2], announced the existence of similarity solutions just for below and above a cooled and heated surface, respectively. Rotem and Claassen [3] and Raju *et al*. [4] discussed on the heated downward-facing or cooled upward-facing plates numerically. Identically, Clifton and Chapman [5] utilized an integral method for this problem. Jones [6], and Pera and Gebhart [7] performed their study on an inclined plate with a isothermal condition. Also Yu and Lin [8], and Lin *et al*.[9] verified an arbitrary inclined plate. Afzal N., [10], Higher order effects in natural convection flow over a uniform flux horizontal surface. Afzal *et al*.[11], Lin and Yu [12], Brouwers[13] and Chen *et al*. [14] studied the influence of suction or blowing on the free convection of a horizontal flat plate.

The previous studies, dealing with the transport phenomena of momentum and heat transfer, have dealt with one component phases which posses a natural tendency to reach equilibrium conditions. However, there are activities, especially in industrial and chemical engineering processes, where a system contains two or more components whose concentrations vary from point to point. In such a system there is a natural tendency for mass to be transferred, minimizing the concentration differences within the system and the transport of one constituent, from a region of higher concentration to that of a lower concentration. This is called mass transfer. Stokes [15] gave the first exact solution to the Navier–Stokes equation for the flow of a viscous incompressible fluid past an impulsively started infinite horizontal plate. Panda et al [16] analyzed an unsteady free convictive flow and mass transfer of a rotating elastic-viscous liquid through porous media past a vertical porous plate. Sattar [17] discussed the free convection and mass transfer flow though a porous medium past an infinite vertical porous plate with time dependent temperature and concentration. The application of boundary layer techniques to mass transfer has been of considerable assistance in developing the theory of separation processes and chemical kinetics. The phenomenon of heat and mass transfer has been the object of extensive research due to its applications in Science and Technology.

The study of flow of electrically conducting fluid called Magneto Hydrodynamics (MHD). It has gained the attention due to its applications in the design of heat exchangers, induction pumps, and nuclear reactors, in oil exploration and in space vehicle propulsion. Thermal radiation in fluid dynamics has become a significant branch of the engineering sciences and is an essential aspect of various scenarios in mechanical, aerospace, chemical, environmental, solar power and hazards engineering. Bhaskara Reddy and Bathaiah [18, 19] analyze the Magnetohydrodynamic free convection laminar flow of an incompressible Viscoelastic fluid. Later, he was studied the MHD combined free and forced convection flow through two parallel porous walls. Elabashbeshy [20] studied heat and mass transfer along a vertical plate in the presence of magnetic field. Samad, Karim and Mohammad [21] calculated numerically the effect of thermal radiation on steady MHD free convectoin flow taking into account the Rosseland diffusion approximaion. Loganathan and Arasu [22] analyzed the effects of thermophoresis particle deposition on non-Darcy MHD mixed convective heat and mass transfer past a porous wedge in the presence of suction or injection. Ghara, Maji, Das, Jana and Ghosh [23] analyzed the unsteady MHD Couette flow of a viscous fluid between two infinite non-conducting horizontal porous plates with the consideration of both Hall currents and ion-slip. The radiation effect on steady free convection flow near isothermal stretching sheet in the presence of magnetic field is investigated by Ghaly et al. [24]. Also, Ghaly [25] analyzed the effect of the radiation on heat and mass transfer on flow and thermal field in the presence of magnetic field for horizontal and inclined plates.

