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Radix-2 Algorithms for realization of Type-II Discrete Sine Transform and Type-IV Discrete Sine Transform

M. N. Murty
(Department of Physics, National Institute of Science and Technology, Palur Hills, Berhampur-761008, Odisha, India)

ABSTRACT
In this paper, radix-2 algorithms for computation of type-II discrete sine transform (DST-II) and type-IV discrete sine transform (DST-IV), each of length $N=2^m (m=2,3,.....)$, are presented. The odd-indexed output components of DST-II can be realized using simple recursive relations. The recursive algorithms are appropriate for VLSI implementation. The DST-IV of length *N* can be computed from type-II discrete cosine transform (DCT-II) and DST-II sequences, each of length *N*/2.
Keywords – Discrete sine transform, discrete cosine transform, radix-2 algorithm, recursive algorithm.

I. INTRODUCTION

Discrete transforms play a significant role in digital signal processing. Discrete cosine transform (DCT) and discrete sine transform (DST) are used as key functions in many signal and image processing applications. There are eight types of DCT and DST. Of these, the DCT-II, DST-II, DCT-IV, and DST-IV have gained popularity. The DCT and DST transform of types I, II, III and IV, form a group of so-called "even" sinusoidal transforms. Much less known is group of so-called "odd" sinusoidal transforms: DCT and DST of types V, VI, VII and VIII.

The original definition of the DCT introduced by Ahmed *et al*. in 1974 [1] was one-dimensional (1-D) and suitable for 1-D digital signal processing. The DCT has found wide applications in speech and image processing as well as telecommunication signal processing for the purpose of data compression, feature extraction, image reconstruction, and filtering. Thus, many algorithms and VLSI architectures for the fast computation of DCT have been proposed [2]-[7]. Among those algorithms [6] and [7] are believed to be most efficient two-dimensional DCT algorithms in the sense of minimizing any measure of computational complexity.

The DST was first introduced to the signal processing by Jain [8], and several versions of this original DST were later developed by Kekre *et al.* [9], Jain [10] and Wang *et al.* [11]. Ever since the introduction of the first version of the DST, the different DST's have found wide applications in several areas in Digital signal processing (DSP), such as image processing[8,12,13], adaptive digital filtering[14] and interpolation[15]. The performance of DST can be compared to that of the discrete cosine transform (DCT) and it may therefore be considered as a viable alternative to the DCT. For images with high correlation, the DCT yields better results; however, for images with a low correlation of coefficients, the DST yields lower bit rates [16]. Yip and Rao [17] have proven that for large sequence length ($N \geq 32$) and low correlation coefficient ($\rho < 0.6$), the DST performs even better than the DCT.

In this paper, radix-2 algorithms for computation of type-II DST and type-IV DST, each of length $N=2^m (m=2,3,.....)$, are presented. The odd-indexed output components of DST-II are realized using simple recursive relations**.** The DST-IV is computed from DCT-II and DST-II sequences, each of length *N*/2**.**

The rest of the paper is organized as follows. The proposed radix-2 algorithm for DST-II is presented in Section-II. An example for realization of DST-II of length *N* = 8 is given in Section-III. The proposed radix-2 algorithm for type-IV DST is presented in Section-IV. Conclusion is given in Section-V.

II. PROPOSED RADIX-2 ALGORITHM FOR DST-II

Let $x(n), 1 \leq n \leq N,$ be the input data array. The type-II DST is defined as

$$Y_{II}(k) = \sqrt{\frac{2}{N}} C_k \sum_{n=1}^{N} x(n) \sin\left[\frac{(2n-1)k\pi}{2N}\right] \quad (1)$$

for $k = 1, 2, ..., N$

where ,

$$C_k = \begin{cases} \frac{1}{\sqrt{2}} \; if \; k = N \\ 1 if \; k = 1, 2, ..., N-1 \end{cases}$$

The $Y_{II}(k)$ values represent the transformed data. Without loss of generality, the scale factors in (1) are ignored in the rest of the paper. After ignoring scale factors, (1) can be written as

$$Y_{II}(k) = \sum_{n=1}^{N} x(n) \sin\left[\frac{(2n-1)k\pi}{2N}\right] \quad (2)$$

for $k = 1, 2, ..., N$.

