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Abstract

This paper investigates the MHD flow and heat transfer of an electrically conducting non-newtonian power-law fluid over a non-linearly stretching surface along with porous plate in porous medium. The governing equations are reduced to non-linear ordinary differential equations by means of similarity transformations. These equations are then solved numerically with the help of Runge-Kutta shooting method. The effect of various flow parameters in the form of dimensionless quantities on the flow field are discussed and presented graphically.

Keywords: Heat Source/Sink Parameter; MHD flow; Power-law Fluid; Stretching sheet; porous medium.

I. Introduction:


Motivated by these analyses and practical applications, the main concern of the present paper is to study the effect of variable thermal conductivity on the power-law fluid flow and heat transfer over a non-linearly stretching sheet along with porous plate in porous medium in the presence of a transverse magnetic field. This extends the work in [11]. The obtain similarity equations were solved numerically to show the effects of the governing parameters.

II. Nomenclature

- $A$ - constant
- $B_0$ - uniform magnetic field
- $b$ - stretching rate, positive constant
- $C_f$ - skin friction
- $f$ - dimensionless stream function
- $h(x)$ - heat transfer co-efficient
- $k$ - thermal conductivity
- $K$ - consistency co-efficient
III. Mathematical Formulation

Let us consider the case of a steady two-dimensional flow and heat transfer of a non-Newtonian power-law fluid over a non-linear stretching sheet. We consider the effect of porous plate in porous medium. Further a Cartesian coordinate system $(x, y)$ is used where $x$ and $y$ are co-ordinates measured along and normal to the surface, respectively. The continuous stretching sheet is assumed to have a non-linear velocity and prescribed temperature of the form $U(x) = bx^m$ and $T_\infty + Ax^r$ respectively, where $b$ is the stretching constant, $x$ is the distance from the slot; $A$ is a constant whose value depends upon the properties of the fluid. Here $m$ and $r$ are the velocity and temperature exponents, respectively. Here we neglect the induced magnetic field, which is small in comparison with the applied magnetic field. Further the external electrical field is assumed to be zero. Under these assumptions the basic equations governing the flow and heat transfer in usual notations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} (3.1)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{K} u$$  \hspace{1cm} (3.2)

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{Q_s}{\rho c_p} (T - T_\infty)$$  \hspace{1cm} (3.3)

Where $u$ and $v$ are the flow velocity components along the $x$ and $y$-axes respectively, $\nu$ is the kinematic viscosity of the fluid, $K$-consistency of the fluid, $n$ is the power-law index, $\rho$ is the fluid density and $c_p$ is the specific heat at constant pressure.

The first term in the right hand side of the equation (3.2) is the shear rate $\left(\frac{\partial u}{\partial x}\right)$ has been assumed to be negative throughout the boundary layer since the stream wise velocity component $u$ decreases monotonically with the distance $y$ from the moving surface (for continuous stretching surface). The flow is driven solely by the stretching surface, which moves with a prescribed velocity $U(x)$. $T$ is the temperature of the fluid and $\alpha$ is the thermal diffusivity of the fluid. The last term containing $Q_s$ in equation (3.3) represents the temperature dependent volumetric rate of heat source when $Q_s > 0$ and heat sink when $Q_s < 0$. Thus the appropriate boundary conditions are

$$u(x,0) = U(x),$$  \hspace{1cm} (3.4)

$$v(x,0) = 0,$$  \hspace{1cm} (3.5)

$$T(x,0) = T_\infty(x),$$  \hspace{1cm} (3.6)

$$u(x,y) \rightarrow 0, T(x,y) \rightarrow T_\infty \text{ as } y \rightarrow \infty$$  \hspace{1cm} (3.7)

Here, boundary condition (3.4) assures no slip at the surface and equation (3.5) signifies the importance of impermeability of the stretching surface. Equation (3.6) is the variable prescribed surface temperature at the wall whereas the equation (3.7) means that the stream velocity and the temperature vanish outside the boundary layer. In order to obtain the similarity solutions of equations (3.1) - (3.7), we assume that the variable magnetic field $B_0(x)$ is of the form

$$B_0(x) = B_0 \left(1 - \frac{x}{l}\right)$$

where $B_0$ is the magnetic field at the slot and $l$ is the distance from the slot to the point where the magnetic field is zero.
\[ B_s(x) = B_0 x^{(m - 1)/2} \]