Chemical reaction can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. The effect of the presence of foreign mass on the free convection flow past a semi-infinite vertical plate was studied by Gebhart *et al*. [26]. The presence of a foreign mass in air or water causes some kind of chemical reaction. During a chemical reaction between two species, heat is also generated (Byron Bird *et al.* [27]). In most of cases of chemical reaction, the reaction rate depends on the concentration of the species itself. A reaction is said to be first order if the rate of reaction is directly proportional to concentration itself (Cussler [28]). Shakhaoath et al. [29] studied the possessions of chemical reaction on MHD Heat and mass transfer of nanofluid flow on a continuously moving surface. Chemical reaction effects on heat and mass transfer laminar boundary layer flow have been discussed by many authors (Apelblat [30]; Das *et al* [31]; Muthucumaraswamy *et al*. [32] in various situations. Anjalidevi and Kandasamy [33] investigated the effects of chemical reaction and heat and mass transfer on a laminar flow along a semi-infinite horizontal plate Moreover, there are a number of physical circumstances where internal heat generation in an otherwise forced convective flow over a flat surface do occur. For instance, convection with internal heat generation plays an important role in the overall heat transfer process, i.e, in the development of a metal waste form from spent nuclear fuel, phase change processes and thermal combustion processes, Crepeau and Clarksean[34] considered the classical problem of natural convection from an isothermal vertical plate and added a heat generation term in the energy equation. They found that for a true similarity solution to exist, the internal heat generation must decay exponentially with the classical similarity variable. This type of model can be used in mixtures where a radioactive material is surrounded by inert alloys and in the electromagnetic heating of materials. Olanrwaju [35] studied the combined effects of an exponentially decaying internal heat Generation and convective surface boundary condition on the thermal boundary layer over a flat plate.

In the present study, an attempt is made to analyze the simultaneous effects of thermal radiation and chemical reaction on hydromagnetic heat and mass transfer flow over a horizontal flat plate with a heat source that decays exponentially and convective surface boundary condition. The governing boundary layer equations are reduced to a system of nonlinear ordinary differential equations using similarity transformations and the resulting equations are then solved numerically using Runge-Kutta fourth order method along with shooting technique. A parametric study is conducted to illustrate the influence of various governing parameters on the velocity, temperature and concentration as well as the skin-friction coefficient (surface shear stress), the local Nusselt number (surface heat transfer coefficient) the plate surface temperature and the local Sherwood (surface concentration gradient) number and discussed in detail.

II. MATHEMATICAL ANALYSIS

A steady two-dimensional steady free convective flow of a viscous incompressible, electrically conducting and radiating fluid adjacent to flat plate is considered. The flow is assumed to be in the direction of x-axis along the plate and y-axis normal to it, with a stream of cold fluid at temperature T_∞ and concentration C_∞ over the upper surface of the flat plate with a uniform velocity U_∞, while the lower surface of the plate is heated by convection from a hot fluid at temperature T_f, which provides heat transfer coefficient h_f as shown in Fig.1. The cold fluid is in contact with the upper surface of the plate generates heat internally at the volumetric rate $\dot{q}$. A uniform magnetic field is applied in the direction perpendicular to the plate. The fluid is assumed to be slightly conducting, so that the magnetic Reynolds number is much less than unity, and hence the induced magnetic field is negligible in comparison with applied magnetic field. It is assumed that there is no applied voltage which implies the absence of electrical field. The fluid is considered to be a gray, absorbing emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the heat flux in the energy equation. The analysis considers a homogeneous first-order chemical reaction.

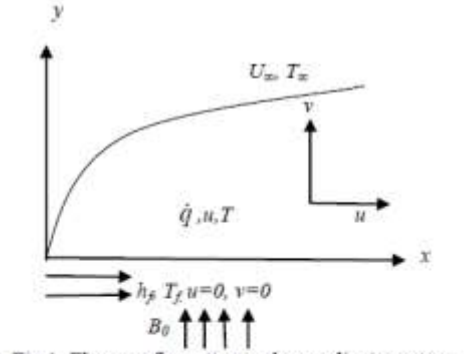

Fig.1. Flow configuration and co-ordinate system

The continuity, momentum, energy, and concentration equations describing the flow can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\dot{q}-\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x}+\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-Kr'C \tag{4}$$

where u and v denote the fluid velocity in x, y - directions respectively, υ - the kinematic viscosity, σ - the electrical conductivity, B_0 - the magnetic induction, ρ - the fluid density, T - the local temperature, C - the local species concentration, C_p - the specific heat at constant pressure, k - the thermal conductivity of fluid, q_r - the radiative heat flux, $\dot{q}$ - the volumetric heat generation, D - the mass diffusivity, and Kr' - the chemical reaction rate on the species concentration.

