RESEARCH ARTICLE OPEN ACCESS

Analytical Development of the Forward and Inverse Kinematics of A Robotic Leg Biologically-Inspired In Insect

Gabriel Duarte Gonçalves Pedro*, Pedro Américo Almeida Magalhães Junior,
Pontifícia Universidade Católica de Minas Gerais.

Abstract
This article proposes an analytical development for the inverseand forward kinematics of a robotic leg for robots biologically-inspired in insects. The forward and inverse kinematics is a part of the solution to models of path planning for robotic legs. The method Denavit-Hartenberg was used for the development of forward kinematics. The inverse kinematics was found using the method of inverse transformation matrices. The developed equations' results were compared tothe results of functions fkine and ikine of the robotics to olbox for Matlab, analyzing numerical and analytic calolution for the inverse kinematics.

I. INTRODUCTION

Robotics has had a special demand, especially after recent events such as the nuclear accident in Fukushima and incentives as DARPA Robotics Challenge, aimed at developing semi-autonomous ground robots dedicated to complex tasks in degraded and dangerous environments. Many of these researched robots own configurations with handlers to interact with the environment and / or locomotion (locomotion by Robots with legs).

Mobile robots, regardless of their mean of locomotion, one can say that are used in unhealthy environments, also known as 3-Ds (Dirty, Dangerous, Difficulty). They are fundamentally vehicles capable of replacing humans in order to avoid endangering life, in all types of dangerous work that requires strong security measures or in areas which humans cannot easily access [1].

The variety of animals that move using legs is immense and their locomotion capability in different terrains and in different conditions is fascinating [1]. In this context, bionics, which has the principle of building robots inspired by biological systems (biologically-inspired robots), has inspired robotics research and the development of mobile machinery adapted to different environments (structured and unstructured).

This article proposes an analytical development for the inverse and forward kinematics of a robotic leg for biologically-inspired robots in insects in which the forward and inverse kinematics is a part of the solution to models of path planning of the feet of the robotic leg.

The method Denavit - Hartenberg [2][3][4][5] was used to find the model of forward kinematics. The inverse kinematics was developed analytically using the method of inverse transformation matrices proposed by [4] [5] [6]. The developed equations` results were compared to the results of functions fkine and ikine of the robotics to olbox for Matlab[7], analyzing numerical and analytical solution for inverse kinematics.

An analysis of biological in spiration for the development of a robotic leg for a robot insect is presented in topic II. In topic III the forward kine matice quations are developed. In topic IV the inverse kinematicse quations are developed. Topic V compared results obtained using analytical equations and functions of the Robotics Toolbox for five points in the work space of the robotic leg and on topic VI a conclusion is reached by analyzing the results and comparing the analytical and numerical method for the solution of inverse kinematics.

II. BIOLOGICAL INSPIRATION

The insects are some of the most successful creatures on Earth, being found in many different terrestrial environments.

The configuration of the presented legisins pired by the legs of insects seeking advantages in locomotion per leginun structure denvironments. Many of the serobots have six legs and some were inspired by the stick in sectand the cockroach [10] [11].The legs of an insect are composed of 5 parts which are: Tarsus, Tibia, Femur, Trochanter and Coxa, Fig 1, and has 5 degrees of freedom, however, reproducing a robotic leg with 5 degrees of freedom increases the cost of its mechanism and control. For the theoretical perspective, three degrees of freedom has been shown to lower number of degrees of freedom necessary for the robot to have an omnidirectional walk [8] apud [9]. As an eventual simplification, Tarsus and trochanter are suppressed because of their size and function. Consequently the leg comprises three segments, Coxa, Femur and Tibia [8].

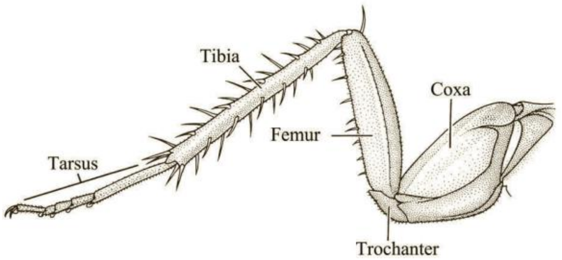

Figure 1. Anatomical structure of a typical insect leg [8] apud [9].

