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Tenser Product of Representation for the Group C_n

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Abstract
The main objective of this paper is to compute the tenser product of representation for the group C_n. Also algorithms designed and implemented in the construction of the main program designated for the determination of the tenser product of representation for the group C_n including a flow-diagram of the main program. Some algorithms are followed by simple examples for illustration.
Key Words: representation for the group, degree of the representation, character of representation, tenser product.

Introduction

The group of invertible $n \times n$ matrices over a field F denoted by GL(n,F). The matrix representation of a group G is a homomorphism $T:G \longrightarrow GL(n,F)$, the degree of this matrix is the degree of that representation [1], the trace for this matrix representation is the character of this representation, [2].

In this paper we consider the group $C_n = \langle x : x^n = 1 \rangle$. In section one the definition of tenser product introduced and apply that the f or representation of this groups by example, the main proposition introduce for the tenser product which we needed it in section two which include the algorithms designed and implemented in the construction of the main program designated for the determination of the tenser product of representation for the group C_n.

§.1 Preliminaries

In this section, we recall definition proposition and remark which we needed in the next section.

Definition 1.1 **: [3]**

Let $A \in M_n(\mathbb{C})$, $B \in M_m(\mathbb{C})$, we defined a matrix $A \otimes B \in M_m(\mathbb{C})$, put

$$A \otimes B = \begin{bmatrix} \alpha_{11}B & \alpha_{12}B & \dots & \alpha_{1n}B \\ \alpha_{21}B & \alpha_{22}B & \dots & \alpha_{2n}B \\ \vdots & \vdots & & \vdots \\ \alpha_{n1}B & \alpha_{n2}B & \dots & \alpha_{nn}B \end{bmatrix}_{nm \times nm}, \quad A = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2n} \\ \vdots & \vdots & & \vdots \\ \alpha_{n1} & \alpha_{n2} & \dots & \alpha_{nn} \end{bmatrix}_{n \times n}, \quad B = \begin{bmatrix} \beta_{11} & \beta_{12} & \dots & \beta_{1m} \\ \beta_{21} & \beta_{22} & \dots & \beta_{2m} \\ \vdots & \vdots & & \vdots \\ \beta_{m1} & \beta_{m2} & \dots & \beta_{mm} \end{bmatrix}_{m \times m}$$

Thus

$$A \otimes B = \begin{bmatrix} \delta_{11} & \delta_{12} & \dots & \delta_{1k} \\ \delta_{21} & \delta_{22} & \dots & \delta_{2k} \\ \vdots & \vdots & & \vdots \\ \delta_{k1} & \delta_{k2} & \dots & \delta_{kk} \end{bmatrix}_{nm \times nm}$$

Where
$$\delta_{11} = \begin{bmatrix} \alpha_{11}\beta_{11} & \alpha_{11}\beta_{12} & \dots & \alpha_{11}\beta_{1m} \\ \alpha_{11}\beta_{21} & \alpha_{11}\beta_{22} & \dots & \alpha_{11}\beta_{2m} \\ \vdots & \vdots & & \vdots \\ \alpha_{11}\beta_{m1} & \alpha_{11}\beta_{m2} & \dots & \alpha_{11}\beta_{mm} \end{bmatrix}_{m \times m}, \dots, \delta_{1k} = \begin{bmatrix} \alpha_{1n}\beta_{11} & \alpha_{1n}\beta_{12} & \dots & \alpha_{1n}\beta_{1m} \\ \alpha_{1n}\beta_{21} & \alpha_{1n}\beta_{22} & \dots & \alpha_{1n}\beta_{2m} \\ \vdots & \vdots & & \vdots \\ \alpha_{1n}\beta_{m1} & \alpha_{1n}\beta_{m2} & \dots & \alpha_{1n}\beta_{mm} \end{bmatrix}_{m \times m}, \dots$$

$$\delta_{kk} = \begin{bmatrix} \alpha_{nn}\beta_{11} & \alpha_{nn}\beta_{12} & \cdots & \alpha_{nn}\beta_{1m} \\ \alpha_{nn}\beta_{21} & \alpha_{nn}\beta_{22} & \cdots & \alpha_{nn}\beta_{2m} \\ \vdots & \vdots & & \vdots \\ \alpha_{nn}\beta_{m1} & \alpha_{nn}\beta_{m2} & \cdots & \alpha_{nn}\beta_{mm} \end{bmatrix}_{m\times m} \quad \text{and } k = nm.$$