The boundary conditions for the velocity, temperature and concentration fields are

$$\begin{gathered} u(x,0)=v(x,0)=0, \\ -k\frac{\partial T}{\partial y}(x,0)=h_f\left[T_f-T(x,0)\right] \\ C(x,0)=C_w=Ax^n+C_\infty, \end{gathered} \tag{5}$$

$$u(x,\infty)=U_\infty,\ T(x,\infty)=T_\infty,\ C(x,\infty)=C_\infty,$$

where C_w is the species concentration at the plate surface, A is the constant, n is the power index of the concentration and C_∞ is the concentration of the fluid away from the plate.

By using the Rosseland approximation (Brewster [36]), the heat flux q_r is given by

$$q_r=\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

where σ^* is the Stephen-Boltzmann constant and k^* the mean absorption. It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If the temperature differences with in the flow are sufficiently small, then equation (3) can be linearized by expanding T^4 into the Taylor series about T_∞, which after neglecting higher order terms takes the form

$$T^4=4T_\infty^3T-3T_\infty^4 \tag{7}$$

In view of the equations (6) and (7), the equation (3) reduces to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\dot{q}}{\rho c_p}+\frac{16\sigma^*T_\infty^3}{3k^*\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{8}$$

In order to write the governing equations and the boundary conditions in the dimensionless form, the following dimensionless quantities are introduced.

$$\begin{gathered} \eta=\frac{y}{x}\sqrt{\mathrm{Re}_x},\quad u=U_\infty f', \\ v=\frac{\upsilon}{2x}\sqrt{\mathrm{Re}_x}\left(\eta f'-f\right),\quad \theta=\frac{T-T_\infty}{T_f-T_\infty}, \\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty},\ Bi_x=\frac{h_f}{k}\sqrt{\frac{\upsilon x}{U_\infty}}, \\ \lambda_x=\frac{\dot{q}x^2e^n}{k\,\mathrm{Re}_x\left(T_f-T_\infty\right)},\ M_x=\frac{\sigma B_0^2x}{\rho U_\infty}, \\ Kr_x=\frac{Kr'x}{U_\infty},\ \mathrm{Pr}=\rho c_p\frac{\upsilon}{k},\quad Sc=\frac{\upsilon}{D}, \\ Nc=\frac{C_\infty}{C_w-C_\infty},\quad R=\frac{4\sigma_sT_\infty^3}{k^*k} \end{gathered} \tag{9}$$

where prime denotes the differentiation with respect to η. $Re_x=U_\infty x/\upsilon$ is the local Reynolds number. The local internal heat generation parameter λ_x is defined so that the internal heat generation $\dot{q}$ decays exponentially with the similarity variable η as stipulated in [37].

In the view of the above similarity transformations, the equations (2), (8) and (4) reduce to

$$f'''+\frac{1}{2}ff''-M_x f'=0 \tag{10}$$

$$\left(1+\frac{4}{3}R\right)\theta''+\frac{1}{2}\Pr f\theta'+\lambda_x e^{-n}=0 \tag{11}$$

$$\varphi''-\frac{1}{2}Scf\varphi'-ScKr_x(\varphi+Nc)=0 \tag{12}$$

The corresponding boundary conditions are

$$f(0)=f'(0)=0, \quad \theta'(0)=Bi_x\left[\theta(0)-1\right],$$
$$\varphi(0)=1, \; f'(\infty)=1, \theta(\infty)=0, \; \varphi(\infty)=0. \tag{13}$$

where η is the similarity variable, $f(\eta)$ - the dimensionless stream function, $f'(\eta)$ - the dimensionless velocity, $\theta(\eta)$ - the dimensionless temperature, $\varphi(\eta)$ - the dimensionless concentration η - the similarity variable, λ_x - the local internal heat generation parameter, M - the magnetic parameter, Bi_x - the local convective heat transfer parameter, Pr - the Prandtl number, Sc - the Schmidt number, Kr_x - the local chemical reaction parameter, Nc - the concentration difference parameter and R- the radiation parameter.