Once the format of the legs is defined, we can think of robots that can use this type of leg, having four or more legs. Sometimes the increase in the number of legsis not interesting, because it increases the complexity of the control and energy consumption, since each joint will have an actuation system. However with the increase on the number of legs brings the guarantee of static balance for the robot, asfor the step of one or more legs, there always will be three or more legs supporting the body. A quadruped robot shows an interesting number of legs, because it has the minimum required quantity for the robot to walk on static balance, and also allows it to walk in dynamic balance when thinking about a walk moving two legs at same time.Most insects have six legs, however , if it's intended to save energy a robot with insect type configuration can have four legs so having a reduction from six actuators having an energy saving, and still being possible for a walk in static balance although an insect with these features does not exist in the wild. We can see an example of the configuration of this robot in figure 2.

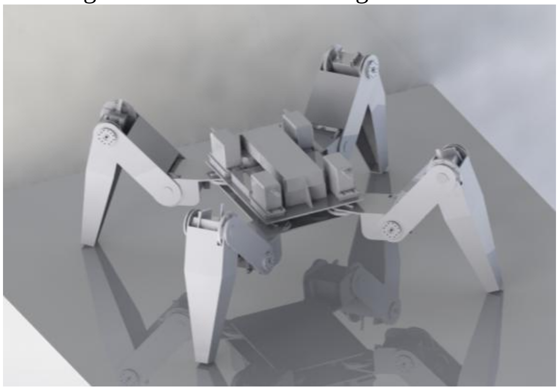

Figure 2. Four leg robot with insect inspiration

III. FORWARDKINEMATICS

Considering that the robot is in static balance, the leg that will take a step will have its kinematic model similar to a robotic manipulator, so the method of Denavit–Hartenberg was used.

Having defined the segments of the robot leg similar toan insect leg, Coxa, Femur and Tibia,the guidelinesof the Denavit-Hartenberg algorithm werefollowed according to [2] ,[3] , [4] and [5], given the position of each coordinate axis of the joints of robotic legs (Figure 3) where there are three rotational joints (J1, J2, and J3) and L1, L2 and L3 links.

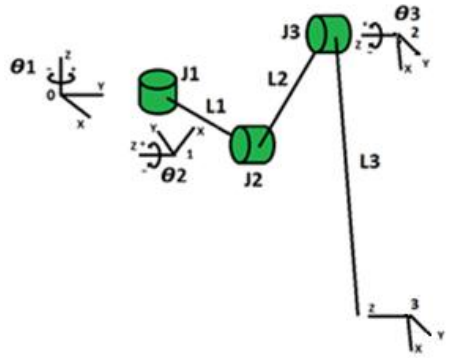

Figure 3. Position of the coordinate axes 0, 1, 2 and 3.

Following the method of Denavit - Hartenberg, the robotic leg is placed in zero position which corresponds to the position where all the angles of the joints are equal to zero and all directions "x" are aligned (Figure 4).

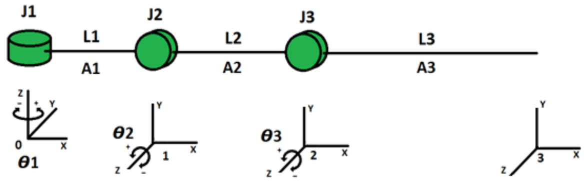

Figure 4. Zero position of Robotics Leg.

