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Windows used in FIR Filters optimized for Far-side Stop band Attenuation (FSA) performance.

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ABSTRACT
It has been proposed that the Exponential window provides better side-lobe roll-off ratio than Kaiser window which is very useful for some applications such as beam forming, filter design, and speech processing. In this paper the second application i.e. design of digital nonrecursive Finite Impulse Response (FIR) filter by using Exponential window is proposed. The far-end stopband attenuation is most significant parameter when the signal to be filtered has great concentration of spectral energy. The filter should be designed in such a way so that it can provide better far-end stopband attenuation (amplitude of last ripple in stopband). Digital FIR filter designed by Kaiser window has a better far-end stopband attenuation than filter designed by the other previously well known adjustable windows such as Dolph-Chebyshev and aramaki, which are special cases of Ultraspherical windows, but obtaining a digital filter which performs higher far-end stopband attenuation than Kaiser window will be useful. In this paper, the design of nonrecursive digital FIR filter has been proposed by using Exponential window. It provides better far-end stopband attenuation than filter designed by well known Kaiser window, which is the advantage of filter designed by Exponential window. The proposed schemes were simulated on commercially available software and the results show the close agreement with proposed theory.

Keywords **-** Digital FIR filter, far-end stop band attenuation, side-lobe roll-off ratio, window techniques, Kaiser Window.

I. INTRODUCTION

Windows are time-domain weighting functions that are used to reduce Gibbs' oscillations resulting from the truncation of a Fourier series. Very recently, windows have been used to facilitate the detection of irregular and abnormal heartbeat patterns in patients in electrocardiograms . Windows have also been employed to improve the reliability of weather predicttion models . With such a large number of applications available for windows that span a variety of disciplines, general methods that can be used to design windows with arbitrary characteristics are especially useful.

In the literature, the Kaiser window is known as flexible window and this is widely used for the applications of Digital filter design and spectrum analysis. This is because it achieves close approximation to the discrete prolate spheroidal functions that have maximum energy concentration in the mainlobe. Window length **N** and shape parameter **α** are two independent parameters of Kaiser window and by adjusting these parameters, the spectral parameters such as mainlobe width, ripple ratio and sidelobe roll off ratio for different applications can be controlled.

The sidelobe roll off ratio is important parameter for some applications such as beamforming , digital filter design and speech processing. For beamforming applications, the large sidelobe roll off ratio is required to reject far end interference better . For filter design applications, it can reduce the far end attenuation for stopband energy. and for speech processing, it reduces the energy leak from one band to another .

It has been discovered that window based on exponential function provides higher sidelobe roll off ratio than Kaiser window. In this paper we explore this fact of Exponential window to design the digital nonrecursive FIR filter which provides the better far-end stopband attenuation among filters designed by well known windows in literature.

The paper is organized as follow; The spectral parameters of window are described in section II. The proposed window is explained in section III. The application of proposed window to design nonrecursive FIR filter is explored in section IV. The simulation results are given in section V. The synthesis and physical design is were given in section VI and this paper is concluded in section VII. The paper is also equipped with the references given at the end of this paper.

II. SPECTRAL PARAMETERS OF WINDOW

A window W(nT) with length of N is a Time-Domain function and it is nonzero for n≤ |(N-1)/2| and zero for otherwise. The frequency spectrum of W(nT) can be found as

$$W(e^{jwt}) = e^{-j\theta(w)} W_0(e^{jwT})$$

Where $Wo(e^{jwT})$ is called the amplitude function. The amplitude and phase spectrums of a window are given by $A(w) = |Wo(e^{jwT})|$ and θ(w), respectively, and $Wo(e^{jwT}) / Wo(e^{0})$ is a normalized version of the amplitude spectrum.

A typical window's normalized amplitude spectrum and some common spectral characteristics are depicted in Figure 1.

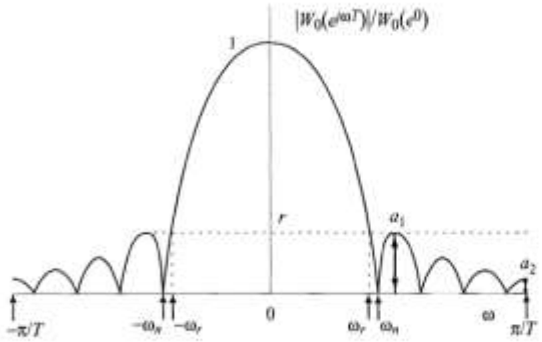

Figure 1. Amplitude spectrum and some common spectral characteristics of a typical normalized window .

