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Feedback Linearization Controller Of The Delta WingRock Phenomena

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ABSTRACT
This project deals with the control of the wing rock phenomena of a delta wing aircraft. a control schemeis proposed to stabilize the system. The controlleris a feedback linearization controller. It is shown that the proposed control scheme guarantee the asymptotic convergence to zero of all the states of the system. To illustrate the performance of the proposed controller, simulation results are presented and discussed. It is found that the proposed control scheme work well for the wing rock phenomena of a delta wing aircraft.
Key words: Wing Rock, Nonlinear Control of Wing Rock, Feedback Control

I. INTRODUCTION

Wing rock phenomena is a limit-cycle rolling motion by flight aircrafts with small aspect-ratio wings, or with long pointed forebodies at angles of attack [1]. thewing rock phenomenais studied by many researchers, because of its importance in the stability of an aircraft during attack maneuvers. It is also reported in [6] that the such phenomena does not have a limit cycle can happen at an 80/65 degree double delta wing.

It is a nonlinear phenomenon experienced by aircraft in which oscillations and unstable behavior is experienced [9]. This problem may affect flight effectiveness or even present a serious damage due to potential instability of the aircraft [1]. This phenomenais extensively studied experimentally, giving in mathematical models that describe the nonlinear rolling motion using simple differential equations as in [7],[8].

The wing rock model for a wing aircraft used in [1] is considered in this project. Wing rock is modeled as a self-induced, rolling motion, which causes rolling moment to be a nonlinear function of the roll angle ϕ and the roll-rate p. The coefficients of such nonlinear function is obtained by curve fitting with experimental data at specific values of angle of muvering. In addition, yawing dynamicis added to the model by adding the yawing rate r = - ($\partial\beta/\partial t$) and ignoring the nonlinear term β due to its small value compared with the other terms

II. MATHIMATICAL MODEL OF THE PHENOMENA

The transformation $z = T(x)$ is defined such that:

$$z_1 = N_p x_1 + N_r x_4 + x_5$$

$$z_2 = -N_\beta x_4$$

$$z_3 = -N_\beta x_5$$

$$z_4 = N_\beta (N_p x_2 + N_\beta x_4 + N_r x_5) \quad \textbf{(2.6)}$$

$$z_5 = N_\beta N_p (-\omega^2 x_1(t) + \mu_1 x_2(t) + \mu_2 x_1{}^2(t) x_2(t) + b_1 x_2{}^3(t) + b_2 x_1(t) x_2{}^2(t)$$

$$+ L_\delta x_3(t) + L_\beta x_4(t) - L_r x_5(t)) + N_\beta^2 x_5 - N_\beta N_r (N_p x_2 + N_\beta x_4 + N_r x_5)$$

The inverse transformation $x = T^{-1}(z)$ exists and it is as follows.

$$x_1 = \frac{1}{N_p}(z_1 + \frac{N_r}{N_\beta} z_2 + \frac{1}{N_\beta} z_3)$$

$$x_2 = \frac{1}{N_\beta N_p}(z_4 + N_\beta z_2 + N_r z_3)$$

$$x_3 = \frac{1}{L_\delta}[z_5 + (\omega^2 N_\beta - \frac{b_2}{N_\beta N_p^2}(z_4 + N_\beta z_2 + N_r z_3)^2)(z_1 + \frac{N_r}{N_\beta} z_2 + \frac{1}{N_\beta} z_3)]$$
$$-\frac{1}{L_\delta}[(\mu_1 + \frac{\mu_2}{N_p^2}(z_1 + \frac{N_r}{N_\beta} z_2 + \frac{1}{N_\beta} z_3)^2 + \frac{b_1}{N_\beta^2 N_p^2}(z_4 + N_\beta z_2 + N_r z_3)^2)(z_4 + N_\beta z_2 + N_r z_3)]$$
$$+\frac{1}{L_\delta}[N_p L_\beta z_2 - N_p L_r z_3 + N_\beta z_3 + N_r z_4]$$

$$x_4 = -\frac{1}{N_\beta} z_2$$

(2.7)

$$x_5 = -\frac{1}{N_\beta} z_3$$

Hence, the dynamic model of the wing rock phenomenon can be written as,

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = z_3$$

$$\dot{z}_3 = z_4 \qquad \textbf{(2.8)}$$

$$\dot{z}_4 = z_5$$

$$\dot{z}_5 = q(x) + g(x)u$$

where:

$$q(x) = [N_\beta N_p(-\omega^2 x_1(t) + \mu_1 x_2(t) + \mu_2 x_1{}^2(t) x_2(t) + b_1 x_2{}^3(t) + b_2 x_1(t) x_2{}^2(t) + +L_\beta x_4(t) - L_r x_5(t))$$
$$+N_\beta^2 x_5 - N_\beta N_r(N_p x_2 + N_\beta x_4 + N_r x_5)] - L_\delta k\, x_3(t)$$

$$g(x) = kN_\beta N_p L_\delta$$

III. FEEDBACK LINEARIZATION CONTROLLER FOR THE WING ROCK PHENOMENON

3.1 Design of the Controller

Recall from the previous chapter that the wing rock phenomenon can be described using the following set of differential equations:

