

RELIABILITY BASED DESIGN OF A GEAR BOX

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ABSTRACT

Reliability is the probability that a system, component or device will perform without failure for a specified period of time under specified operating conditions. The concept of reliability is of great importance in the design of various machine members. Conventional engineering design uses a deterministic approach. It disregards the fact that the material properties, the dimensions of the components and the externally applied loads are statistical in nature. In conventional design this uncertainties are covered with a factor of safety, which is not always successful. The growing trend towards reducing uncertainty and increasing reliability is to use the probabilistic approach.

In the present work a three shaft four speed gear box and six speed gear box are designed using reliability principles. For the specified reliability of the system (Gear box), component reliability (Gear pair) is calculated by considering the system as a series system. Design is considered to be safe and adequate if the probability of failure of gear box is less than or equal to a specified quantity in each of the two failure modes. . All the parameters affecting the design are considered as random variables and all the random variables are assumed to follow normal distribution. A computer program in C++ is developed to calculate the face widths in bending and surface failure modes. The larger one out of the two values is considered. By changing the variations in the design parameters, variations in the face widths are studied.

KEYWORDS: Reliability Based Design, four speed gear box, six speed gear box, probabilistic approach.

I. INTRODUCTION:

Gears are critical mechanical elements that are used to transmit torque or power from one rotating shaft to another. This is accomplished by successively engaging teeth. Generally power transmission can be made by means of belts, ropes, chains and gears. In precession machines in which a definite velocity ratio is of greater importance, the only positive drive is by means of toothed wheels or gears. Therefore, to get perfect velocity ratio gears are found to be efficient. Hence, gears constitute an important element in many machines. When a gear fails the consequences are serious, like it stops the functioning of entire machine. Replacing a gear is usually a time consuming major over haul. This process is costly and hence needs to be avoided. Being an important element the gears are required to be designed for a failure free operation, for a given time and under given working conditions. The above need cannot be fulfilled by deterministic design, these conditions force to follow probabilistic approach to design. The basic gear dimensions that are required to specify a gear are pitch circle diameter, face width, and module. The gear dimension will depend upon the factors such as properties of gear material, the power transmitted, and rpm of the gear shaft, the velocity ratio and the pressure angle.

II. DETERMINISTIC DESIGN:

2.1. FAILURE DUE TO BENDING STRESS:

The allowable stress in bending is $S_b/F.S$

Induced bending stress assuming the tooth as a cantilever beam with end load is

$$s_b = \beta M_t / A t_1$$

Where

$$M_t = 72735 p/n_w$$

$$\beta = [K_c K_d (i+1)] / i m y \cos \alpha$$

$$y = 0.52(1 + 20/T_w)$$

$$A = (T_w + T_p) m / 2$$

The face width (t_1) due to bending is calculated as

$$t_1 = (\beta M_t / (A s_b))$$

2.2. FAILURE DUE TO WEAR STRESS:

The allowable stress in wear is $S_w / F.S$.

The relationship between induced wear stress and tooth thickness is

$$s_w = \gamma/A(\sqrt{(M_t/t)})$$

Where

$$\gamma = \frac{0.59(i+1)\sqrt{(i+1)EK_cK_d}}{i\sqrt{\sin 2\alpha}}$$

The face width due to wear is calculated as

$$t_2 = (\gamma/A)^2 (M_t/s_w^2)$$

The greater of t_1, t_2 is taken as the face width of the gear pair.

III. PROBABILISTIC DESIGN:

Probabilistic design is a concept where by engineering variables are treated statistically. Each variable is modeled to reflect a spectrum of possible values. Classical equations are then adopted to yield meaningful results in terms of probability of failure. In probabilistic design, the power transmitted, speed of the input gear, allowable and induced stresses in

bending and surface wear and center distance of the gear pair are assumed as random variables. All the random variables are assumed to follow normal distribution. Deterministic data like number of teeth on gears (T_w, T_p), K_c, K_d, E , module and pressure angle are assumed to be known.

In the present work probability of failure of a gear is defined as the probability that induced bending stress or wear stress exceeds the corresponding strength of the material. Two failure modes namely the bending and surface wear modes are considered. The design parameter is the face width of gear and is taken as larger one out of the two values obtained in bending and surface wear failure modes. Approximate mean, standard deviation and coefficient of variation of induced shear stress can be obtained from relationships

$$f \approx f(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n)$$

$$\text{And } \sigma_f \approx [\sum_{x=1}^n (\partial f / \partial x_i)^2 (\sigma_{x_i})^2]^{1/2}$$

Which holds when the dispersion of each random variable, $C = \sigma_x / \bar{x}$ is less than 0.2.