Important window parameter is the ripple ratio r which is defined as

$$r = \frac{\text{maximum sidelobe amplitude}}{\text{main lobe amplitude}}$$

The ripple ratio is a small quantity less than unity and, in consequence, it is convenient to work with the reciprocal of *r* in dB, i.e.

$$R = 20\log_{10} 1/r$$

where R can be interpreted as the minimum side-lobe attenuation (minimum stopband attenuation) relative to the main lobe and -R is the ripple ratio in dB. Another parameter used to describe the side-lobe pattern of a window is the side-lobe roll-off ratio, s, which is defined as

$$s = \frac{a_1}{a_2}$$

Where, a_1 and a_2 are the amplitudes of the side lobe nearest and furthest, respectively, from the main lobe (see Figure 1). If S is the side-lobe roll-off ratio in dB, then s is given by

$$s = 10^{s/20}$$

For the side-lobe roll-off ratio to have meaning, the envelope of the side-lobe pattern should be monotonically increasing or decreasing. The side-lobe roll-off ratio provides a description of the distribution of energy throughout the side lobes, which can be of importance if prior knowledge of the location of an interfering signal is known.

III. PROPOSED WINDOW

From Fig. 2, it can be seen that exp(x) and Io(x) have the same shape characteristic. Therefore, a new window can be proposed as

$$W_{ex}(k) = \frac{\exp\left(\propto_{ex}\sqrt{1-\left(\frac{2n}{N-1}\right)2}\right)}{\exp(\propto_{ex})}, \qquad |n| \le \frac{N-1}{2}$$

The normalized spectrum of proposed window (in dB) can be obtained by

$$W_N(e^{jwT}) = 20\log_{10}\left(\frac{|A(w)|}{|A(w)|_{max}}\right)$$

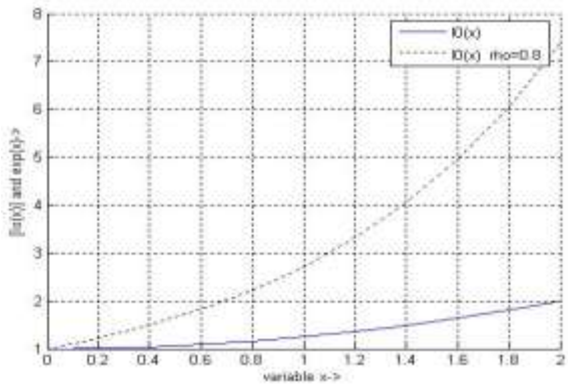

Figure 2. Similar shape characteristics of function exp(x) and Io(x).

Figure 3 shows the frequency spectrum of proposed window for a fixed value of filter length N = 51. The parameter $\propto_{ex}$ =0 corresponds to the rectangular window. From figure 3, it can be easily seen that, when $\propto_{ex}$ increases then the mainlobe width increases and ripple ratio decreases.

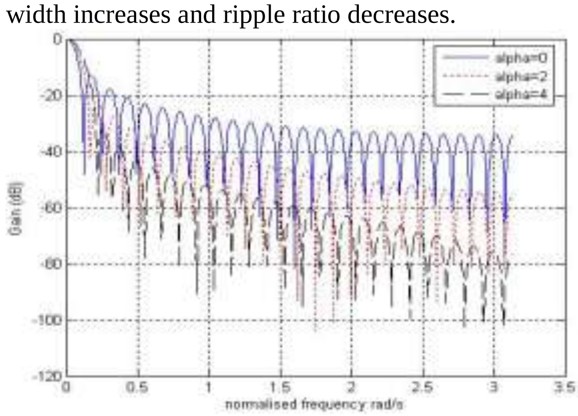

Figure 3. Proposed window spectrum in dB for □ex= 0, 2, and 4 and N=51

IV. APPLICATION TO DESIGN FIR FILTER DESIGN

A. Filter Design using Exponential window

Fourier series method with windowing is the most straightforward technique to design FIR filters and involves a minimal amount of computation compared to the optimization methods. The aim to use a window in Fourier series method is to truncate and smooth the infinite duration impulse response of the ideal prototype filter. The impulse response of a realizable noncasual FIR filter using a window function, w(nT), is obtained as

$$h_{nc}(nT) = w(nT)\, h_{id}(nT)$$

where h_{id} (nT) is the infinite duration impulse response of the ideal filter. For a low pass filter with a cut off frequency, wc,and sampling frequency, ws, it can be found as

$$h_{id}(nT) = \begin{cases} \frac{w_c T}{\pi} \; ; \; n = 0 \\ \frac{\sin w_c nT}{n\pi} \; ; \; w_c \leq |w| \leq w_s/2 \end{cases}$$