When $N = 2^m (m = 2, 3, ...)$, (2) can be written as

$$Y_{II}(k) = \sum_{n=1}^{N/2} \left[x(n) + (-1)^{k+1} x(N+1-n)\right] \sin\left[\frac{(2n-1)k\pi}{2N}\right] \quad (3)$$

for $k = 1, 2, ..., N$

Let

$$P(n) = x(n) - x(N+1-n) \quad \text{for even } k \quad (4)$$

and

$$Q(n) = x(n) + x(N+1-n) \quad \text{for odd } k \quad (5)$$

Using (4) and (5) in (3), the even output components $Y_{II}(2k)$ and odd output components $Y_{II}(2k-1)$ of DST-II are given by

$$Y_{II}(2k) = \sum_{n=1}^{N/2} P(n) \sin\left[\frac{(2n-1)k\pi}{N}\right] \quad (6)$$

for $k = 1, 2, ..., \frac{N}{2}$.

$$Y_{II}(2k-1) = \sum_{n=1}^{N/2} Q(n) \sin\left[\frac{(2n-1)(2k-1)\pi}{2N}\right] \quad (7)$$

for $k = 1, 2, ..., \frac{N}{2}$.

Equation (7) can also be written as

$$Y_{II}(2k+1) = \sum_{n=1}^{N/2} Q(n) \sin\left[\frac{(2n-1)(2k+1)\pi}{2N}\right] \quad (8)$$

for $k = 0, 1, 2, ..., \frac{N}{2} - 1.$

Suppose

$$R(k) = Y_{II}(2k+1) - Y_{II}(2k-1) \quad (9)$$

Using (7) and (8) in (9), we obtain

$$R(k) = \sum_{n=1}^{N/2} Q(n) \left[\sin\left\{\frac{(2n-1)(2k+1)\pi}{2N}\right\} - \sin\left\{\frac{(2n-1)(2k-1)\pi}{2N}\right\}\right] \quad (10)$$

As $\sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$, (10) can be written as

$$R(k) = 2\sum_{n=1}^{N/2} Q(n) \sin\left[\frac{(2n-1)\pi}{2N}\right] \cos\left[\frac{(2n-1)k\pi}{N}\right] \quad (11)$$

From (9), we have

$$Y_{II}(2k+1) = R(k) + Y_{II}(2k-1) \quad (12)$$

The odd output components of DST-II can be realized using the recursive relation (12) and the even output components can be realized using (6).

III. EXAMPLE FOR REALIZING DST-II OF LENGTH *N* = 8

To clarify the proposal, the output data sequence $\{Y_{II}(k); k = 1, 2, ..., 8\}$ is realized from the input data sequence $\{x(n); n = 1, 2, ..., 8\}$ for DST-II of length $N = 8$.

3.1 Procedure for realizing odd output components of DST-II

Putting $k = 0$ in (9), we get

$$R(0) = Y_{II}(1) - Y_{II}(-1) \quad (13)$$

From (7), we have for $k = 0$

$$Y_{II}(-1) = -\sum_{n=1}^{N/2} Q(n)\sin\left[\frac{(2n-1)\pi}{2N}\right] \quad (14)$$

Putting $k = 0$ in (8), we obtain

$$Y_{II}(1) = \sum_{n=1}^{N/2} Q(n)\sin\left[\frac{(2n-1)\pi}{2N}\right] \quad (15)$$

From (14) and (15), we get

$$Y_{II}(-1) = -Y_{II}(1) \quad (16)$$

Using (16) in (13), we have

$$Y_{II}(1) = \frac{1}{2}R(0) \quad (17)$$

Putting $k = 1, 2 \,\&\, 3$ in (12), we obtain

$$Y_{II}(3) = R(1) + Y_{II}(1) \quad (18)$$

$$Y_{II}(5) = R(2) + Y_{II}(3) \quad (19)$$

$$Y_{II}(7) = R(3) + Y_{II}(5) \quad (20)$$

For *k* =0 and *N* = 8, (11) can be expressed as

$$R(0) = 2\sum_{n=1}^{4} Q(n)\sin\left[\frac{(2n-1)\pi}{16}\right]$$

$$= 2\left[Q(1)\sin\frac{\pi}{16} + Q(2)\sin\frac{3\pi}{16} + Q(3)\sin\frac{5\pi}{16} + Q(4)\sin\frac{7\pi}{16}\right] \quad (21)$$