From the zero position of the robotic leg, homogeneous transformation matrices were found: A1, A2 and A3 (1, 2 and 3) using the parameters of the Denavit-Hartenberg method (Table 1). The following matrices were obtained:

TABLE 1.PARAMETERSOF DENAVIT-HATEMBERG

Link	θ (°)	α (°)	L (m)	D (m)
1	θ 1	90	L1	0
2	θ 2	0	L2	0
3	θ 3	0	L3	0

$$A1 = \begin{bmatrix} \cos(\theta 1) & 0 & \sin(\theta 1) & L1 * \cos(\theta 1) \\ \sin(\theta 1) & 0 & -\cos(\theta 1) & L1 * \sin(\theta 1) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$A2 = \begin{bmatrix} \cos(\theta 2) & -\sin(\theta 2) & 0 & L2 * \cos(\theta 2) \\ \sin(\theta 2) & \cos(\theta 2) & 0 & L2 * \sin(\theta 2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$A3 = \begin{bmatrix} \cos(\theta 3) & -\sin(\theta 3) & 0 & L3 * \cos(\theta 3) \\ \sin(\theta 3) & \cos(\theta 3) & 0 & L3 * \sin(\theta 3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

The global transformation matrix ${}^{0}T_3$ (4)was found by multiplying the matrices A1, A2, and A3.

$$ {}^{0}T_3 = A_1 * A_2 * A_3 \tag{4}$$

Getting a matrix (5):

$$ {}^{0}T_3 = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Where the terms "n", "s", and "a" are the unit direction vectors of the coordinate system three,and terms p_x,$p_y$and$p_z$are the terms of the position coordinate system threerelative to the coordinate system zero [4] [5],thereforep_x, $p_y$andp_z(6, 7 and 8) has the forward kinematic equations of the robot leg which are as follows:

$$ px = L1 * \cos(\theta 1) + L2 * \cos(\theta 2) * \cos(\theta 1) - L3 * \sin(\theta 3) * \sin(\theta 2) * \cos(\theta 1) + L3 * \cos(\theta 3) * \cos(\theta 2) * \cos(\theta 1) \tag{6}$$

$$ py = L1 * \sin(\theta 1) + L2 * \cos(\theta 2) * \sin(\theta 1) - L3 * \sin(\theta 3) * \sin(\theta 2) * \sin(\theta 1) + L3 * \cos(\theta 3) * \cos(\theta 2) * \sin(\theta 1) \tag{7}$$

$$ p = L2 * \sin(\theta 2) + L3 * \sin(\theta 3) * \cos(\theta 2) + L3 * \cos\theta 3 * \sin(\theta 2) \tag{8}$$

Where L1, L2 and L3 are the distances of each link; □ 1, □ 2 and □ 3 are the angles of the joints and px, py and pz is the Cartesian position X, Y and Z, respectively, of the tip of the robotic leg.

It can be seen that these equations represent the position of the tip of the robotic leg, because calculating the zero position for all angles equal to zero, the expected result is obtained (10, 11, and 12):

$$ px = L1 + L2 + L3 \tag{10}$$

$$ py = 0 \tag{11}$$

$$ pz = 0 \tag{12}$$

IV. INVERSE KINEMATICS

The forward kinematics results in direct expressions that get the position of the tip of the robotic leg given angle values while inverse kinematics seeks to determine a set of values of the angles to suit a coordinated robotic leg tip [2], [3], [4] and [5].

The inverse kinematics generally does not have an analytic solution and sometimes isn’t even a solution. Also do not have a single methodology for its solution. [5].

As the leg has three degrees of freedom it was possible to find inverse kinematics analytically using the method of the inverse transformation matrix proposed by [4], [5] and [6].

Its methodology consists in using the inverse transformation in or derto obtain expressions with an easy solution [5].

