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Static and Dynamic Reanalysis of Tapered Beam

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ABSTRACT
Beams are one of the common types of structural components and they are fundamentally categorized as uniform and non-uniform beams. The non-uniform beams has the benefit of better distribution of strength and mass than uniform beam. And non-uniform beams can meet exceptional functional needs in aeronautics,robotics,architecture and other unconventional engineering applications. Designing of these structures is necessary to resist dynamic forces such as earthquakes and wind.
The present paper focuses on static and dynamic reanalysis of a tapered cantilever beam structure using multipolynomial regression method. The method deals with the characteristics of frequency of a vibrating system and the procedures that are available for the modification of physical parameters of vibrating system. The method is applied on a tapered cantilever beam for approximate structural static and dynamic reanalysis. Results obtained from the assumed conditions of the problem indicate the high quality approximation of stresses and natural frequencies using ANSYS and Regression method.

I. INTRODUCTION

A beam is a structural element and it can be able to withstand load predominately by resisting the bending. Beams can be categorized into different classes depending on different attributes such as shape of cross-section, geometric profile, boundary conditions etc. In present problem we are using the tapered beam which are being increasingly using in various engineering applications. The benefits of these beams are structural efficiency, ability to meet architectural requirements and less fabrication costs. These tapered beams can be used in high-rise structures, bridges, commercial building applications.

II. LITERATURE

P.Nagalatha and P.sreenivas[1] done the reanalysis of structural modification of abeam element based on natural frequencies using polynomial regression method. The results obtained from both FEM and Regression method are compared and the error obtained is very small.B.RamaSanjeevasresta and Dr.Y.V.MohanReddy[2] presented a paper which focuses on dynamic reanalysis of simple beam structure using a polynomial regression method. E Ozkaya [3] considered linear transverse vibrations of simply supported Euler-Bernoulli beam carrying masses.For different number of masses, mass ratios and mass locations natural frequencies are calculated. R Vasudevan and B Parthasaradhi [4] studied the free vibration responses of a rotating tapered composite beams with tip mass. The natural frequencies of rotating tapered composite beam at all the modes considered are increased with increase in hub radius.Dhyai Hassan Jawad [5] presented the free vibration and buckling behaviour of non-uniform Euler-Bernoulli beam under variation of tapered parameter and degree of flexural bending by using Finite Element Method and it is linked with Mat lab Program.S C Mohanty[6] investigated the free vibration of a functionally graded ordinary(FGO)pre-twisted cantilever Timoshenko beam. Increase in the value of power law index decreases the first two modal frequencies of beam.

III. STATIC REGRESSION REANALYSIS

We use the principle of minimum potential energy for deriving the equilibrium equations for a three-dimensional problem. Since the nodal degrees of freedom are treated as unknowns in the present formulation, the potential energy π_p has to be first expressed in terms of nodal degrees of freedom. Then the necessary equilibrium equations can be obtained by setting the first partial derivatives of π_p with respect to each of the nodal degrees of freedom equal to zero. The various steps involved in the derivation of equilibrium equations are given below.

Step 1: The solid body is divided into E finite elements.

Step 2: The displacement model within an element "e" is assumed as

$$U = \begin{Bmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{Bmatrix} = [N]Q^{(e)}$$

Where $Q^{(e)}$ is the vector of nodal displacement degrees of freedom of the element and [N] is the matrix of shape functions.

Step 3: The element characteristic (stiffness) matrices and characteristic (load) vectors are to be derived from the principle of minimum potential energy. For this the potential energy functional of the body π_p is written as (by considering only the body and surface forces).

$$\pi_p = \sum_{e=1}^{E} \pi_p^{(e)}$$

Where $\pi_p^{(e)}$is the potential energy of element e given by

Where $V^{(e)}$ is the volume of the element, $S_1^{(e)}$ is the portion of the surface of the element over which distributed surface forces or tractions, φ ,are prescribed and φ is a vector of body forces per unit volume.

The strain vector ϵ appearing in equation can be expressed in terms of the nodal displacement vector $Q^{(e)}$ by differentiating equation suitably as

The stress can be obtained from the strains$\vec{\varepsilon}$ using

$$\vec{\sigma} = [D](\vec{\varepsilon} - \overrightarrow{\varepsilon_0}) = [D][B]\overrightarrow{Q^{(e)}} - [D]\overrightarrow{\varepsilon_0}$$

$$\pi_p^{(e)} = \frac{1}{2}\iiint_{V^{(e)}} \overrightarrow{Q^{(e)^T}}\,[B]^T[D][B]\overrightarrow{Q^{(e)}}dV - \iiint_{V^{(e)}} \overrightarrow{Q^{(e)^T}}\,[B]^T[D]\overrightarrow{\varepsilon_0}dV - \iint_{S_1^{(e)}} \overrightarrow{Q^{(e)^T}}\,[N]^T\overrightarrow{\overline{\emptyset}}dS_1 - \iiint_{V^{(e)}} \overrightarrow{Q^{(e)^T}}\,[N]^T\overrightarrow{\overline{\emptyset}}dV$$

In the above equations only the body and surface forces are considered. But generally some external concentrated forces also will be acting at various nodes. If denotes the vector of nodal forces (acting in the directions of the nodal displacement vector of the total structure or body ,the total potential energy of the structure or body can be expressed as

$$\pi_p = \sum_{e=1}^{E} \pi_P^{(e)} - \vec{Q}^T\overrightarrow{P_C}$$

The static equilibrium configuration of the structures can be found by solving the following necessary conditions (for the minimization of potential energy).