Using (21) in (17), we get

$$Y_{II}(1) = \frac{R(0)}{2} = Q(1)S_1 + Q(2)S_3 + Q(3)S_5 + Q(4)S_7 \quad (22)$$

where

$$S_n = \sin\frac{n\pi}{16}$$

Putting *k*=1 and *N* = 8 in (11), we obtain

$$R(1) = \left[Q(1)\sin\frac{\pi}{16} - Q(4)\sin\frac{7\pi}{16}\right]2\cos\frac{\pi}{8} + \left[Q(2)\sin\frac{3\pi}{16} - Q(3)\sin\frac{5\pi}{16}\right]2\cos\frac{3\pi}{8}$$

$$= \left[Q(1)S_1 - Q(4)S_7\right]C_1 + \left[Q(2)S_3 - Q(3)S_5\right]C_3 \quad (23)$$

where

$$C_1 = 2\cos\frac{\pi}{8} \text{ and } C_3 = 2\cos\frac{3\pi}{8}$$

Putting k = 2 and N = 8 in (11), we have

$$R(2) = \left[Q(1)S_1 - Q(2)S_3 - Q(3)S_5 + Q(4)S_7\right]C_2 \tag{24}$$

where

$$C_2 = 2\cos\frac{2\pi}{8} = 2\cos\frac{\pi}{4} \text{ and in general } C_n = 2\cos\frac{n\pi}{8}$$

Similarly, putting k = 3 and N = 8 in (11), we obtain

$$R(3) = \left[Q(1)S_1 - Q(4)S_7\right]C_3 + \left[Q(3)S_5 - Q(2)S_3\right]C_1 \tag{25}$$

For $N = 8 \,\&\, n = 1, 2, 3, 4$, we have from (5)

$$Q(1) = x(1) + x(8)$$
$$Q(2) = x(2) + x(7) \tag{26}$$
$$Q(3) = x(3) + x(6)$$
$$Q(4) = x(4) + x(5)$$

The odd output components $Y_{II}(1), Y_{II}(3), Y_{II}(5) \,\&\, Y_{II}(7)$ of DST-II can be realized using (22),(23),(24),(25) and (26)along with the recursive relations (18),(19) and (20) as shown in the data flow diagram of Fig.1.

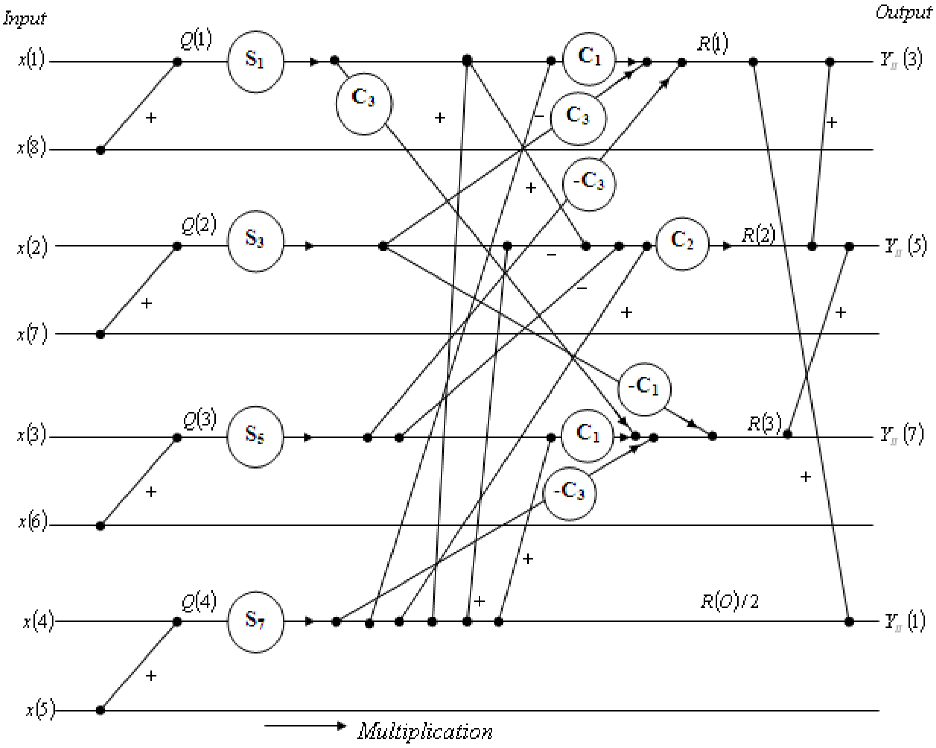

Figure 1: Signal flow graph for odd output components of DST-II of length N=8.