Example 1.2 **:**

$$A = \begin{bmatrix} 1 & -3 \\ 2 & 0 \end{bmatrix}_{2\times 2}, \quad B = \begin{bmatrix} 1 & -2 & -1 \\ 3 & 1 & 2 \\ 6 & 4 & 5 \end{bmatrix}_{3\times 3}$$

$$A \otimes B = \left[\begin{array}{ccc:ccc} 1 & -2 & -1 & -3 & 6 & 3 \\ 3 & 1 & 2 & -9 & -3 & -6 \\ 6 & 4 & 5 & -18 & -12 & -15 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 2 & -4 & -2 & 0 & 0 & 0 \\ 6 & 2 & 4 & 0 & 0 & 0 \\ 12 & 8 & 10 & 0 & 0 & 0 \end{array}\right]$$

Proposition 1.3 **: [4]**

Let A, A′, B, B′ ∈ $M_m(K)$, then

(1) (A + A′) ⊗ B = (A ⊗ B) + (A′ ⊗ B)

(2) (A ⊗ B) (A′ ⊗ B′) = AA′ ⊗ BB′

Remark 1.4 **:**

Let S and T be two representations of degree n and m of the group C_n, for each x ∈ C_n define U(x) = S(x) ⊗ T(x). Then U is representation of degree nm, we write U = S ⊗ T.

Now, let χ_S, χ_T be two character of S and T respectively then $\chi_U = \chi_S\chi_T$.

§.2 The Algorithms

This section contains a collection of the computer ready Fortran algorithms for many standard methods of number theory installed in our main program.

Algorithm (1)**: The Number of Degree of Representation for the Group C_n**

Input: n (the degree of the group C_n)

Step 1: To evaluate m where $T:C_n \longrightarrow M(K)$,

$$M_m(K) = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix}_{m\times m}$$

```
Step 2: Do I = 1 to m
          Do J = 1 to m
          Print IA(I,J)
          End J-loop
          End I-loop
```

Output: The number of degree of representation for groups C_n is m.

Example 2.1 **:**

The representation $T:C_4 \longrightarrow M_3(\mathbb{R})$, the degree of this representation for the group C_4 is 3.

$C_4 = <x:x^4 = 1> = \{1,x,x^2,x^3\}$

$$T(1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad T(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \quad T(x^2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad T(x^3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

Algorithm (2)**:The Tenser Product of Two Representations for the Group C_n**

Input: n (the degree of the group C_n)

Step 1: Do C is the matrix of dimension mn×mn
    C(0,0) = 0
    Do I = 1 to n
    Do J = 1 to n
    T(x) = A(I,J)
    End J-loop
    End I-loop

Step 2: Do I = 1 to m
    Do J = 1 to m
    Set T(x) = B(I,J)
    End J-loop
    End I-loop

Step 3: call algorithm 1

Step 4: To evaluate C where C(I,J) = A(I,J)∗B

Step 5: Set C(1,1) = A(1,1)∗B
    C(1,2) = A(1,2)∗B
    ⋮
    C(I,n) = A(I,n)∗B

where
$$B = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1m} \\ B_{21} & B_{22} & \cdots & B_{2m} \\ \vdots & \vdots & & \vdots \\ B_{m1} & B_{m2} & \cdots & B_{mm} \end{bmatrix}_{m\times m}$$

Step 6: Set
$$C = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1nm} \\ C_{21} & C_{22} & \cdots & C_{2nm} \\ \vdots & \vdots & & \vdots \\ C_{nm1} & C_{nm2} & \cdots & C_{nmmn} \end{bmatrix}_{nm\times nm}$$

Output: The tenser product of two representations of C_n is C(mn,mn)