The solutions generated whenever Bi_x and λ_x are defined as in equations (9) are the local similarity solutions. In order to have a true similarity solution the parameters Bi_x and λ_x must be constants and not depend on x. This condition can be met if the heat transfer coefficient h_f is proportional to $x^{1/2}$ and the internal heat generation $\dot{q}$ is proportional to x^{-1}. In this case, we assume

$$h_f=cx^{-\frac{1}{2}}, \; \dot{q}=lx^{-1} \tag{14}$$

where c and l are constants but have the appropriate dimensions. In view of (14), we obtain

$$Bi=\frac{c}{k}\sqrt{\frac{\upsilon}{U_\infty}}, \; \lambda=\frac{l\upsilon e^n}{kU_\infty\left(T_f-T_\infty\right)} \tag{15}$$

The Biot number (Bi) lumps together the effects of convection resistance of the hot fluid and the conduction resistance of the flat plate. The parameter λ is a measure of the strength of the internal heat generation.

It can also be noted that the local parameters M_x and Kr_x and in (9) are functions of x and generate local similarity solution. In order to have a true similarity solution we assume the following relation [38].

$$\sigma=\frac{b}{x}, \quad Kr'=\frac{m}{x} \tag{16}$$

where b and m are the constants with appropriate dimensions. In view of relation (9) the parameters, M_x and Kr_x are now independent of x and henceforth, we drop the index "x" for simplicity.

From the technological point of view, the physical quantities of interest are the skin friction coefficient $f''(0)$, the local Nusselt number $\theta'(0)$ and the Sherwood number $\varphi'(0)$ which represent the wall shear stress, the heat transfer rate and the mass transfer rate respectively. Our task is to investigate how the values of $f''(0)$, $\theta'(0)$, $\varphi'(0)$ and $\theta(0)$ the plate surface temperature vary with the radiation parameter R, the magnetic parameter M and the Prandtl number Pr, the internal heat generation λ, local biot number Bi, the Schmidt number Sc, the local chemical reaction parameter Kr, and the concentration difference parameter Nc and radiation parameter R.

III. METHOD OF SOLUTION

The system of equations governing the flow field are non-linear. Therefore analytical solutions are not possible. Hence, numerical solutions were obtained by using Runge-Kutta fourth order technique along with shooting method. Using a similarity variable, the governing non-linear partial differential equations have been transformed into non- linear ordinary differential equations. First of all, higher order non-linear differential equations (10), (11) and (12) are converted into simultaneous linear differential Equations of first order and they are further transformed into initial value problem applying the shooting technique(Jain *et al*.[39]. Once the problem is reduced to initial value problem, then it is solved using Runge-Kutta fourth order technique. The step size $\Delta\eta$ =0.0001 is used to obtain the numerical solution with six decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient, the Nusselt number the Sherwood number and plate surface temperature which are respectively proportional to $f''(0)$, $\theta'(0)$, $\varphi'(0)$ and $\theta(0)$ are also sorted out and their numerical values are presented in a tabular form.

IV. RESULTS AND DISCUSSION

In order to get a physical insight into the problem, a parametric study is conducted to illustrate the effects of different governing parameters viz local magnetic field parameter M, local internal heat generation parameter λ_x, the local convective heat transfer parameter Bi, the Prandtl number Pr, the Schmidt number Sc, the local chemical reaction parameter Kr, and the concentration difference parameter *Nc* and radiation parameter *R* upon the nature of flow and transport, the numerical results are depicted graphically in Figs.2-11. Throughout the calculations, the parametric values are chosen as for *M*=0.1, *Bi*=0.1, λ=10, *Kr*=0.1, *Sc*=0.62, *Nc*=0.01, *Pr*=0.72 and *R*=1.0. All the graphs therefore correspond to these values unless specifically indicated on the appropriate graph. Numerical results for the skin-friction coefficient, the Nusselt number, the Sherwood number and the plate surface temperature for various values of physical parameters are reported in Tables.

The influence of the magnetic parameter (*M*) on the velocity is shown in Fig.2. It is seen that as the magnetic parameter increases, the velocity decreases and this qualitatively agrees with the expectations, since the magnetic field exerts a retarding force which opposes the velocity. Fig.3 display the effect of the magnetic parameter on the temperature. It is noticed that the temperature of the fluid increases as the magnetic parameter increases. This is due to the fact that the applied magnetic field tends to heat the fluid, and thus reduces the heat transfer from the wall. The effect of the magnetic parameter on the concentration field is illustrated in Fig.4. It can be seen that an increase in magnetic parameter produces significant increase in the concentration boundary layer.