3.1 FAILURE DUE TO BENDING STRESS:

The mean and standard deviation of s_b are given by

$$\bar{s}_b = \beta(\bar{M}_t) / (\bar{A} \bar{t}_1) \dots \dots \dots (1)$$

$$\sigma_{s_b} = (1/\bar{A} \bar{t}) [(\beta^2 \bar{M}_t^2 (\bar{t}^2 \sigma_A^2 + \bar{A}^2 \sigma_t^2) + \beta^2 \bar{A}^2 \bar{t}^2 \sigma_{M_t}^2) / \bar{A}^2 \bar{t}^2]^{1/2}$$

$$\text{Where } M_t = 72735 \bar{p} / \bar{n}_w$$

$$\sigma_{M_t} = \frac{72735}{\bar{n}_w} \sqrt{[\bar{P}^2 \sigma_{n_w}^2 + \bar{n}_w^2 \sigma_p^2] / \bar{n}_w^2}$$

The coefficient of variation of torque and induced bending stress can be expressed as:

$$C_{M_t}^2 = C_p^2 + C_{n_w}^2$$

$$C_{s_b}^2 = C_t^2 + C_A^2 + C_{M_t}^2$$

3.2 FAILURE DUE TO WEAR STRESS:

The mean and standard deviation of s_w are given as

$$\bar{s}_w = \gamma/\bar{A} \sqrt{(\bar{M}_t/\bar{t})} \dots \dots \dots (2)$$

$$\sigma_{s_w} = \frac{(1/\bar{A} \bar{t}) [4\bar{M}_t^2 \gamma^2 \bar{t}^2 \sigma_A^2 + \gamma^2 \bar{A}^2 \bar{M}_t^2 \sigma_t^2 + \bar{A}^2 \bar{t}^2 \gamma^2 \sigma_{M_t}^2]^{1/2}}{(4\bar{M}_t \bar{A}^2 \bar{t}^2)^{1/2}}$$

The coefficient of variation of induced wear stress can be expressed as

$$C_{sw}^2 = \frac{C_t^2 + C_{Mt}^2 + 4C_A^2}{4}$$

Standard normal variate in bending mode and surface wear mode are

$$Z_b = -[\bar{S}_b - \bar{s}_b]/[\sigma_{sb}^2 + \sigma_{sb}^2]^{1/2}$$

$$Z_w = -[\bar{S}_w - \bar{s}_w]/[\sigma_{sw}^2 + \sigma_{sw}^2]^{1/2}$$

By rearranging the standard normal variate equations introducing the coefficient of variations, the mean values of induced bending and surface wear stress are

$$\bar{s}_b = \frac{\bar{S}_b \pm [\bar{S}_b^2 - \bar{S}_b^2(1 - Z_b^2 C_{sb}^2)(1 - Z_b^2 C_{sb}^2)]^{1/2}}{(1 - Z_b^2 C_{sb}^2)}$$

$$\bar{s}_w = \frac{\bar{S}_w \pm [\bar{S}_w^2 - \bar{S}_w^2(1 - Z_w^2 C_{sw}^2)(1 - Z_w^2 C_{sw}^2)]^{1/2}}{(1 - Z_w^2 C_{sw}^2)}$$

By taking the smaller values of s_b , Z , s_w the mean values of t_1 & t_2 can be calculated by using equation (1) and equation (2). For the specified reliability of system (Gear box), component reliability (Gear pair) is calculated by considering the system as a series system. The face width of each gear pair is calculated for different output speeds and maximum out of these from two failure modes is considered as the face width of the gear pair. A program is developed in 'C++' to calculate the face width of the gear pair using the above two equations.