If the equation (4) is true, then the equation (13) is true too:

$$ A_1^{-1} * {}^{0}T_3 = A_2 * A_3 \tag{13}$$

Considering ${}^{0}T_3$ as the equation (5),and the inverse of A1 being (14):

$$ A_1^{-1} = \begin{bmatrix} \cos(\theta 1) & \sin(\theta 1) & 0 & -L1 \\ 0 & 0 & 1 & 0 \\ \sin(\theta 1) & -\cos(\theta 1) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

And the matrices A2 and A3 of the same equations (2) and (3) respectively.Solve the equation (13) and comparing the result of the elements of the fourth column of the matrices we obtain the following equations (15, 16 e 17):

$$ p_x * \cos(\theta 1) + p_y * \sin(\theta 1) - L1 = L2 * \cos(\theta 2) + L3 * \cos(\theta 2) * \cos(\theta 3) - L3 * \sin(\theta 2) * \sin(\theta 3) \tag{15}$$

$$ p_z = L2 * \sin(\theta 2) + L3 * \cos(\theta 2) * \sin(\theta 3) + L3 * \cos(\theta 3) * \sin(\theta 2) \tag{16}$$

$$ p_x * \sin(\theta 1) - p_y * \cos(\theta 1) = 0 \tag{17}$$

Using equation (17) we find the equation for the angle $\theta 1$, that is:

$$ \theta 1 = \tan^{-1}\left(\frac{p_y}{p_x}\right) \tag{18}$$

To find the equation of the angle $\theta 3$, call the function of the left side of the equation (15) of “v”.

$$ v = p_x * \cos(\theta 1) + p_y * \sin(\theta 1) - L1 \tag{19}$$

And equation (15) becomes:

$$ v = L2 * \cos(\theta 2) + L3 * \cos(\theta 2) * \cos(\theta 3) - L3 * \sin(\theta 2) * \sin(\theta 3) \tag{20}$$

Then simplified equations (16) and (20) trigonometric and we obtain equations (21) and (22):

$$ v = L2 * \cos(\theta 2) + L3 * \cos(\theta 2 + \theta 3) \tag{21}$$

$$ p_z = L2 * \sin(\theta 2) + L3 * \sin(\theta 2 + \theta 3) \tag{22}$$

To simplify the equations (21) and (22) becomes the sum of $v^2 + p_z^2$and apply some trigonometric properties, we obtain the equation (23).

$$ v^2 + p_z^2 = L2^2 + L3^2 + 2 * L2 * L3 * \cos(\theta 3) \tag{23}$$

And isolating$\theta 3$we obtain the equation (24).

$$\theta 3 = \pm\cos^{-1}\left(\frac{v^2+p_z^2-L2^2-L3^2}{2*L2*L3}\right) \quad (24)$$

To find the equation $\theta 2$, we use the VZ plane (Figure 5)

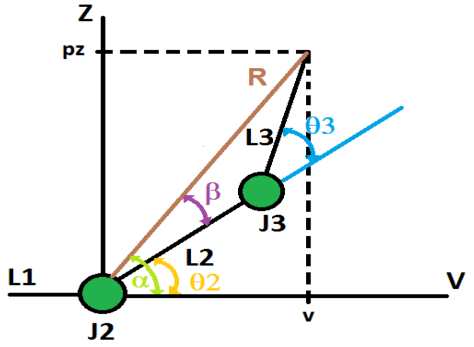

Figure 5. Plane VZ and angles θ2, θ3, **α e β**

Observing figure we find the trigonometric relations (25), (26) and (27):

$$\theta 2 = \alpha - \beta \quad (25)$$

$$\tan(\alpha) = \frac{p_z}{v} \quad (26)$$

$$\tan(\beta) = \frac{L3*\sin(\theta 3)}{L2+L3*\cos(\theta 3)} \quad (27)$$

And using the trigonometric property (28):

$$\tan(A - B) = \frac{\tan(A)-\tan(B)}{1+\tan(A)*\tan(B)} \quad (28)$$

We find the equation (29) Substituting equations (25), (26) and (27) in equation (28) and performing some simplifications:

$$\tan(\alpha - \beta) = \tan(\theta 2) = \frac{p_z * (L2 + L3 * \cos(\theta 3)) - v * L3 * \sin(\theta 3)}{v * (L2 + L3 * \cos(\theta 3)) + p_z * L3 * \sin(\theta 3)} \quad (29)$$

Then we find Equation (30) which is the equation for the angle $\theta 2$:

$$\theta 2 = \tan^{-1}\left(\frac{p_z * (L2 + L3 * \cos(\theta 3)) - v * L3 * \sin(\theta 3)}{v * (L2 + L3 * \cos(\theta 3)) + p_z * L3 * \sin(\theta 3)}\right) \quad (30)$$

Then the inverse kinematics equations are (18), (24) and (30) and the value of "v" in equation (19).