The influence of the internal heat generation (λ) on the temperature is shown in Fig.5. Fig.5 reveals that only for weak internal heat generation i.e. λ= 0.1, the plate surface temperature is less 1 and heat is able to flow from the lower surface of plate into the fluid on the upper face of the plate. For all other values of λ, the heat flows back into the plate. It is seen that as the internal heat generation increases, the temperature increases.

The effect of the Biot number (*Bi*) on the temperature distribution is depicted in Fig.6. Because of strong internal heat generation (λ=10), Fig.6 shows that the plate surface temperatures exceed the temperature of the fluid on the lower surface of the plate and the direction of heat flow is reversed as noted in the earlier discussion. The peak temperature occurs in the thermal boundary in a region close to the plate. Although the temperature reduces as the local Biot number increases but the back heat flow persists.

Fig.7 shows the effect of the Prandtl number on the temperature (*Pr*). It is observed that, an increase in the Prandtl number results in a decrease of the thermal boundary layer, and hence temperature decreases. The reason is that, smaller values of *Pr* are equivalent to increase the thermal conductivities, and therefore heat is able to diffuse away from the heated surface more rapidly than for higher values of *Pr*.

Fig.8 shows the effect of the radiation parameter (*R*) on the temperature. It is noticed that the temperature descends near the wall, while it ascends in the free stream with an increase in the radiation parameter. The effect of the radiation parameter *R* is to reduce the temperature significantly in the flow region. An increase in the radiation parameter implies the release of heat energy from the flow region, and the fluid temperature decreases as the thermal boundary layer thickness becomes thinner.

The effect of chemical reaction parameter (*Kr*) on the concentration is presented in Fig.9. It is observed that the species concentration decreases with an increase in the chemical reaction parameter *Kr*. Physically this shows that an increase in chemical reaction parameter breaks up the bonds between the atoms and thus the density of the species dilutes and thereby decreases the concentration of the fluid.

The effect of the Schmidt number (*Sc*) on the concentration are shown in Fig.10. The Schmidt number embodies the ratio of momentum to mass diffusivity. The Schmidt number quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases.

Fig.11 shows the effect of the concentration difference parameter (*Nc*) on the concentration. It is observed that, an increase in the concentration difference parameter decreases the concentration.

Table 1 presents a comparison of the local Nusselt numbers $\theta'(0)$, the plate surface temperature $\theta(0)$ between the present results and the results obtained by Olanrewaju [38] for the reduced case *M*=*R*=*Sc*=*Kr*=*Nc*=0. It is found that there is a good agreement, Table -2 and Table-3 shows the analysis of the local skin friction coefficient, the local Nusselt number, the plate surface temperature, and the Sherwood number for various values of the physical parameters. For all values of physical parameters embedded in the system, except for magnetic parameter the value of local skin friction coefficient represented by $f''(0)$=0.132016. Note that we have tabulated the values of $\theta'(0)$ and not - $\theta'(0)$ as in Table 3, because except for one case $\theta'(0)$ is positive means heat flows into the flat plate.

From Table 2, it is observed that as the local convective heat transfer parameter (Bi_x) increases the plate heating becomes stronger, leads to an increase in the Nusselt number and plate surface temperature decreases rapidly. As the Prandtl number (*Pr*) increases (from 0.72 Air to 7.1water) the plate surface temperature decreases rapidly curtailing the back heat flow into the plate and hence it leads to a decrease in the Nusselt number. It is observed that as the local internal heat generation increases i.e as λ increases from 5 to 10, both the Nusselt number and the plate surface temperature increase. It is noticed that as the radiation parameter (*R*) increases, both the local Nusselt number and the plate surface temperature decrease.

It is found from Table 3, with an increase in the magentic parameter (*M*) the skin-friction coefficient, the local Nusselt number and the Sherwood number decrease, while the plate surface temperature increases rapidly.

It is clearly seen from Table 4, that with an increase in the local chemical reaction parameter (*Kr*), the concentration difference parameter (*Nc*) and Schmidt number (*Sc*) leads to an increase in the Sherwood number.