Example 2.2 **:**

The representation $T:C_3 \longrightarrow M_3(\mathbb{R})$, the degree of this representation for the group C_3 is 3.
$C_3 = <x:x^3 = 1> = \{1,x,x^2\}$

$$T(1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}, \quad T(x) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}_{3\times 3}, \quad T(x^2) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}_{3\times 3}$$

$$T(1)\otimes T(x)=\begin{bmatrix} 0 & 0 & 1 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 1 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots & & & \\ 0 & 0 & 0 & \vdots & 0 & 0 & 1 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 1 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 1 & 0 & \vdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots & & & \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 1 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 1 & 0 \end{bmatrix}_{9\times 9}, \quad T(1)\otimes T(x^2)=\begin{bmatrix} 0 & 1 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 1 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 1 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots & & & \\ 0 & 0 & 0 & \vdots & 0 & 1 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 1 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 1 & 0 & 0 & \vdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots & & & \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 1 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 1 & 0 & 0 \end{bmatrix}_{9\times 9}$$

$$T(x)\otimes T(1)=\begin{bmatrix} 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots & & & \\ 1 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 1 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots & & & \\ 0 & 0 & 0 & \vdots & 1 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 1 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 1 & \vdots & 0 & 0 & 0 \end{bmatrix}_{9\times 9}, \quad T(x)\otimes T(x^2)=\begin{bmatrix} 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 1 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 1 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots & & & \\ 0 & 1 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 1 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 1 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots & & & \\ 0 & 0 & 0 & \vdots & 0 & 1 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 1 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 1 & 0 & 0 & \vdots & 0 & 0 & 0 \end{bmatrix}_{9\times 9}$$

$$T(x^2)\otimes T(1)=\begin{bmatrix} 0 & 0 & 0 & \vdots & 1 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 1 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 1 & \vdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots & & & \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots & & & \\ 1 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 1 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \end{bmatrix}_{9\times 9}, \quad T(x^2)\otimes T(x)=\begin{bmatrix} 0 & 0 & 0 & \vdots & 0 & 0 & 1 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 1 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 1 & 0 & \vdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots & & & \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 1 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 1 & 0 \\ \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots & & & \\ 0 & 0 & 1 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 1 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \vdots & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \end{bmatrix}_{9\times 9}$$

***Algorithm (3)*:The Tenser Product of Three Representations for the Group C_n**

Input: n (the degree of the group C_n)
Step 1: Call algorithm 2
Step 2: Do I = 1 to k
    Do J = 1 to k
    D(I,J)
    End J-loop
    End I-loop

Step 3: To evaluate R where R(I,J) = C(I,J) ∗ D
Step 4: Set
R(1,1) = C(1,1) ∗ D
R(1,2) = C(1,2) ∗ D

$$\text{where } D = \begin{bmatrix} D_{11} & D_{12} & \dots & D_{1k} \\ D_{21} & D_{22} & \dots & D_{2k} \\ \vdots & \vdots & & \vdots \\ D_{k1} & D_{k2} & \dots & D_{kk} \end{bmatrix}_{k\times k}$$

Step 5: Set

$$R = \begin{bmatrix} R_{11} & R_{12} & \dots & R_{1s} \\ R_{21} & R_{22} & \dots & R_{2s} \\ \vdots & \vdots & & \vdots \\ R_{s1} & R_{s2} & \dots & R_{ss} \end{bmatrix}_{s\times s} \quad \text{where } s = nm\times k$$

Step 6: Do I = 1 to s
Do J = 1 to s
Print R(I,J)
End J-loop
End I-loop
Output: The tenser product of three representations of C_n is R(s,s)

Example 2.3 **:**

The representation $T:C_4 \longrightarrow M_2(\mathbb{R})$, the degree of this representation for the group C_4 is 2.