The leg has a kinematic redundancy at the third joint, and for configuring the type of insect legs is desired the negative results of the equation (24).

V. COMPARISONOFRESULTS

With the purpose of comparing the results of the equations developed, the distances of joints (L1, L2 and L3) were determined and two programs in Matlab were made. One of the programs calculates the forward and inverse kinematics using the equations developed in this article, being considered the negative results of the equation (24) since it ensures configuration like insect leg desired for the robotic leg.

The other program calculates the forward and inverse kinematics using the fkine and ikine functions of the robotics toolbox for Matlab [7] where a virtual model of the robotic leg was created using the function SerialLink(L) where L corresponds to a matrix with parameters of Denavit-Hartenberg of robotic leg.

The result of the calculation was compared to a set of five points that are present in the workspace of the robotic leg in the plane x=0.15m describinga step, and calculated inverse kinematics. With the result of the inverse kinematics, forward kinematics was calculated to determine whether it corresponds to the given point.

The distances of joints used are: L1=0.06m; L2=0.09 and L3=0.15m. The five specific points are shown in Table 2. This procedure was done with the two mentioned programs in Matlab. Results of these calculations are presented in Tables 3 and 4.

TABLE 2.POINT PATH OF A STEP

Points	X(m)	Y(m)	Z(m)
A	0,15	0,1	-0,1
B	0,15	-0,1	-0,1
C	0,15	-0,06	-0,064
D	0,15	-0,1	-0,032
E	0,15	0,06	-0,064

TABLE3.CALCULATIONS ANALYTICAL EQUATIONS

	Inverse Kinematics		
Points	□1(°)	□2(°)	□3(°)
A	33,6901	29,3102	-103,1299
B	-33,6901	29,3102	-103,1299
C	-21,8014	57,7559	-126,8449
D	-33,6901	72,3110	-124,0284
E	21,8014	57,7559	-126,8449
	Forward kinematics		
Points	px(m)	py(m)	pz(m)
A	0,15	0,1	-0,1
B	0,15	-0,1	-0,1
C	0,15	-0,06	-0,064
D	0,15	-0,1	-0,032
E	0,15	0,06	-0,064

TABLE 4.CALCULATION WITHTOOLBOX OF ROBOTICS FOR MATLAB

	Inverse Kinematics(ikine)			
Points	□1(°)	□2(°)	□3(°)	Iterations
A	33,6901	29,3102	-103,1299	327
B	-33,6901	-108,7912	103,1299	294
C	-21,8014	-122,1942	126,8449	291
D	-33,6901	-287,6890	235,9716	329
E	21,8014	57,7559	-126,8449	320
	Forward kinematics(fkine)			
Points	px(m)	py(m)	pz(m)	-
A	0,15	0,1	-0,1	-
B	0,15	-0,1	-0,1	-
C	0,15	-0,06	-0,064	-
D	0,15	-0,1	-0,032	-
E	0,15	0,06	-0,064	-

From the results of Tables 3 and 4,noticed that for points A and E results were equal and corresponds to the same configuration of the leg.For point D noticed that the values of the angles are different,but corresponds to the same configuration, because the angles □ 2 and □ 3 arecomplements angled at 360 ° however starting from different directions, The result of the function ikinefound the greatest distance to the point D and may be mechanically limited in practice by following their direction of rotation.The points B and C values of □2 are different and □ 3 have opposite signs because the corresponding redundant solution caused by kinematic redundancy present in the joint 3, being the solution of the functionikineanundesired configuration for leg robotics.

The functionikinesolves the inverse kinematics by an iterative method using the pseudo inverse of the Jacobian[7].