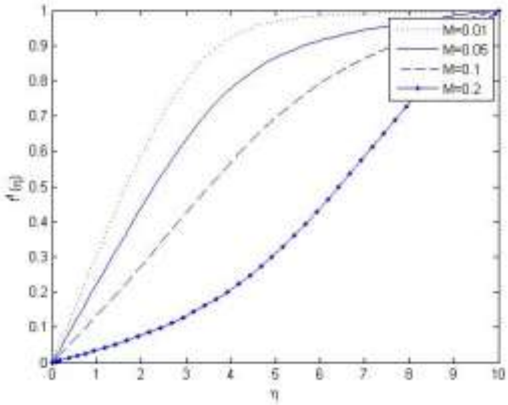

Fig.2: Velocity profiles for different values of *M*

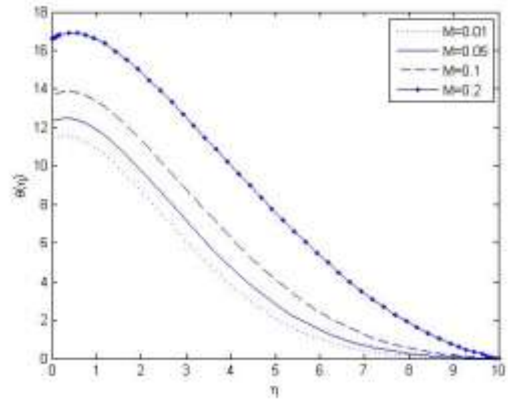

Fig.3: Temperature profiles for different values of *M*

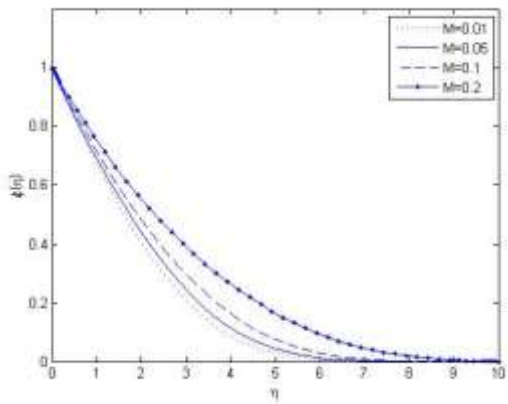

Fig.4: Concentration profiles for different values of *M*

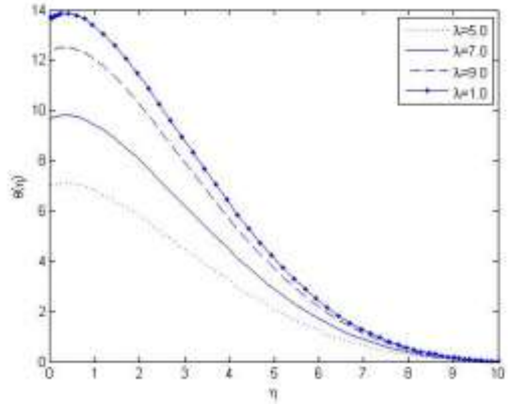

Fig5: Temperature profiles for different values of *λ*

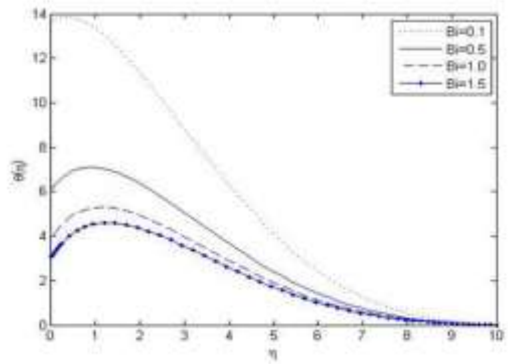

Fig.6: Temperature profiles for different values of *Bi*

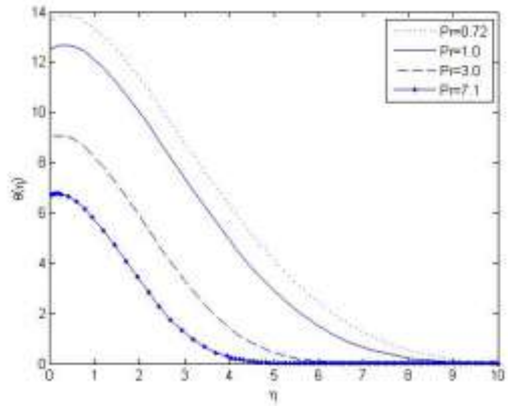

Fig.7: Temperature profiles for different values *Pr*

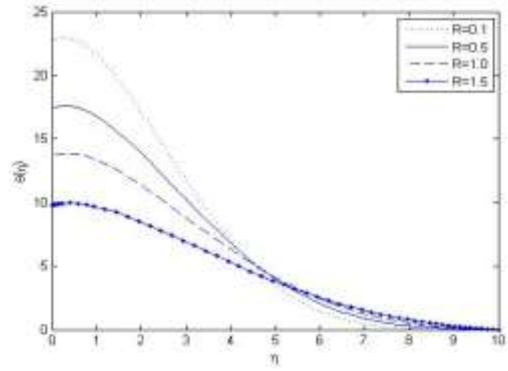

Fig.8: Temperature profiles for different values of *R*

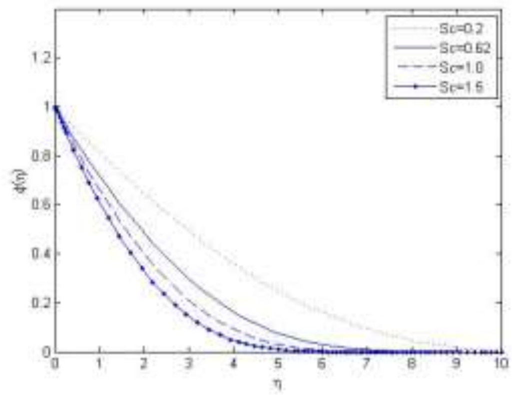

Fig.10: Concentration profiles for different values *Sc*