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$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -\omega^2 x_1(t) + \mu_1 x_2(t) + \mu_2 x_1^2(t) x_2(t) + b_1 x_2^3(t) + b_2 x_1(t) x_2^2(t) + L_\delta x_3(t) + L_\beta x_4(t) - L_r x_5(t)$$
$$\dot{x}_3(t) = -k x_3(t) + ku \quad \dot{x}_4(t) = x_5(t)$$
$$\dot{x}_5(t) = -N_p x_2(t) - N_\beta x_4(t) - N_r x_5(t) \quad \textbf{(3.1)}$$

Using the transformation defined in chapter 2, the above equations can be written as:

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = z_3$$
$$\dot{z}_3 = z_4 \quad \textbf{(3.2)}$$
$$\dot{z}_4 = z_5$$
$$\dot{z}_5 = q(x) + g(x)u$$

With,

$$q(x) = [N_\beta N_p(-\omega^2 x_1(t) + \mu_1 x_2(t) + \mu_2 x_1^2(t) x_2(t) + b_1 x_2^3(t) + b_2 x_1(t) x_2^2(t) + + L_\beta x_4(t) - L_r x_5(t)) + N_\beta^2 x_5 - N_\beta N_r (N_p x_2 + N_\beta x_4 + N_r x_5)] - L_\delta k\, x_3(t)$$

$$g(x) = k N_\beta N_p L_\delta$$

In this chapter, a feedback linearization controller will be designed to control the wing rock phenomenon.

Let the scalars $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ be chosen such that the polynomial $P_1(s) = s^5 - \alpha_5 s^4 - \alpha_4 s^3 - \alpha_3 s^2 - \alpha_2 s - \alpha_1$ is a Hurwitz polynomial (i.e. , the roots of $P_1(s) = 0$ are located in the left half plane).

Proposition 3.1:
The feedback linearization controller

$$u = \frac{1}{g(x)}[-q(x) + \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3 + \alpha_4 z_4 + \alpha_5 z_5] \quad \textbf{(3.3)}$$

guarantees the asyptotic convergence of z_1, z_2, z_3, z_4, z_5 to zero as $t \to \infty$.

Proof:
The closed loop system when the controller (3.3) is used is as follows:

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = z_3$$

$$\dot{z}_3 = z_4 \qquad \textbf{(3.4)}$$

$$\dot{z}_4 = z_5$$

$$\dot{z}_5 = \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3 + \alpha_4 z_4 + \alpha_5 z_5$$

Define the vector z such that $z = [z_1 \quad z_2 \quad z_3 \quad z_4 \quad z_5]^T$.
The closed loop system given by (3.4) can be written in compact form as:

$$\dot{z} = A_z z \qquad \textbf{(3.5)}$$

where the matrix A_z is such that:

$$\therefore A_z = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \end{bmatrix}$$

The solution of the differential equation given by (3.5) is $z(t) = \exp(A_z t) z(0)$. Since the matrix A_z is a stable matrix, the vector *z(t)* will converge to zero asymptotically as $t \to \infty$.

Remark
The feedback linearization controller

$$u = \frac{1}{g(x)}[-q(x) + \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3 + \alpha_4 z_4 + \alpha_5 z_5]$$

can be written in the original coordinates by using the transformation:

$$z_1 = N_p x_1 + N_r x_4 + x_5$$

$$z_2 = -N_\beta x_4$$

$$z_3 = -N_\beta x_5$$

$$z_4 = N_\beta (N_p x_2 + N_\beta x_4 + N_r x_5)$$

$$z_5 = N_\beta N_p (-\omega^2 x_1(t) + \mu_1 x_2(t) + \mu_2 x_1^2(t) x_2(t) + b_1 x_2^3(t) + b_2 x_1(t) x_2^2(t)$$

$$+ L_\delta x_3(t) + L_\beta x_4(t) - L_r x_5(t)) + N_\beta^2 x_5 - N_\beta N_r (N_p x_2 + N_\beta x_4 + N_r x_5)$$

(3.6)

3.2 Simulation results

The poles of $P_1(s) = 0$ are chosen to be -1, -2, -3, -4, -5, then

$$P_1(s) = s^5 - \alpha_1 s^4 - \alpha_2 s^3 - \alpha_3 s^2 - \alpha_4 s^1 - \alpha_5$$

$$= (s+1)(s+2)(s+3)(s+4)(s+5)$$

$$s^5 - \alpha_1 s^4 - \alpha_2 s^3 - \alpha_3 s^2 - \alpha_4 s^1 - \alpha_5 = s^5 + 15s^4 + 85s^3 + 225s^2 + 274s^1 + 120$$

Hence,

$$\alpha_1 = -15, \alpha_2 = -85, \alpha_3 = -225, \alpha_4 = -274, \alpha_5 = -120$$

The performance of the closed loop system is simulated using the MATLAB software and the results are plotted for the states with initial values = [0.2 0 0 0 0]. Figure 8 – Figure 12 show the plots of $\phi, p, \delta, \beta, \frac{\partial \beta}{\partial t}$ respectively.