$C_4 = \langle x:x^4 = 1\rangle = \{1,x,x^2,x^3\}$

$$T(1) = T(x^2) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad , \quad T(x) = T(x^3) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Now present some tenser product for these representations of the group C_4

$$T(1)\otimes T(x)\otimes T(1) = \left[\begin{array}{ccccc|ccccc} 0 & 0 & \vdots & 1 & 0 & 0 & 0 & \vdots & 0 & 0 \\ 0 & 0 & \vdots & 0 & 1 & 0 & 0 & \vdots & 0 & 0 \\ \dots & & & & \dots & \dots & & & & \dots \\ 1 & 0 & \vdots & 0 & 0 & 0 & 0 & \vdots & 0 & 0 \\ 0 & 1 & \vdots & 0 & 0 & 0 & 0 & \vdots & 0 & 0 \\ \hline 0 & 0 & \vdots & 0 & 0 & 0 & 0 & \vdots & 1 & 0 \\ 0 & 0 & \vdots & 0 & 0 & 0 & 0 & \vdots & 0 & 1 \\ \dots & & & & \dots & \dots & & & & \dots \\ 0 & 0 & \vdots & 0 & 0 & 1 & 0 & \vdots & 0 & 0 \\ 0 & 0 & \vdots & 0 & 0 & 0 & 1 & \vdots & 0 & 0 \end{array}\right]_{8\times 8} , \quad T(x)\otimes T(x^2)\otimes T(1) = \left[\begin{array}{ccccc|ccccc} 0 & 0 & \vdots & 0 & 0 & 1 & 0 & \vdots & 0 & 0 \\ 0 & 0 & \vdots & 0 & 0 & 0 & 1 & \vdots & 0 & 0 \\ \dots & & & & \dots & \dots & & & & \dots \\ 0 & 0 & \vdots & 0 & 0 & 0 & 0 & \vdots & 1 & 0 \\ 0 & 0 & \vdots & 0 & 0 & 0 & 0 & \vdots & 0 & 1 \\ \hline 1 & 0 & \vdots & 0 & 0 & 0 & 0 & \vdots & 0 & 0 \\ 0 & 1 & \vdots & 0 & 0 & 0 & 0 & \vdots & 0 & 0 \\ \dots & & & & \dots & \dots & & & & \dots \\ 0 & 0 & \vdots & 1 & 0 & 0 & 0 & \vdots & 0 & 0 \\ 0 & 0 & \vdots & 0 & 1 & 0 & 0 & \vdots & 0 & 0 \end{array}\right]_{8\times 8}$$

$$T(x)\otimes T(x^2)\otimes T(x)=\begin{bmatrix} 0\,0 \vdots 0\,0 & | & 0\,1 \vdots 0\,0 \\ 0\,0 \vdots 0\,0 & | & 1\,0 \vdots 0\,0 \\ \cdots\cdots & | & \cdots\cdots \\ 0\,0 \vdots 0\,0 & | & 0\,0 \vdots 0\,1 \\ 0\,0 \vdots 0\,0 & | & 0\,0 \vdots 1\,0 \\ \hline 0\,1 \vdots 0\,0 & | & 0\,0 \vdots 0\,0 \\ 1\,0 \vdots 0\,0 & | & 0\,0 \vdots 0\,0 \\ \cdots\cdots & | & \cdots\cdots \\ 0\,0 \vdots 0\,1 & | & 0\,0 \vdots 0\,0 \\ 0\,0 \vdots 1\,0 & | & 0\,0 \vdots 0\,0 \end{bmatrix}_{8\times 8},\quad T(x^2)\otimes T(x)\otimes T(x^3)=\begin{bmatrix} 0\,0 \vdots 0\,1 & | & 0\,0 \vdots 0\,0 \\ 0\,0 \vdots 1\,0 & | & 0\,0 \vdots 0\,0 \\ \cdots\cdots & | & \cdots\cdots \\ 0\,1 \vdots 0\,0 & | & 0\,0 \vdots 0\,0 \\ 1\,0 \vdots 0\,0 & | & 0\,0 \vdots 0\,0 \\ \hline 0\,0 \vdots 0\,0 & | & 0\,0 \vdots 0\,1 \\ 0\,0 \vdots 0\,0 & | & 0\,0 \vdots 1\,0 \\ \cdots\cdots & | & \cdots\cdots \\ 0\,0 \vdots 0\,0 & | & 0\,1 \vdots 0\,0 \\ 0\,0 \vdots 0\,0 & | & 1\,0 \vdots 0\,0 \end{bmatrix}_{8\times 8}$$