VI. Conclusion

In this study, equations of forward and inverse kinematics for a robotic leg inspired in insect were found. The method of Denavit-Hartenberg shows itself as an easy application to obtain forward kinematic of any configuration of the leg. On the other hand, inverse kinematics is always a difficult problem to be solved. In this work, it was possible to obtain analytically the inverse kinematics for being a system with three degrees of freedom and the use of the method of inverse matrices. Obtaining the equations of the inverse kinematics showed certain advantages compared to the iterative solution used by the function ikine of the Toolbox of robotics. Iterative Solutions provides a possible solution that may not be the desired one, being sometimes necessary to be analyzed the result to obtain the desired solution. With the analytical equations is easy to find the solution and maintain the desired configuration, in this case using negative values of the equation (24), avoiding problems of changing configuration of the leg in a defined trajectory. Another advantage is that by not using an iterative method for its solution, analytical equations do not need high processing power.

VII. Acknowledgment

The authorsthankthegeneroussupportofthe “Pontifícia Universidade Católica de Minas Gerais–PUCMINAS”, the “Coordenação de Aperfeiçoamento de Pessoal de Nível Superior– CAPES,”the “Conselho Nacional de Desenvolvimento Científico e Tecnológico–CNPq,” andthe “Fundação de Amparo a Pesquisa de Minas Gerais–FAPEMIG”.

References

[1] SILVA, Manuel Fernando dos Santos; *Sistema Robótico de Locomoção Multipernas; 2005. Dissertação de Doutorado*. Faculdade de Engenharia da Universidade do Porto - FEUP.

[2] CRAIG, John J. *Introduction to Robotics – Mechanics and Control*. 2º Ed – 1989 – Addison –Wesley Publishing Company.

[3] SPONG, Mark W.; Hutchinson, Seth; Vidyasagar, M.; *Robot Modeling and Control*; 2006. Editora. JOHN WILEY & SONS, INC.

[4] FILHO, Alberto Adade. *Fundamentos de Robótica- Cinemática, Dinâmica e Controle de Manipuladores Robóticos*. 2001 – Centro Técnico Aeroespacial – Instituto Tecnológico de Aeronáutica - São José dos Campos – SP

[5] SANTOS, Víctor M. F.; Robótica Industrial, 2004. *Departamento de Engenharia Mecânica da Universidade de Aveiro – Portugal.*

[6] PAUL, R. P.; Robot Manipulators: Mathematics, *Programming and Control.* 1981. MIT Press.

[7] CORKE, Peter. *Robotics Toolbox for MatlabRelese9*. 2011. Disponível em <http://www.petercorke.com/Robotics_Toolbox.html>

[8] CHEN, Jie. LIU, Yubin. ZHAO, Jie. ZHANG, He. JIN, Hongzhe: Biomimetic *Design and Optimal Swing of a Hexapod Robot Leg*, 2014. State Key Laboratory of

Robotics and System, Harbin Institute of Technology, Harbin 150001, P. R. China

[9] Klaassen B, Linnemann R, Spenneberg D, Kirchner F. Biomimetic *walking robot SCORPION: Control and modeling*. *Robotics and Autonomous Systems*, 2002, **41**, 69-76.

[10] PASKARBEIT, Jan. SCHMITZ, Josef. SCHILLING, Malte. SCHNEIDER, Axel. - *Layout and Construction of a Hexapod Robot with Increased Mobility*. 2010. Proceedings of the 2010 3rd IEEE RAS & EMBS International Conference on Biomedical Robotics and Biomechatronics, The University of Tokyo, Tokyo, Japan, September 26-29, 2010

[11] ROENNAU, A.; KERSCHER, T.; DILLMANN, R. - *Design and Kinematics of a Biologically-Inspired Leg for a Six-Legged Walking Machine*. 2010- Proceedings of the 2010 3rd IEEE RAS & EMBS International Conference on Biomedical Robotics and Biomechatronics, The University of Tokyo, Tokyo, Japan, September 26-29, 2010