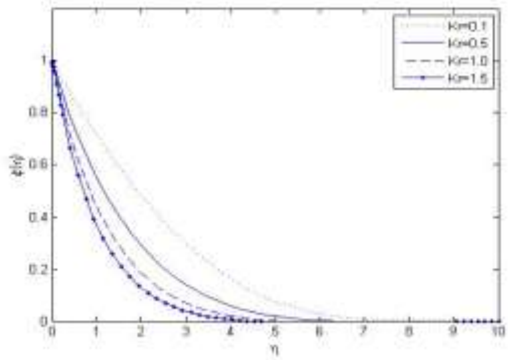

Fig.9: Concentration profiles for different values of *Kr*

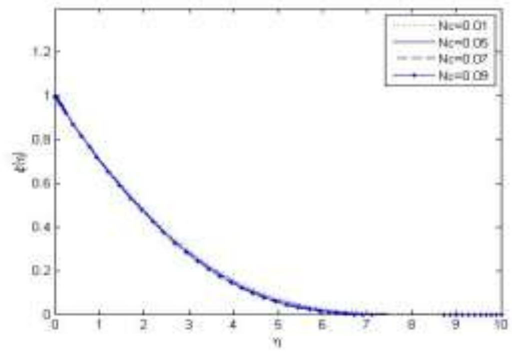

Fig.11: Concentration profiles for different values of *Nc*

TABLE 1: Comparison of $\theta'(0)$ and $\theta(0)$ for different values of Bi_x, Pr, λ_x

Bi	*Pr*	λ	Olanrewaju		Present results	
			$\theta'(0)$	$\theta(0)$	$\theta'(0)$	$\theta(0)$
0.1	0.72	1.0	0.1154879	2.15487958	0.115492	2.154917
1.0	0.72	1.0	0.3526541	1.35265410	0.352660	1.352660
1.0	0.72	1.0	0.4437910	1.04437910	0.443796	1.044380
0.1	3.00	1.0	0.272290	1.27229008	0.027234	1.272345
0.1	7.10	1.0	-0.0101008	0.89899201	-0.010099	0.899006
0.1	0.72	5.0	0.8763365	9.76336572	0.876353	9.763531
0.1	0.72	10	1.8273973	19.273973	1.827430	19.274299