The momentum and energy equations can be transformed to the corresponding ordinary differential equations by the following transformations (111)

\[ \eta = \frac{y}{x} (Re_s)^{1/\eta} \]

\[ \theta(\eta) = \frac{T_T - T_n}{T_w - T_n} \]

\[ \psi(x, y) = Ux (Re_s)^{-1/\eta} f(\eta), \]

Where \( \eta \) is the similarity variable, \( \psi(x, y) \) is the stream function and \( \theta \) are the dimensionless similarity function and temperature, respectively. The velocity components \( u \) and \( v \) given by

\[ n(-f')^n - m f'^2 + \left( \frac{2mn - m + 1}{n + 1} \right) \]

\[ \theta' + Npr \left( \frac{2mn - m + 1}{n + 1} \right) f' \theta' + Npr (\beta \theta - rf \theta') = 0 \]

The boundary conditions (3.4) - (3.7) now becomes

\[ f(\eta) = 0, f'(\eta) = 1, \theta(\eta) = 1, \alpha \eta = 0 \]

\[ f'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \]

Where \( M_n = \sigma B_s^2/\rho \) is the magnetic parameter, \( \delta = K_b/\nu \) is the permeability parameter, \( Npr = Npe_s/(Re_s)^{2/\eta} \) is the modified Prandtl number for power – law fluids, \( Npe_s = c_p \rho U x/\kappa \) is the convectional Peclet number (111), \( \beta = Q_s / c_p \rho b \) is the heat source/sink parameter. Here primes denote the differentiation with respect to \( \eta \).

The physical quantities of interest are the skin-friction coefficient \( C_f \) and the local Nusselt number \( \text{Nu}_x \), which are defined as

\[ C_f = \frac{2 \tau_w}{\rho U_x^2} \]

\[ \text{Nu}_x = \frac{x q_w}{k (T_T - T_n)} \]

respectively, where the wall shear stress \( \tau_w \) and heat transfer from the sheet \( q_w \) are given by

\[ \tau_w = \mu_0 \left( \frac{\partial u}{\partial y} \right)_{y=0} \]

\[ q_w = -\kappa_1 \left( \frac{\partial T}{\partial y} \right)_{y=0} \]

with \( \mu_0 \) and \( \kappa_1 \) being the dynamic viscosity and thermal conductivity, respectively. Using the non-dimensional variable (2.7), we obtain

\[ u = \frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x} \]

The local Reynolds number is defined by

\[ \text{Re}_x = \frac{U x^n}{\nu} \]

The mass conservation equation (3.1) is automatically satisfied by equation (3.10). By assuming the similarity function \( f(\eta) \) to depend on the similarity variable \( \eta \), the momentum equation (3.2) and the heat equation (3.3) transform into the coupled non-linear ordinary differential equation of the form

\[ C_f = \left( \frac{-2 \tau_{xy}}{\rho (bx)^2} \right)_{y=0} = 2 \left( -f'(0) \right)' \text{Re}_x^{-1} \]

\[ \text{Nu}_x = -\text{Re}_x^{-1} \delta \theta(0) \]

\[ \tau_{xy} \] is the shear stress and \( \text{Re}_x \) is the local Reynolds number.

IV. Results and Discussion

The ordinary differential equations (3.12)-(3.13) with the boundary conditions (3.14)-(3.15) have been solved by Runge-Kutta shooting method. Numerical results are obtained to study the effect of the various non-dimensional parameters namely, the magnetic parameters \( M_n \), permeability parameter \( \delta \), the velocity exponent parameter \( m \) and the temperature exponent parameter \( r \), the modified Prandtl number \( \text{Npr} \) and the heat source/sink parameter \( \beta \) on the flow and heat transfer are shown graphically in the Figures 1-9.

