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A Stochastic Model by the Fourier Transform of Pde for the Glp - 1

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Abstract
The peptide hormone glucagon like peptide GLP -1 has most important actions resulting in glucose lowering along with weight loss in patients with type 2 diabetes. As a peptide hormone, GLP -1 has to be administered by injection. A few small-molecule agonists to peptide hormone receptors have been described.
Here we develop a model for credit risk based on a model with stochastic eigen values called principal component stochastic covariance. The fourier transform of the Green function of the pricing PDE is,

$$B(\psi,k)=\delta.\frac{1-e^{\Psi\xi}}{k(1+\lambda)+k(-1+\lambda)e^{-\psi\xi}}$$

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I. INTRODUCTION

GLP-1 is one of the incretine, is a natural postprandial hormone released in response to nutrient intake and acts to stimulate insulin secretion. GLP -1 has attracted much interest as a future treatment for type -2 diabetes because it has multiple antidiabetic actions and at the same time, lowers body weight. The compounds were not antagonized by the selective GLP-1 receptor antagonist, exendin. Exendin is a fragment of a close analog of GLP-1 and must be expected to bind at the orthosteric agonist binding site.

The mechanism behind the phenomenon was investigated further in a saturation binding experiment measuring the affinity and number of binding sites for GLP-1 in the absence or presence of compound 2.

II. NOTATIONS

D_i → Symmetric positive definite matrix
W_i → Brownian motion
r(t) → Risk free interest rate
$d_i(t)$ → Dividend yield for the i^{th} firm
Tr → Trace of a matrix
η_i → Stopping time
$\|_{(\eta_i>t)}$ → Truncating factor

III. THE CREDIT GRADES WISHART PROCESS:

In Credit Grades model, we assumes that volatility is deterministic. By means of stochastic covariance wishart process, we extend credit grades model. A wishart process with integer degree of freedom k is a sum of k independent n-dimensional Ornstein –uhlenbeck process.

Consider $\{V^k\}_{k=1}^{k}$ as an independent set of Ornstein – Uhlenbeck process.

$$dV_t^{(k)}=BV_t^{(k)}dt+RdW_t^{(k)}$$

Where B and R are (n,n) matrices with R invertible[1,2].
Then a wishart process of degree k is defined as

$$Y_t = \sum_{k=1}^{k} V_t^{(k)} V_t^{(k)'} ,$$

Where $V_t^{(k)}$ is the transpose of the Vector $V_t^{(k)}$.

Here By using a Ito's lemma, to find a diffusion SDE for the process Y_t

$$dY_t = \sum_{k=1}^{K(k)} \left[dV_t^{(k)} V_t^{(k)'} + V_t^{(k)}.dV_t^{(k)'} + dV_t^{(k)}.dV_t^{(k)'} \right]$$

$$= (KR'R + BY_t + Y_t B')dt + \sum_{k=1}^{k} \left[RdW_t^{(k)} V_t^{(k)'} + V_t^{(k)} dW_t^{(k)'} R' \right]$$

Here the term of the SDE contains Y_t, but the diffusion part contains the terms $V_t^{(k)}$ and $V_t^{(k)'}$. Also Y_t satisfies the following matrix SDE.,[3]

$$dY_t = \left(KR'R + BY_t + Y_t B' \right) dt + RdW_t Y_t^{1/2} + Y_t^{1/2} dW_t R' , \qquad \text{.......(1)}$$

Where W_t is an n x n standard Brownian motion matrix.

IV. THE DYNAMICS OF THE ASSETS :

The assets are defined on a probability space (Ω, F, R) Where $\{F_t\}_{t \geq 0}$ is the information up to time t and R is the risk – natural measure equivalent to the measure P.
Let us assume that the i[th] firm's asset price per share is given by $A_i(t)$.
Here we review the results regarding the dynamics of the assets with stochastic covariance wishart process.
Assume the asset's prices follow the multivariate real – world model,

$$d \ln A_t = \left(\mu_i + (Tr(D_i Y_t)) \right) dt + Y_t^{1/2} dW_t^A$$

$$dY_t = \Omega\Omega' + MY_t + Y_t M_t^{\sigma} Y_t^{1/2} + Y_t^{1/2} dw^{\sigma} q'$$

$$dY_t = (\Omega\Omega' + MY_t + Y_t M')dt + RdW_t^{\sigma} Y_t^{1/2} dw_{\sigma} R' \qquad \text{......(2)}$$

Here the vector $\mu = (\mu_1 ....\mu_n)$ is constant and D_i is a symmetric positive definite matrix, the log price process has the drift term,
$E_t(d \ln A_t) = \left(\mu_i + \left(T_\gamma (D_i Y_t) \right) \right) dt$ and the Quadratic variation $V_t(d \ln A_t) = Y_t dt$

Also we assume that the Brownian motion driving the assets and the Brownian motions driving the wishart proves are uncorrelated.
$Tr(D_i Y_t) > 0$ accounts for risk premium. For the transition distribution of A_{t+h} given A_t and (Y_t) We have

$$\ln A_{t+h} / A_t, (Y_t) \sim N \left(\ln A_t + \int_t^{t+h} \mu + Tr(D_i Y_u) du, \int_t^{t+h} Y_u du \right),$$

and the unconditional probability function can be found by Integration over the distribution function of $\int_t^{t+h} Y_u du$