TABLE 2: Numerical values of $\theta'(0)$ and $\theta(0)$, for M=0.1, Kr=0.1, Nc=0.01, Sc=0.62

Bi	*Pr*	λ	*R*	$\theta'(0)$	$\theta(0)$
0.1	0.72	10	1.0	0.696258	13.792030
0.5	0.72	10	1.0	1.987599	6.436097
1.0	0.72	10	1.0	2.361447	4.306528
1.4	0.72	10	1.0	2.511604	3.451182
0.1	0.72	10	1.0	0.696258	13.792030
0.1	1.0	10	1.0	0.582509	12.638357
0.1	3.0	10	1.0	0.231234	9.075582
0.1	7.1	10	1.0	0.003590	6.766688
0.1	0.72	5.0	1.0	0.316561	7.075836
0.1	0.72	7.0	1.0	0.468440	9.762314
0.1	0.72	9.0	1.0	0.620318	12.448791
0.1	0.72	10	1.0	0.696258	13.792030
0.1	0.72	10	0.1	0.997953	22.919388
0.1	0.72	10	0.5	0.837926	17.521028
0.1	0.72	10	1.0	0.696258	13.792030
0.1	0.72	10	2.0	0.517243	9.892633

TABLE 3: Numerical values of $f''(0)$, $\theta'(0)$, $\theta(0)$ and $-\varphi'(0)$ for Bi=0.1, Pr=0.72, λ=10, Kr=0.1, Nc=0.01, Sc=0.62, R=1.0

M	$f''(0)$	$\theta'(0)$	$\theta(0)$	$-\varphi'(0)$
0.05	0.308256	0.789805	11.488105	0.351403
0.1	0.220988	0.782246	12.397426	0.331091
0.2	0.132015	0.696258	13.792030	0.304323
0.3	0.034259	0.234338	16.918056	0.255852

TABLE 4: Numerical values of $\theta(0)$ and $-\varphi'(0)$ for M=0.1, Bi=0.1, Pr=0.72, λ=10, R=1.0

Kr	Nc	Sc	$-\varphi'(0)$
0.1	0.01	0.62	0.304323
0.5	0.01	0.62	0.536209
1.0	0.01	0.62	0.719398
1.5	0.01	0.62	0.854658
0.1	0.01	0.62	0.304323
0.1	0.05	0.62	0.311264
0.1	0.07	0.62	0.314734
0.1	0.09	0.62	0.318204
0.1	0.01	0.02	0.193699
0.1	0.01	0.62	0.304323
0.1	0.01	1.0	0.370460
0.1	0.01	1.5	0.438049

V. CONCLUSIONS

A steady two-dimensional boundary layer model has been developed for the boundary layer flow of a hydromagnetic, viscous, incompressible radiating fluid along a flat plate in the presence of exponentially decaying internal heat generation, thermal radiation and chemical reaction with convective surface boundary condition. The governing boundary layer equations are reduced to nonlinear ordinary differential equations using similarity transformations and the resulting equations are then solved numerically using Runge-Kutta fourth order method along with shooting technique.

A parametric study is conducted to illustrate the behavior of various physical quantities for different values of the governing parameters and the results are summarized as follows.

- As the Magnetic parameter increases, the velocity boundary layer thickness, the skin friction coefficient, the Nusselt number and the Sherwood number decrease while the plate surface temperature, the temperature and the concentration increase.
- There is a decrease in the thermal boundary layer thickness and the plate surface temperature with an increase in the local Prandtl number or the local Biot number.
- With an increase in the local Biot number, the Nusselt number increases.
- An increase in the local Prandtl number leads to a decrease in Nusselt number.
- An increase in the internal heat generation prevents the rapid flow of heat from the lower surface to the upper surface of the plate
- As radiation parameter increases, the temperature decends near the wall, while it ascends in the free stream where as the Nusselt number and plate surface temperature decreases rapidly.
- An increase in the local chemical reaction parameter (Kr), the concentration difference parameter (Nc) and Schmidt number (Sc) leads to an increase in the Sherwood number and decrease in the concentration.

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