$$\left[S_n = \sin\left(\frac{n\pi}{16}\right) \,\&\, C_n = 2\cos\left(\frac{n\pi}{8}\right)\right]$$

3.2 Procedure for realizing even output components of DST-II

Putting successively $k = 1, 2, 3, 4$ in (6), we get the following expressions for $N = 8.$

$$Y_{II}(2) = \left[P(1) + P(4)\right]S_2 + \left[P(2) + P(3)\right]S_6 \tag{27}$$

$$Y_{II}(4) = [P(1)+P(2)-P(3)-P(4)]S_4 \quad (28)$$

$$Y_{II}(6) = [P(1)+P(4)]S_6 - [P(2)+P(3)]S_2 \quad (29)$$

$$Y_{II}(8) = P(1)-P(2)+P(3)-P(4) \quad (30)$$

For $N = 8 \,\&\, n = 1,2,3,4$, we have from (4)

$$\begin{aligned} P(1) &= x(1)-x(8) \\ P(2) &= x(2)-x(7) \\ P(3) &= x(3)-x(6) \\ P(4) &= x(4)-x(5) \end{aligned} \quad (31)$$

The even output components $Y_{II}(2), Y_{II}(4), Y_{II}(6) \,\&\, Y_{II}(8)$ of DST-II can be realized using (27),(28),(29),(30) and (31) as shown in the data flow diagram of Fig. 2.