Figures (1(a)-(c)) respectively, depict the effect of shear thinning \( (n < 1) \), Newtonian \( (n = 1) \) and shear thickening \( (n > 1) \) fluids on the horizontal velocity profiles \( f' \) with \( \eta \) for different values of velocity exponent parameter \( m \) and magnetic parameter \( M_n \) and constant permeability parameter \( \delta \). The effect of flatter bow of horizontal velocity as a consequence of increasing the strength of the magnetic field is observed for all values of velocity exponent parameter \( m \). The effect of increasing values of the velocity exponent parameter \( m \) is to
reduce the momentum boundary layer thickness, which tends to zero as the space variable $\eta$ increases from the boundary surface. Physically, $m < 0$ implies that the surface is decelerated from the slot, $m = 0$ the continuous movement of a flat surface, and $m > 0$ implies the surface is accelerated from the extruded slit.

Horizontal velocity profiles $f'$ decreases and disclosing the fact that the effect of stretching of the exponent parameter $m$ from negative values to positive values is to decelerate the velocity and hence reduces the momentum boundary layer thickness. Further it is observed from these figures that the horizontal velocity profiles $f'$ decrease with increasing values of power-law index $n$.

Figures 2((a)-(c)) respectively demonstrates the horizontal velocity profiles for different values of permeability parameter $\delta$ in the presence/absence of magnetic parameter $Mn$. It is observed that the velocity of the fluid increases as permeability parameter increases.

Figures 3((a)-(c)) respectively demonstrates the temperature profiles for different values of permeability parameter $\delta$ in the presence/absence of magnetic parameter $Mn$. It is observed that the temperature of the fluid decreases as permeability parameter increases.

Figures 4((a)-(c)) respectively depicts temperature profiles for different values of velocity exponent parameter $(m)$ in the presence/absence of magnetic parameter $Mn$. The effect of increasing values of velocity exponent parameter $m$ is to decrease the temperature profiles.

Figures 5((a)-(c)) shows respectively, the shear thinning, Newtonian and shear thickening fluids on the temperature profiles $\theta(\eta)$ for different values of temperature exponent parameter $r$ and velocity exponent parameter $m$ in presence of magnetic parameter $Mn$. It is seen that the temperature of the fluid decreases as temperature exponent parameter $r$ increases. The same result is obtained for absence of magnetic field also.

Figures 6((a)-(c)) demonstrates the temperature profiles for several sets of values of the modified Prandtl number $Npr$ in the absence/presence of magnetic parameter $Mn$ for shear thinning, Newtonian and shear thickening fluids. It is clear that the temperature of the fluid decreases as Prandtl number increases.

Figures 7((a)-(c)) shows respectively, the shear thinning, Newtonian and shear thickening fluids on the temperature profiles for different values of heat source/sink parameter $\beta$ in the presence of magnetic field $Mn$. It is clear that temperature profiles decreases as heat source/sink parameter $\beta$ increases. The same result is obtained for absence of magnetic field also.
**Fig. 1.** Non dimensional velocity profiles for different values of stretching parameter \( m \)

**Fig. 2.** Non dimensional velocity profiles for different values of \( \delta \) with presence/absence of magnetic field \( Mn \)
Fig. 3. Effect of temperature distribution for different values of $\delta$ with presence/absence of magnetic parameter

$Mn = 0$, $Mn = 1$
Fig.4. Effect of stretching parameter $m$ over temperature distribution

- $r = 0$
- $r = 1$
- $r = -1$

Fig.5. Temperature profiles for different values of stretching parameter $m$ and wall temperature $r$ with $Mn = 1$
Fig. 6. Temperature profiles for different values of modified Prandtle number $Npr$

$m = 0, m = 1$

$r = 0, r = 1, r = -1$

$n = 0.8, n = 1$
Fig. 7. Temperature profiles for different values of $\beta$ and wall temperature parameter ($r$) with $Mn = 1$

V. Conclusions:

- The effect of increasing values of velocity exponent parameter $m$ is to reduce the horizontal velocity.
- The increasing values of magnetic parameter $Mn$ results in flattering the horizontal velocity profiles and increase the temperature profiles.
- The effects of Permeability parameter $\delta$ is to increase the velocity profiles and decrease the temperature profiles.
- The effect of modified Prandtl number $Npr$ is to decrease the wall temperature gradient.
- The internal heat source/sink parameter $\beta$ increases the temperature profiles.

References