$$T(x^3)\otimes T(1)\otimes T(x^2)=\begin{bmatrix} 0\,0 \vdots 0\,0 & | & 1\,0 \vdots 0\,0 \\ 0\,0 \vdots 0\,0 & | & 0\,1 \vdots 0\,0 \\ \cdots\cdots & | & \cdots\cdots \\ 0\,0 \vdots 0\,0 & | & 0\,0 \vdots 1\,0 \\ 0\,0 \vdots 0\,0 & | & 0\,0 \vdots 0\,1 \\ \hline 1\,0 \vdots 0\,0 & | & 0\,0 \vdots 0\,0 \\ 0\,1 \vdots 0\,0 & | & 0\,0 \vdots 0\,0 \\ \cdots\cdots & | & \cdots\cdots \\ 0\,0 \vdots 1\,0 & | & 0\,0 \vdots 0\,0 \\ 0\,0 \vdots 0\,1 & | & 0\,0 \vdots 0\,0 \end{bmatrix}_{8\times 8},\quad T(x)\otimes T(x^3)\otimes T(x^2)=\begin{bmatrix} 0\,0 \vdots 0\,0 & | & 0\,0 \vdots 1\,0 \\ 0\,0 \vdots 0\,0 & | & 0\,0 \vdots 0\,1 \\ \cdots\cdots & | & \cdots\cdots \\ 0\,0 \vdots 0\,0 & | & 1\,0 \vdots 0\,0 \\ 0\,0 \vdots 0\,0 & | & 0\,1 \vdots 0\,0 \\ \hline 0\,0 \vdots 1\,0 & | & 0\,0 \vdots 0\,0 \\ 0\,0 \vdots 0\,1 & | & 0\,0 \vdots 0\,0 \\ \cdots\cdots & | & \cdots\cdots \\ 1\,0 \vdots 0\,0 & | & 0\,0 \vdots 0\,0 \\ 0\,1 \vdots 0\,0 & | & 0\,0 \vdots 0\,0 \end{bmatrix}_{8\times 8}$$

***Algorithm (4)*: The Character of Representations for the Group C_n**

Input: n (the degree of the group C_n)
Step 1: $\chi(0) = 0$
Step 2: Do I = 1 to m
  χ_I
  End I-loop
Step 3: Do J = 1 to n
  χ_J
  End J-loop
Step 4: Do I = 1 to m
  Do J = 1 to n
  $\chi_{(k)} = \chi_I * \chi_J$
  End J-loop
  End I-loop
  Print χ_k

Step 5: Set $\chi_k = \begin{bmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \vdots \\ \chi_s \end{bmatrix}$, s = (nm)/2

Step 6: Call algorithm 3
Step 7: Call algorithm 4
Output: The character of representation for C_n is $\chi(k)$, k = 1 to s.