Now that we have identified the dynamics of the assets, we explain the mechanism of the Credit Grades model, As before, we assume that the ith firm's value $A_t(t)$ is driven by the dynamics

$$\begin{cases} dA_i(t) = diag(A_i(t))[r(t) - d_i(t))Idt + \sqrt{Y_t} dW_t] \\ dY_t = (\Omega\Omega + MY_t + Y_t M')dt + \sqrt{Y_t} dZ_t R + R_t'' \sqrt{Y_t} \end{cases} \qquad \text{....... (3)}$$

Where $\Omega\Omega' = \beta R'R'$ for some $\beta > n-1$ and M is a negative definite matrix.

V. EQUITY CALL OPTIONS

The price of a European call option on the equity is calculated by discounting the risk – neutral expectation of the payoff at maturity.

Since $\left(S_{i,T} \, ||_{\{\eta_i > T\}} - K\right)^+ = \left(S_{i,T} - K\right)^+ \, ||_{\{\eta_i > T\}}$ the price of the call option could be rewritten as V_{call} (t, Y, S_i, k)

$$= E^Q_{(t,Y,S_i)}\left(\exp\left(-\int_t^T r(s)ds \right)\left(S_{i,T} \, \|_{\eta_i > T} - K\right)^+ \right)$$

$$= E^Q_{(t,Y,S_i)}\left(\exp\left(-\int_t^T r(s)ds \right)(S_{i,T} - K)^+ \, \|_{\eta_i > T} \right)$$

The price of a single name derivative on one of the equities satisfies the partial differential equation[4,5].

$$\mathbf{U_t} + A_{(Y,S_i)} - rU = 0$$

Here the price of an equity call option is given by the PDE is

$$U_t + \frac{1}{2} Y_{ii} \left(S_i + D_i(t)\right)^2 U_{S_i S_i} + \left(r(t) - d_i(t)\right) S_i V_{s_i} + A_Y U - rU = 0 \quad \ldots\ldots (4)$$

$$U(t,0) = 0, \ U(T,S) = (S-k)^+,$$

where $A_{(Y,S)}$ is the infinitesimal generator of the joint process (S, Y) .

We first joint change the variable by

$$x_i = \ln\left(\frac{S_i + D_i(t)}{D_i(t)} \right) \quad a_i = \ln\left(\frac{D_i(t) + k}{D_i(t)} \right) \text{ and}$$

G(t,x_i) = exp ($\int_t^T r(s)ds \, \frac{U(t,S_i)}{D_i(T)}$ to transform the PDE to

$$G_t + \frac{1}{2} Y_{ii} \left(Gx_i x_i - Gx_i \right) + A_Y G = 0,$$
$$G(t,0) = 0, U(T, x_i) = \left(e^{xi} - e^{ai}\right)^+ \quad \ldots..(5)$$

The Fourier transform of the Green's function of PDE is given by[6]

$$q_j(\psi, Y, X) = \int_{-\infty}^{\infty} e^{iky_j + A(\psi,k) + T_r(B(\psi,k)Y} dk, \quad \ldots..(6)$$

Where, $B(\psi,k) = \left(\Lambda_{22}(\psi,k)\right)^{-1} \left(\Lambda_{21}(\psi,k)\right)$

$$A(\psi,k) = Tr\left(RR' \int_0^{\psi} B(u,k)du \right)$$

With $\Lambda = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{pmatrix}$

$$= \exp\psi \begin{pmatrix} M & -2R'R \\ -\frac{1}{2}\left(k^2 + \frac{1}{4}\right) I & -M' \end{pmatrix}$$

Now that we have found the fourier transform of the Green's function of the pricing PDE, we solve the pricing problem for an equity call option by the method of images.

VI. RESULT

The price of a call option on $S_j(t)$ with maturity date T and strike price k is given by,

$$V(t,S_j)= \left(D(T)+K\right)\exp\left(-\int_t^T r(s)ds\right)Z(\psi,y)$$

$$y = \ln(S+D(t)) - \ln(D(t)+K)+ \int_t^T (r(s)-d(s))ds$$

$$b = \ln(D(t)) - \ln(D(t)+K)+ \int_t^T (r(s)-d(s))ds$$

and the function z is defined by

$$Z(\psi,y)=e^y - e^b - \frac{e^{\sqrt{y}}}{\pi}\int_0^\infty \frac{e^{A(\psi,k)+T_r(B(\psi,k)Y)}(\cos(yk)-\cos((y-2b)k))}{\frac{\sqrt{\delta}}{2}}ds \quad \text{....(7)}$$

If in the dynamics of the asset (3), We assume n=1 and for the parameters $M=-\frac{k}{2}$, $R=\frac{\sigma}{2}$ and $\Omega\Omega = k\theta$, we have $B(\psi,k) = \frac{\Lambda_{12}}{\Lambda_{22}}$