Delaying the noncasual impulse response h_{nc}(nT) by a period (N-1)/2, a casual filter can be obtained as

$$h(nT) = h_{nc}\left[(n - \frac{N-1}{2})T\right] \; ; \; \leq n \leq N-1$$

The ripples in both passband and stopband regions of the filters designed by the window method are approximately equal to each other.

The frequency response of digital FIR filter designed by exponential window is shown in figure 4 which shows the effect of parameter ($\propto_{ex}$) on the filter characteristic. It can be seen from figure 4, an increase in $\propto_{ex}$ results in better minimum stopband attenuation (As) but a worse transition width.

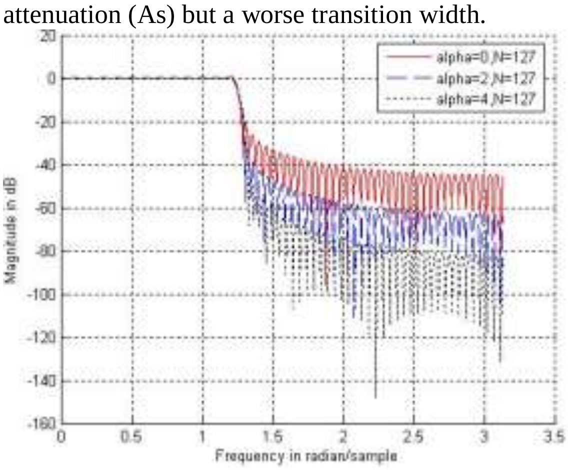

Figure 4. Amplitude spectrums of the filters designed by the exponential window for various $\propto_{ex}$ with N=127

B. Filter Design Equation for Proposed Window

To find the suitable window which satisfies the given prescribed filter specification, we obtain the relation between the window parameters and filter parameters.

Figure 5 shows the relation between Exponential window parameter, αex, and the minimum stopband attenuation, As, for N=127. From figure, it is clear that as the window parameter increases, the minimum stopband attenuation As also increases. By using the quadratic polynomial curve fitting method, the first design equation is obtained as

$$\propto_{ex} = -0.0004275 A_s^{\,2} + 0.1808$$
$$A_s - 3.516 \; ; \quad A_s \leq 20.77$$

The second filter design equation is the relation between minimum stopband attenuation As and normalized width, D, which is required to find the minimum length of the filter. The normalized width parameter can be calculated by the following equation.

$$D = \frac{\Delta w(N-1)}{w_s}$$

where Δw is the transition bandwidth. The relation between D and As is shown in figure 6. The comparison between the filters designed by Kaiser window and proposed window is also shown in figure 6. By using quadratic curve fitting method, an approximate expression for D can be found as

$$D = 5.188 \times 10^{-5} A_s^{\,2} + 0.06617 A_s - 0.1518 \; ; \quad A_s \leq 20.77$$

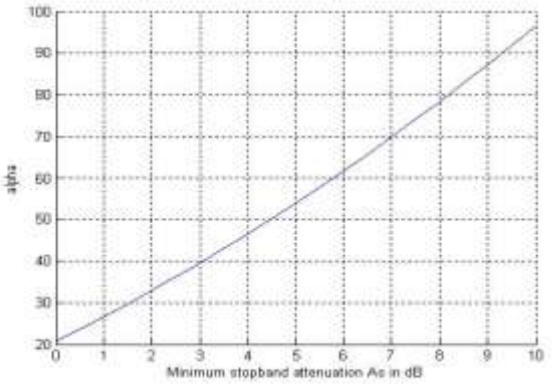

Figure 5. Relation between αex and the minimum stopband attenuation for exponential window with N=127.