Example 2.4 **:**

Consider the character table of C_3, where $\omega = e^{\frac{2\pi i}{3}}$

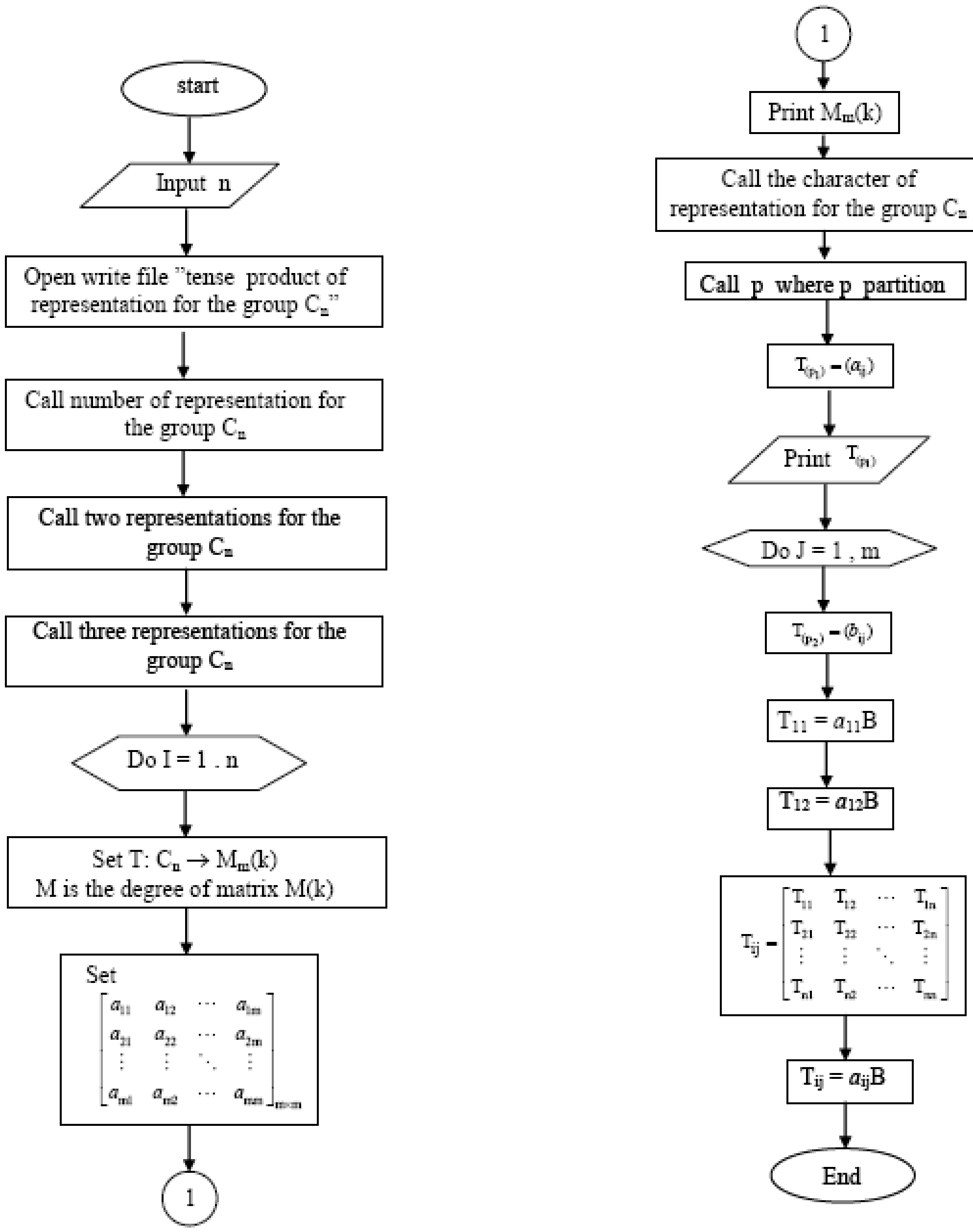
Class	1	x	x^2
Order	1	1	1
χ_1	1	1	1
χ_2	1	ω	ω^2
χ_3	1	ω^2	ω

In 1
$\chi_1 \otimes \chi_2 = (1)(1) = 1$, $\chi_1 \otimes \chi_3 = (1)(1) = 1$, $\chi_2 \otimes \chi_3 = (1)(1) = 1$
In x
$\chi_1 \otimes \chi_2 = (1)(\omega) = \omega$, $\chi_1 \otimes \chi_3 = (1)(\omega^2) = \omega^2$, $\chi_2 \otimes \chi_3 = (\omega)(\omega^2) = 1$
In x^2
$\chi_1 \otimes \chi_2 = (1)(\omega^2) = \omega^2$, $\chi_1 \otimes \chi_3 = (1)(\omega) = \omega$, $\chi_2 \otimes \chi_3 = (\omega^2)(\omega) = 1$
In 1
$\chi_1 \otimes \chi_2 \otimes \chi_3 = (1)(1)(1) = 1$
In x
$\chi_1 \otimes \chi_2 \otimes \chi_3 = (1)\,(\omega)(\omega^2) = 1$
In x^2
$\chi_1 \otimes \chi_2 \otimes \chi_3 = (1)(\omega^2)(\omega) = 1$

$$\chi = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \omega \\ \omega^2 \\ 1 \\ \omega^2 \\ \omega \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

The Algorithm of the Main Program**:**
The Tenser Product of Representations for Group C_n
Input: n (the degree of the group C_n)
Step 1: Call algorithm 1
Step 2: Call algorithm 2
Step 3: Call algorithm 3
Step 4: Call algorithm
Output: (T(I), I = 1 to m) To evaluate the tenser product of representation for the group C_n
End

Flow Diagram of the Main Program



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