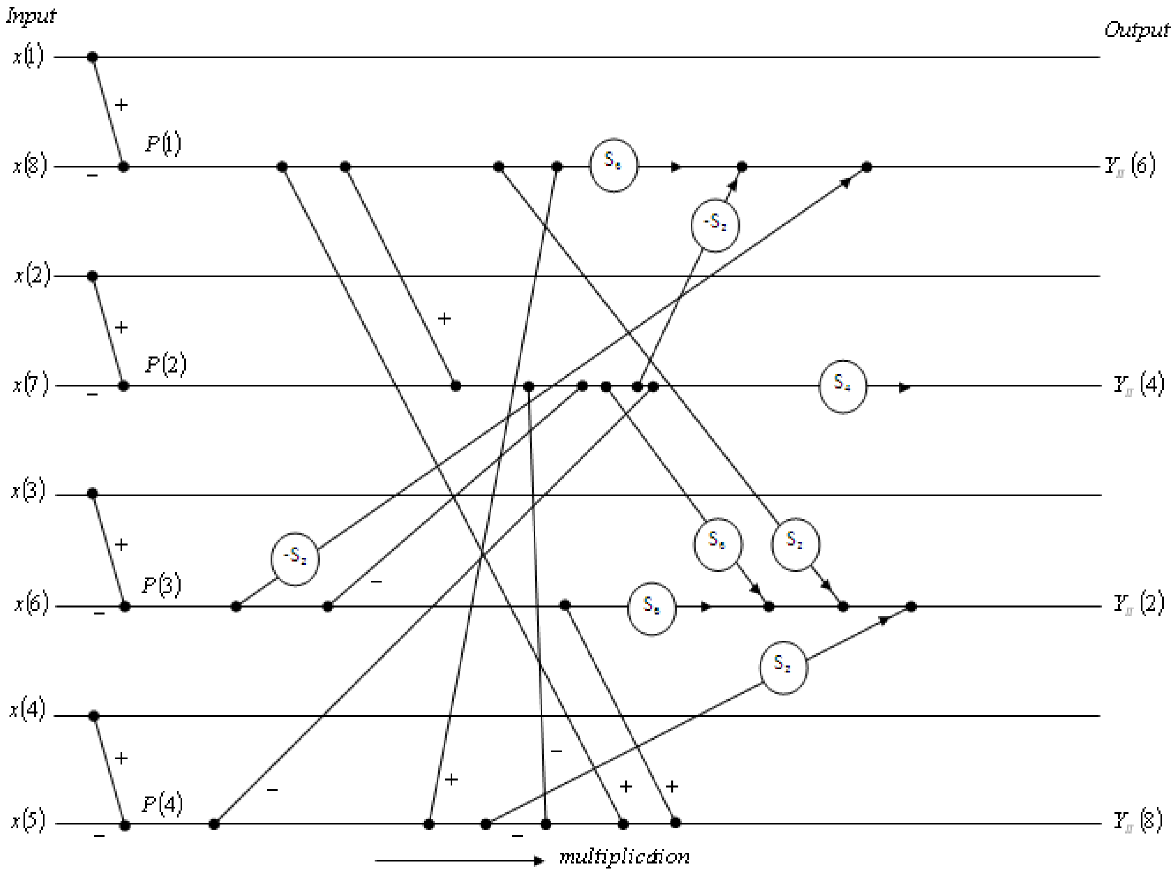

Figure 2: Signal flow graph for even output components of DST-II of length *N*=8. $\left[S_n = \sin\left(\frac{n\pi}{16}\right) \right]$

IV. PROPOSED RADIX-2 ALGORITHM FOR DST-IV

The type-IV DST for the input data sequence $\{x(n); n = 1,2,...., N\}$ is defined as

$$Y_{IV}(k) = \sum_{n=1}^{N} x(n) \sin\left[\frac{(2n-1)(2k-1)\pi}{4N}\right] \quad (32)$$

for $k = 1,2,...., N$.

The $Y_{IV}(k)$ values represent the output data.

The type-II DCT for the input data sequence $\{x(n); n=1,2,....,N\}$ is defined as

$$X(k)=\sum_{n=1}^{N} x(n)\cos\left[\frac{(2n-1)\pi k}{2N}\right] \tag{33}$$

for $k=0,1,2,....,N-1$.

Taking $N=2^m (m \geq 2)$ in (32), even and odd output components of DST-IV can be written as

$$Y_{IV}(2k)=\sum_{n=1}^{N/2}\left\{x(n)\sin\left[\frac{(2n-1)(4k-1)\pi}{4N}\right]-x(N+1-n)\cos\left[\frac{(2n-1)(4k-1)\pi}{4N}\right]\right\} \tag{34}$$

for $k=1,2,....,\frac{N}{2}$.

$$Y_{IV}(2k+1)=\sum_{n=1}^{N/2}\left\{x(n)\sin\left[\frac{(2n-1)(4k+1)\pi}{4N}\right]+x(N+1-n)\cos\left[\frac{(2n-1)(4k+1)\pi}{4N}\right]\right\}$$

for $k=0,1,2,....,\frac{N}{2}-1$. (35)

Define $u(n)$ and $v(n)$ as

$$u(n)=x(n)\sin\left[\frac{(2n-1)\pi}{4N}\right]+x(N+1-n)\cos\left[\frac{(2n-1)\pi}{4N}\right] \tag{36}$$

for $n=1,2,....,\frac{N}{2}$.

$$v(n)=x(n)\cos\left[\frac{(2n-1)\pi}{4N}\right]-x(N+1-n)\sin\left[\frac{(2n-1)\pi}{4N}\right] \tag{37}$$

for $n=1,2,....,\frac{N}{2}$.

Define $T(k)$ and $W(k)$ as

$$T(k)=\sum_{n=1}^{N/2} u(n)\cos\left[\frac{\pi(2n-1)k}{N}\right] \tag{38}$$

for $k=0,1,2,....,\frac{N}{2}-1$.

$$W(k)=\sum_{n=1}^{N/2} v(n)\sin\left[\frac{\pi(2n-1)k}{N}\right] \tag{39}$$

for $k=1,2,....,\frac{N}{2}$.

Where $T(k)$ represents the DCT-II of $u(n)$ of length $N/2$ and $W(k)$ represents DST-II of $v(n)$ of length $N/2$.

Using (36) in (38) and (37) in (39), we obtain

$$W(k)-T(k)=\sum_{n=1}^{N/2}\left\{x(n)\cos\left[\frac{(2n-1)\pi}{4N}\right]-x(N+1-n)\sin\left[\frac{(2n-1)\pi}{4N}\right]\right\}\sin\left[\frac{\pi(2n-1)k}{N}\right]$$
$$-\sum_{n=1}^{N/2}\left\{x(n)\sin\left[\frac{(2n-1)\pi}{4N}\right]+x(n+1-n)\cos\left[\frac{(2n-1)\pi}{4N}\right]\right\}\cos\left[\frac{\pi(2n-1)k}{N}\right]$$

$$= \sum_{n=1}^{N/2} \left\{ x(n) \sin\left[\frac{(2n-1)(4k-1)\pi}{4N}\right] - x(N+1-n)\cos\left[\frac{(2n-1)(4k-1)\pi}{4N}\right]\right\} \quad (40)$$