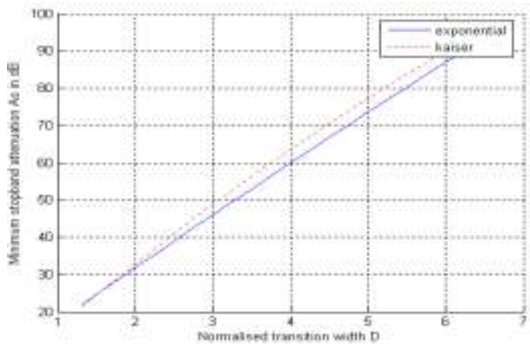

Figure 6. Comparison of the filters designed by exponential and Kaiser windows in terms of the minimum stopband attenuation with N=127.

Figure 7 shows that as the transition width increases, the filters designed by exponential window performs better far end suppression than the filters designed by Kaiser window.

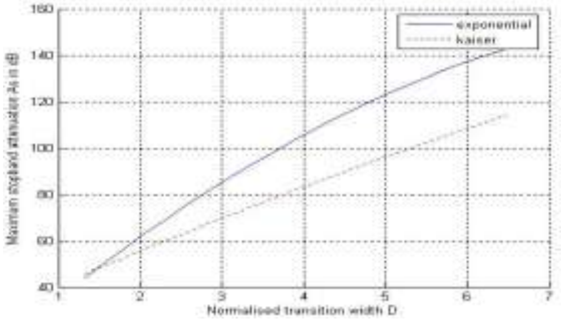

Figure 7. Comparison of the filters designed by exponential and Kaiser windows in terms of the maximum stopband attenuation with N=127.

V. COMPARISON EXAMPLE AND SIMULATION RESULTS

Based on the findings of the previous section, a lowpass nonrecursive filter that would satisfy the specifications

(i) passband edge: ω_p(radian/sample) or f_p (hertz))

(ii) stopband edge: ω_{st} (radian/sample) or f_{st}(hertz)

(iii) passband ripple: A_p

(iv) stopband ripple: A_s

(v) sampling frequency: ω_s (radian/sample) or F_s

can be designed using Algorithm given in [6]. Here an example is being presented which shows that the FIR Filter designed by Exponential window

Example I: Design a lowpass FIR filter by exponential window which meets the following specifications.

Sampling Frequency Fs=5 KHz.
Passband Edge Frequency =100 Hz.
Stopband Edge Frequency = 150 Hz.
Passband attenuation = 10 dB.
Stopband attenuation = 60 dB.
and compare the results of FIR filter designed by exponential window with the FIR filter designed by Kaiser and cosine hyperbolic windows.

For this example, the following simulation results are tabulated in TABLE I. In TABLE I, FSA and MSA are far end stopband attenuation and minimum stopband attenuation respectively.

The frequency response of filters designed by Exponential , Kaiser, Cosh, Rectangular, Triangular, and Hamming Window are shown in figure 8

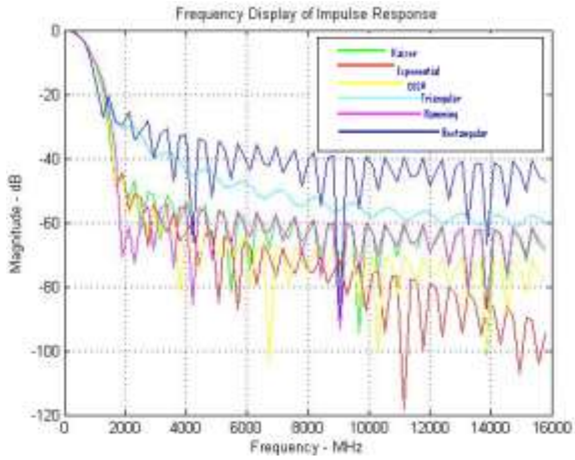

Figure 8. Frequency response of different filters

VI. CONCLUSION

In this paper, the application of the proposed window in the nonrecursive digital FIR filter design is presented in which it is seen that the FIR filter designed by proposed window provides better far end attenuation than filter designed by well known Kaiser Window. The better far end stopband attenuation in case of Exponential window shows the figure of merit and it is used for some applications such as sub band coding and speech processing . The comparison example compare this proposed window with Kaiser and previously proposed Cosh window on the basis of Normalized Transition width D, Filter Length N Design Parameter Į, far end stopband attenuation and minimum stopband attenuation and shows that the far end stopband attenuation is maximum in FIR filters designed by Exponential window than Kaiser and Cosh windows.

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