Now to find Λ_{12} and Λ_{22}, we have[8,9,10],

$$E=\begin{bmatrix} -\frac{k}{2} & -\frac{1}{2}(\delta^2+\frac{1}{4}) \\ -\frac{\sigma^2}{2} & -\frac{k}{2} \end{bmatrix}$$

$\Lambda = e^{\psi E}$ is a 2 x2 matrix with

$$\Lambda_{12}(\psi,k)=\frac{(k^2-\xi^2)\left(-e^{\frac{\psi\xi}{2}}+e^{-\frac{4\xi}{2}}\right)}{-2\sigma^2\xi}$$

$$\Lambda_{22}(\psi,k)=\frac{\sigma^2(-(k+\xi)e^{\frac{\psi\xi}{2}}+(k-\xi)e^{-\frac{\psi\xi}{2}}}{-2\sigma^2\xi}$$

$$\xi=\sqrt{k^2+\sigma^2\delta}$$

Therefore $B(\psi,k)$ = e $= \delta\frac{1-e^{-\psi\xi}}{k(1+\lambda)+k(-1+\lambda)e^{-\psi\xi}}$ …… (8)

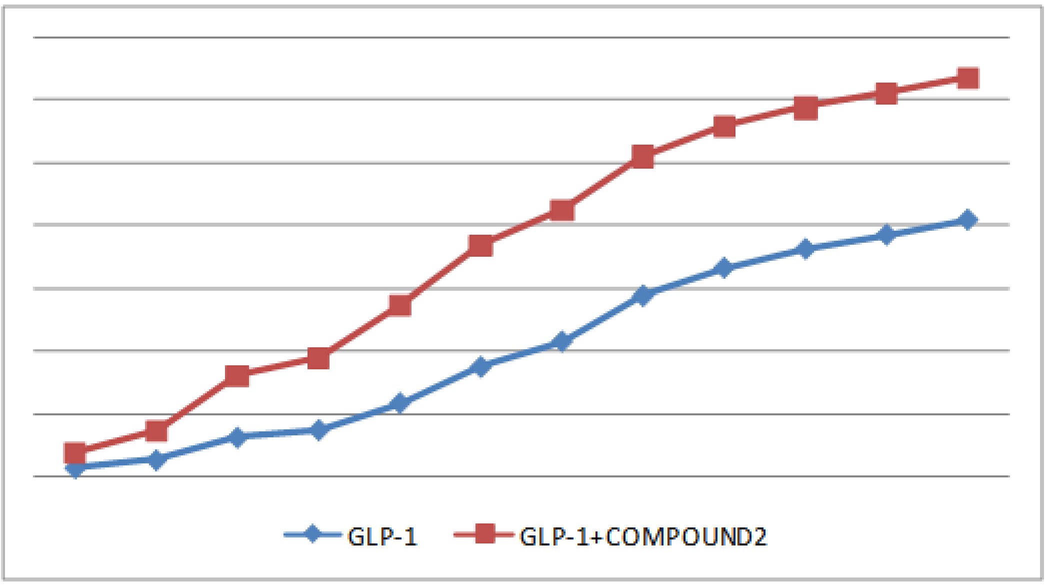

Fig. 1

Example

As per the data the saturation plot and scratched analysis of GLP-1 radio ligand binding to the cloned human[7]GLP1 receptor in the absence or presence of compound 2.

In the saturation plot the presence of compound 2 in GLP-1 is increased then GLP -1 alone is given in fig 2

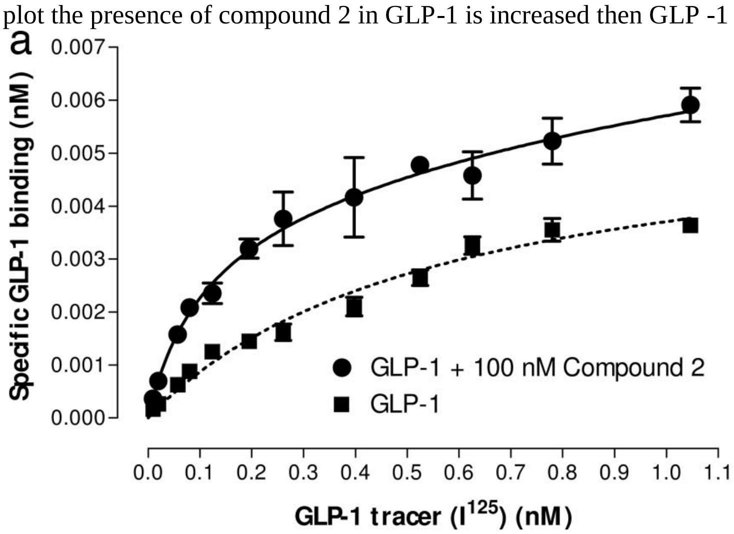

Fig. 2

VII. CONCLUSION

The mathematical model also reflects the same effects of GLP-1 receptor in the absence or presence of compound 2 in fig 2 which are beautifully fitted with fourier transform of the Greens function of PDE is obtained in fig (1). The results matching with the mathwematical and medical report.

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