From (34) and (40), we have

$$Y_{IV}(2k) = \left[W(k) - T(k)\right] \quad \text{for } k = 1, 2, ..., \frac{N}{2} - 1. \quad (41)$$

Similarly, we obtain

$$W(k) + T(k) = \sum_{n=1}^{N/2} \left\{ x(n) \sin\left[\frac{(2n-1)(4k+1)\pi}{4N}\right] + x(N+1-n)\cos\left[\frac{(2n-1)(4k+1)\pi}{4N}\right]\right\} \quad (42)$$

From (35) and (42), we get

$$Y_{IV}(2k+1) = \left[W(k) + T(k)\right] \quad \text{for } k = 1, 2, ..., \frac{N}{2} - 1. \quad (43)$$

Putting $k = 0$ in (35), we have

$$Y_{IV}(1) = \sum_{n=1}^{N/2} \left\{ x(n) \sin\left[\frac{(2n-1)\pi}{4N}\right] + x(N+1-n)\cos\left[\frac{(2n-1)\pi}{4N}\right]\right\} \quad (44)$$

Using (36) in (44), we obtain

$$Y_{IV}(1) = \sum_{n=1}^{N/2} u(n) \quad (45)$$

For $k = 0$, (38) can be expressed as

$$T(0) = \sum_{n=1}^{N/2} u(n) \quad (46)$$

From (45) and (46), we get

$$Y_{IV}(1) = T(0) \quad (47)$$

Putting $k = N/2$ in (39), we have

$$W\left(\frac{N}{2}\right) = \sum_{n=1}^{N/2} v(n) \sin\left[\frac{(2n-1)\pi}{2}\right]$$

Using the value of $v(n)$ from (37) in the above expression, we obtain

$$W\left(\frac{N}{2}\right) = \sum_{n=1}^{N/2} \left\{ x(n) \cos\left[\frac{(2n-1)\pi}{4N}\right] - x(N+1-n)\sin\left[\frac{(2n-1)\pi}{4N}\right]\right\} \sin\left[\frac{(2n-1)\pi}{2}\right] \quad (48)$$

Putting $k = N/2$ in (34), we get

$$Y_{IV}(N) = \sum_{n=1}^{N/2} \left\{ x(n) \sin\left[\frac{(2n-1)(2N-1)\pi}{4N}\right] - x(N+1-n)\cos\left[\frac{(2n-1)(2N-1)\pi}{4N}\right]\right\} \quad (49)$$

Taking a simple example for $N = 4$, it can easily be proved from (48) and (49) that

$$Y_{IV}(N) = W\left(\frac{N}{2}\right) \quad (50)$$

$T(k)$ in (38) and $W(k)$ in (39) can be computed using $u(n)$ and $v(n)$ given in (36) and (37) respectively. Then the even and odd components of DST-IV of length $N = 2^m (m \geq 2)$ can be realized using (41),(43),(47) and (50) as shown in the block diagram of Fig. 3.