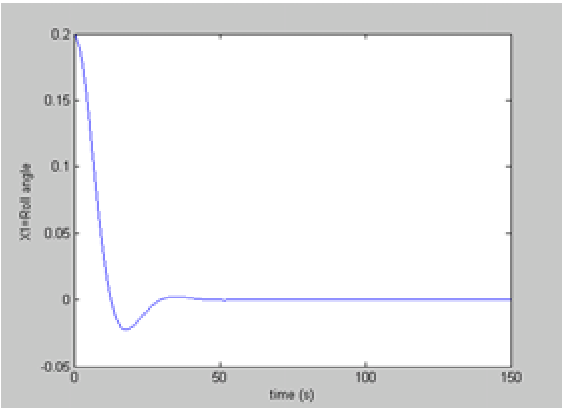

Figure 8 Roll angle (rad.)

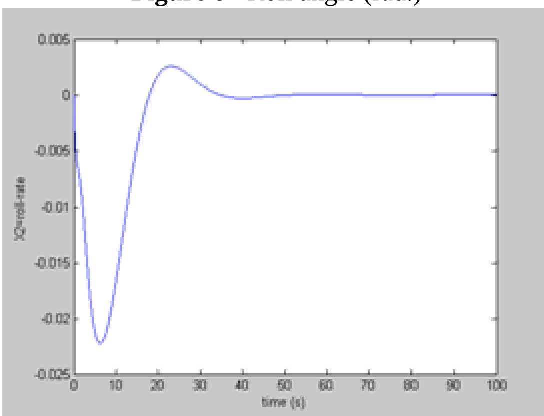

Figure 9 Roll-rate (rad/sec)

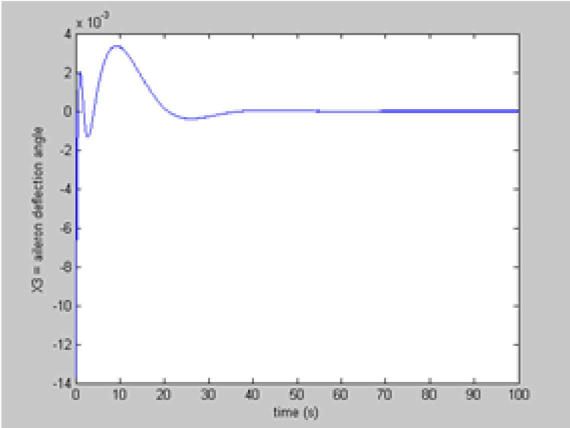

Figure 10 Aileron deflection angle (rad.)

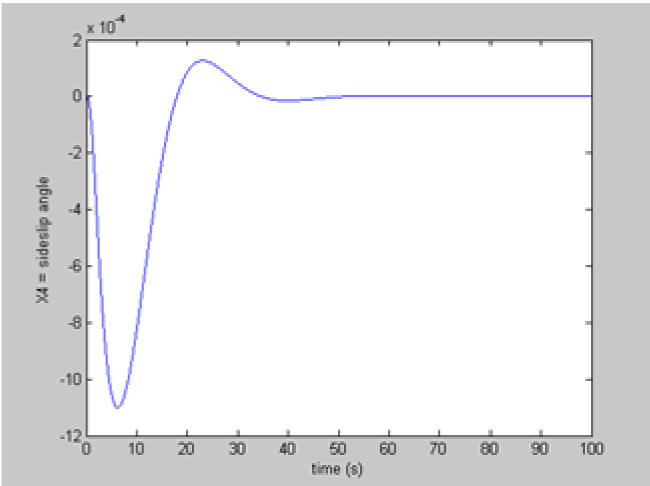

Figure 11 sideslip angle (rad)

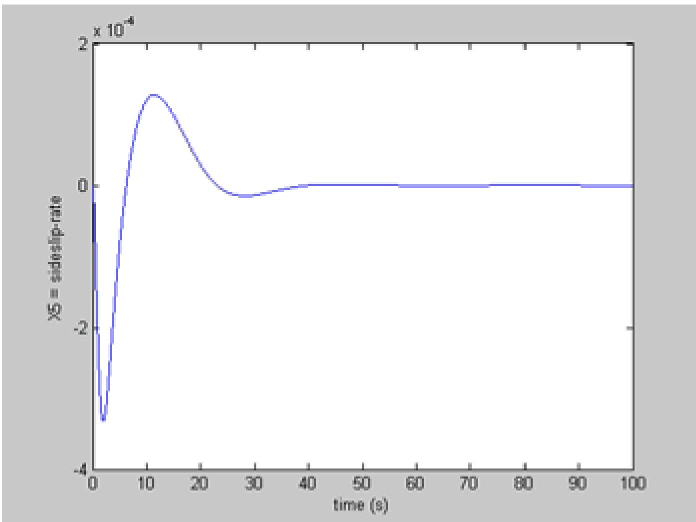

Figure 12 sideslip rate (rad/sec)

IV. CONCLUSION

The figures show that all the states converge to zero asymptotically. Hence, it can be concluded that the proposed feedback linearization controller works well for the stabilization of the wing rock phenomenon.

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