$\frac{\partial \pi_P}{\partial \vec{Q}} = \vec{0}or\frac{\partial \pi_p}{\partial Q_1} = \frac{\partial \pi_P}{\partial Q_2} = \cdots = \frac{\partial \pi_P}{\partial Q_M}$=0

Step 4: The desired equilibrium equations of the overall structure or body can now be expressed using

$$[K]\vec{Q} = \vec{P}$$

Step 5 and 6: The required solution for the nodal displacements and element stresses can be obtained later.

Regression

The actual term "regression" is derived from the Latin word "*regredi*," and means "to go back to" or "to retreat." Thus, the term has come to be associated with those instances where one "retreats" or "resorts" to approximating a response variable with an estimated variable based on a functional relationship between the estimated variable and one or more input variables. In regression analysis, the input (independent) variables can also be referred to as "regressor" or "predictor" variables.

Numerical Example

The tapered cantilever beam of 0.75m length is divided into 5 elements equally. Element stiffness matrix and mass matrix are extracted. Natural frequencies of tapered beam at each node are found from ANSYS.

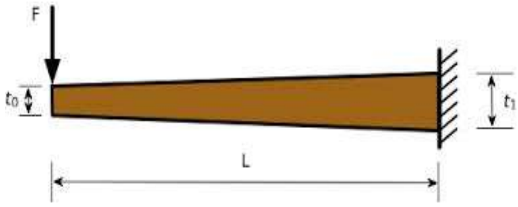

Fig 1.Tapered cantilever Beam

The values of young's modulus(E), density(ρ), length(l) breadth(b), depth(h) of tapered beam are as follows.

Reanalysis of the beam is done by using polynomial regression method and the percentage errors are listed in the tabular column. Stresses of tapered beam by increasing width and depth of the beam. The polynomial regression equation is given by

$S_n=c_1b^2-c_2bh-c_3h^2-c_4b-c_5h+c_6$

$c_1=1.817382813*10^{-1}$

$c_2=0.72265625$

$c_3=4.482421845*10^{-1}$

$c_4=43.640625$

$c_5=1.03125$

$c_6=1513$

Young's modulus(E)	$2.109*10^{11}$ N/m^2
Density(ρ)	7995.74 Kg/m^3
Length(l)	0.75m
Breadth(b)	b1=0.025m b2=0.045m
Depth(h)	h1=0.035m h2=0.055m

Table 1.Decreasing both width and depth of tapered beam.

Width (B)	Height (H)vgh	Stress ANSYS (N/mm^2)	Stress Regression (N/mm^2)	% Error
40.7	50.6	107.809	106.932	0.877
38.6	48.6	120.230	120.112	0.118
36.7	46.7	133.392	132.228	1.164
35.4	45.4	156.589	156.224	0.365
32.4	42.4	202.600	201.920	0.680
30.2	40.2	238.811	238.231	0.580
28.9	38.9	277.613	276.220	1.393
27.6	37.6	302.032	301.996	0.036

IV. DYNAMIC REGRESSION REANALYSIS

In dynamic problems the displacements, velocities, strains, stresses and loads are all time-dependent. The procedure involved in deriving the finite element equations of a dynamic problem can be stated by the following steps.

Step 1: Idealize the body into E finite elements.

Step 2: Assume the displacement model of element e as

$$\vec{U}(x,y,z,t) = \begin{Bmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{Bmatrix} = [N(x,y,z,)]\vec{Q}^{(e)}(t)$$

Where $\vec{U}$ is the vector of displacements, [N] is the matrix of shape function and$\vec{Q}^{(e)}$ is the vector of nodal displacements which is assumed to be a function of time t.

Step 3: Derive the element characteristic (stiffness and mass) matrices and characteristic (load) vector. The strains can be expressed as

$$\vec{\varepsilon} = [B]\vec{Q}^{(e)}$$

And stresses as

$$\vec{\sigma} = [D]\vec{\varepsilon} = [D][B]\vec{Q}^{(e)}$$

By differentiating the displacement equation with respect to time, the velocity field can be obtained as

$$\vec{\dot{U}}(x,y,z,t) = [n(x,y,z)]\vec{\dot{Q}}^{(e)}(t)$$

Where$\vec{\dot{Q}}^{(e)}$ is the vector of nodal velocities. To derive the dynamic equations of motion of a structure, we can either Lagrange equations or Hamilton's principle stated before.

The Lagrange equations are given by

$$\frac{d}{dt}\left\{\frac{\partial L}{\partial \dot{Q}}\right\} - \left\{\frac{\partial L}{\partial Q}\right\} + \left\{\frac{\partial R}{\partial \dot{Q}}\right\} = \{0\}$$

Where

$$L = T - \pi_P$$

is called Lagrangian function, T is the kinetic energy,π_P is potential energy, R is the dissipation function, Q is the nodal displacement.