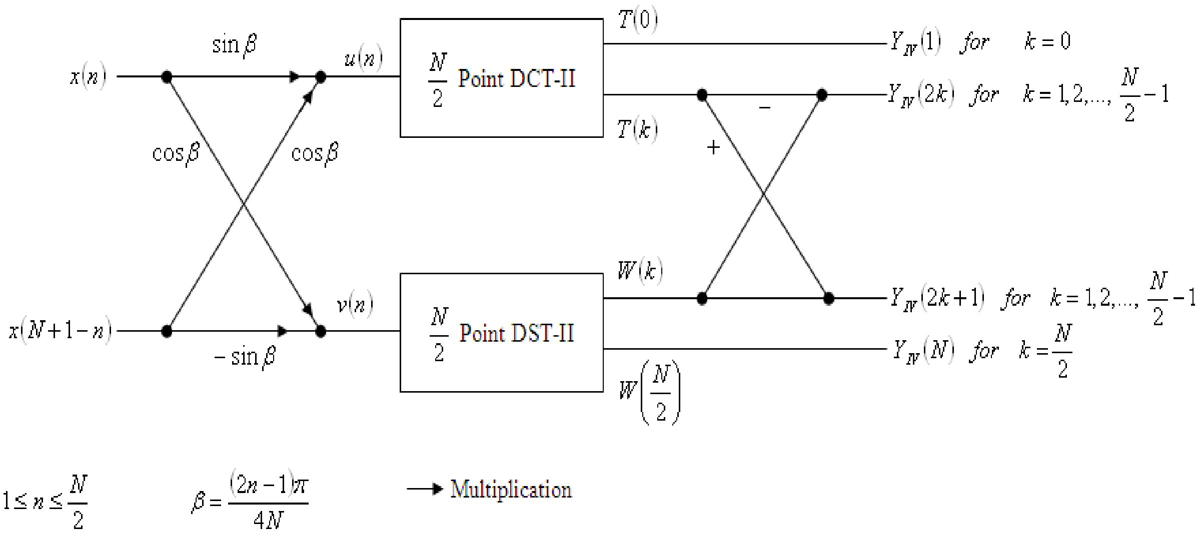

Figure 3. Block diagram of realization of DST-IV of length $N = 2^m (m \geq 2)$

V. CONCLUSION

Radix-2 algorithms for computing type-II DST and type-IV DST, each of length N = 2^m (m=2,3,….), are presented in this paper. In the proposed method for DST-II, the odd-indexed output components are realized using simple recursive relations. The recursive structures require less memory and are suitable for parallel VLSI implementation. Signal flow graph for realization of DST-II of length N = 2^3 is given. The DST-IV of length N is computed using DST-II and DCT-II sequences, each of length N/2. A block diagram for computation of radix-2 DST-IV algorithm is shown.

REFERENCES

[1] N.Ahmed, T.Natarajan, and K.R.Rao, Discrete cosine transform, IEEE Trans. Comput., vol. C-23, 1974, 90-93.
[2] R.K.Rao and P.Yip, Discrete cosine transform: algorithm, advantages, and applications (New York: Academic, 1990).
[3] M.J.Narasimha and A.M.Peterson, On the computation of the discrete cosine transform, IEEE Trans. Communications, vol.COM-26,no.6, June 1978, 934-936.
[4] H.S.Hou, A fast recursive algorithms for computing the discrete cosisine transform, IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-35(10), Oct.1987, 1455-1461.
[5] PeiZong Lee and Fang-Yu Huang, Reconstructed recursive DCT and DST algorithms, IEEE Trans. Signal Processing, vol.42, no.7, Jul.1994, 1600-1609.
[6] P.Duhamel and C.Guillemot, Polynomial transform computation of 2-D DCT ,in Proc. ICASSP'90, Apr.1990, 1515-1518.
[7] E.Feig and S.Winograd, Fast algorithms for the discrete cosine transform, IEEE Trans. Signal Processing, vol.40, no.9, Sept.1992, 2174-2193.
[8] A.K. Jain, A fast Karhunen-Loeve transform for a class of random processes, IEEE Trans. Commun., vol. COM-24, September 1976, 1023-1029.
[9] H.B. Kekre and J.K. Solanka, Comparative performance of various trigonometric unitary transforms for transform image coding, Int. J. Electron., vol. 44, 1978, 305-315.
[10] A.K. Jain, A sinusoidal family of unitary transforms, IEEE Trans. Patt. Anal. Machine Intell., vol. PAMI-I, September 1979,356-365.
[11] Z. Wang and B. Hunt, The discrete W transform, Applied Math Computat., vol. 16, January 1985, 19- 48.
[12] S. Cheng, Application of the sine transform method in time of flight positron emission image reconstruction algorithms,IEEE Trans. BIOMED. Eng., vol. BME-32, March 1985,185-192.
[13] K. Rose, A. Heiman, and I. Dinstein, DCT/DST alternate transform image coding, Proc. GLOBE COM 87, vol. I, November 1987, 426-430.
[14] J.L. Wang and Z.Q. Ding, Discrete sine transform domain LMS adaptive filtering, Proc. Int. Conf. Acoust., Speech, Signal Processing, 1985, 260-263.
[15] Z. Wang and L. Wang, Interpolation using the fast discrete sine transform, Signal Processing, vol. 26, 1992, 131-137.
[16] K. Jain, Fundamentals of Digital Image Processing (Englewood Cliffs, NJ: Prentice - Hall, 1989)
[17] P. Yip and K.R. Rao, on the computation and the effectiveness of discrete sine transform, Comput. Electron, vol. 7, 1980, 45-55.