The kinetic and potential energies of an element ''e'' can be expressed as

$$T^{(e)} = \frac{1}{2}\iiint_{V^{(e)}} \rho \vec{\dot{U}}^T \vec{\dot{U}} dV$$

And

$$\pi_P^{(e)} = \frac{1}{2}\iiint_{V^{(e)}} \vec{\varepsilon}^T \vec{\sigma} dV - \iint_{S_1^{(e)}} \vec{U}^T \vec{\vec{\phi}} dS_1 - \iiint_{V^{(e)}} \vec{U}^T \vec{\vec{\phi}} dV$$

Where V$^{(e)}$ is the volume,ρ is the density and$\vec{\dot{U}}$ is the vector of velocities of element e. By assuming the existence of dissipative forces proportional to the relative velocities, the dissipation function of the element e can be expressed as

$$R^{(e)} = \frac{1}{2}\iiint_{V^{(e)}} \mu \vec{\dot{U}}^T \vec{\dot{U}} dV$$

$$[M^{(e)}] = \iiint_{V^{(e)}} \rho [N]^T [N] dV = elementmassmatrix$$

$$[K^{(e)}] = \iiint_{V^{(e)}} [B]^T [D][B] dV = elementstiffnessmatrix$$

$$[C^{(e)}] = \iiint_{V^{(e)}} \mu [N]^T [N] dV = elementdampingmatrix$$

Step 4: Assemble the element matrices and vectors and derive the overall system equations of motion. we can obtain the dynamic equations of motion of the structure or body as

$$[M]\vec{\ddot{Q}}(t) + [C]\vec{\dot{Q}}(t) + [K]\vec{Q}(t) = \vec{P}(t)$$

Where$\vec{\ddot{Q}}$ is the vector of nodal accelerations in the global system. If damping is neglected, the equations of motion can be written as

$$[M]\vec{\ddot{Q}} + [K]\vec{Q} = \vec{P}$$

Step 5 and 6: Solve the equations of motion by applying the boundary and initial conditions.

Frequencies of Tapered Cantilever Beam

After discretizing to 5 numbers of elements, natural frequencies of the tapered beam are calculated using MATLAB program and ANSYS.

Taper ratio α	Fundamental Frequency	Second Harmonic	Third Harmonic	Fourth Harmonic	Fifth Harmonic
0.1	42.039	100.63	367.64	466.87	937.78
0.2	47.379	89.247	341.95	440.09	907.46
0.3	52.878	82.683	324.36	421.50	870.93
0.4	58.524	76.165	306.09	402.70	832.57
0.5	64.497	69.823	288.37	383.89	795.08
0.6	70.416	63.605	270.58	364.96	757.06
0.7619	80.608	57.519	252.70	345.89	718.49

Table 2.Frequencies of linearly tapered cantilever beam for different values of α.

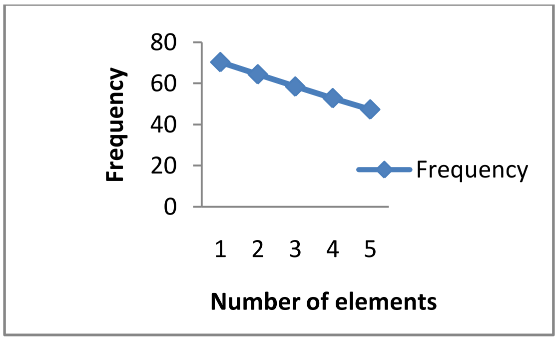

Figure 1.Convergence of fundamental natural frequency for Cantilever beam.

Fundamental natural frequencies of tapered beam by increasing width and depth of beam. The polynomial regression equation is given by

$Fn=c_1b^2-c_2bh+c_3h^2+c_4b+c_5h-c_6$

$c_1=1.240234375*10^{-1}$

$c_2=0.1328125$

$c_3=2.197265625*10^{-2}$

$c_4=0.7890625$

$c_5=1.140625$

$c_6=16.5$

Table 3. Comparison of ANSYS and Regression results for fundamental frequencies.

Width (b)	Height (h)	Natural Frequency ANSYS(Hz)	Natural Frequency Regression(Hz)	% Error
45	55	80.608	79.969	0.639
43.5	53.5	76.223	76.143	0.08
41.5	51.5	72.972	71.986	0.986
37	47	61.623	61.240	0.383
34.8	44.8	56.592	55.978	0.614
33.6	43.6	54.448	54.243	0.205
32.4	42.4	51.907	51.003	0.904
30.2	40.2	47.379	46.944	0.435
28.9	38.9	44.674	44.223	0.451
26.5	36.5	40.562	39.942	0.620

V. CONCLUSION

From this work the dynamic reanalysis of tapered beam is done by obtaining natural frequencies from ANSYS and Polynomial Regression method. By varying the width and depth of beam different natural frequencies are obtained. When the ANSYS results are compared with the results obtained from regression method the error percentage secured is negligible.

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