IV. EXAMPLES:

4.1 4 speed gear box:

4.1.1 kinematic arrangement of a gear box:

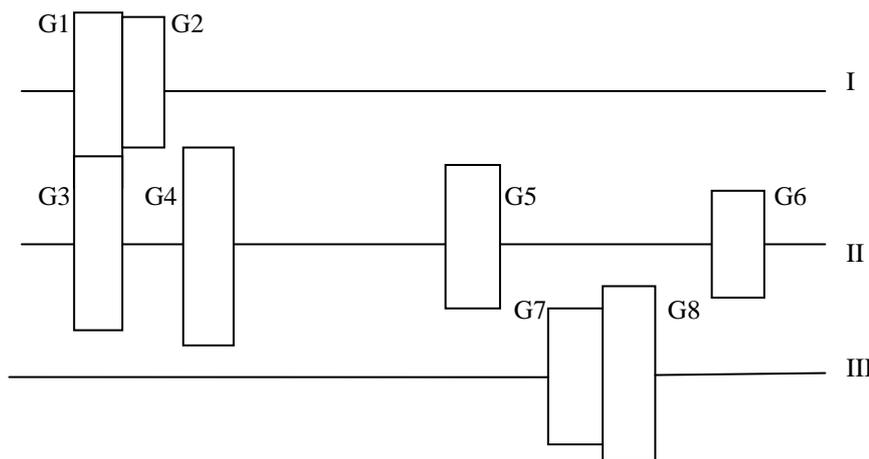


Fig1 kinematic arrangement of 4 speed gear box

4.1.2 The following data is considered while designing a 4 speed gear box:

The values of power(P), bending strength(S_b), wear strength(S_w) were taken as 10 H.P, 2500 Kg/cm² and 17500 Kg/cm² respectively and $K_c = 1.5, K_d = 1.1, \alpha = 20^\circ, C_p = C_{nw} = C_{sw} = 0.1, C_t = C_a = 0.01, E_g = E_p = 2.1 \times 10^6$ Kg/cm². Speeds are 400rpm, 560rpm, 800rpm, 1120rpm. Number of gear teeth of a gearbox G1=24, G2=20, G3=24, G4=28, G5=27, G6=18, G7=27, G8=36 Torques's of 4 gear pairs is 17.9kg-m, 12.79kg-m, 8.95kg-m, 6.39kg-m, and assume the standard module $m=4$ mm

4.2 Six speed gear box:

4.2.1 kinematic arrangement of gear box:

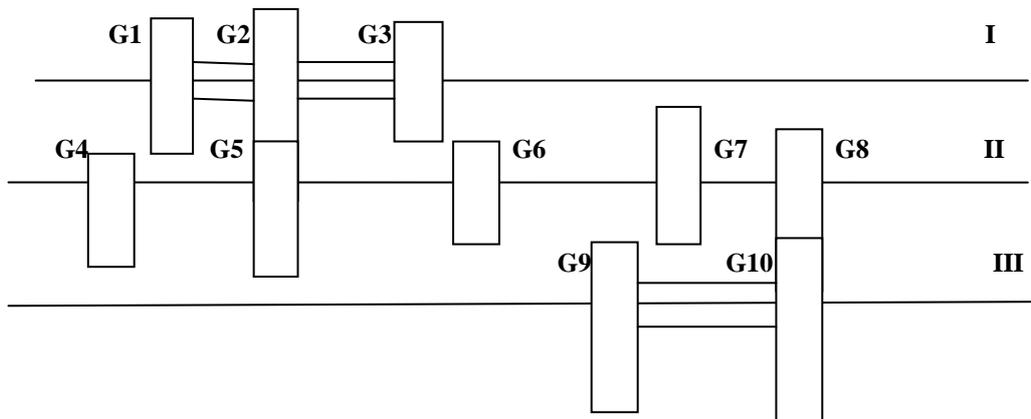


Fig2 kinematic arrangement of 6 speed gear box

4.2.2 The following data is considered while designing a 4 speed gear box:

Power=10HP, $S_w=17500 \text{ kg/cm}^2$, $S_b=2500 \text{ kg/cm}^2$ and other data is assumed same as that of 6 speed gear box. Speeds are 100rpm, 140rpm, 200rpm, 280rpm, and 400rpm, Number of gear teeth of a gearbox $G1=25$, $G2=20$, $G3=30$, $G4=35$, $G5=40$, $G6=30$, $G7=38$, $G8=20$, $G9=38$, $G10=56$. Torques's of a 5 gear pairs is 71.62kg-m, 51.2kg-m, 35.8kg-m, 25.6kg-m, 17.9kg-m, 12.8kg-m and assume the standard module $m=5\text{mm}$

V. RESULTS:

The values of the face widths of the gear pairs for four speed and six speed gear boxes are calculated using a C++ program.

- 1 The power is taken as 10 HP and the coefficient of variation of speed is varied from 0.01 to 0.1. The face widths obtained for these values are represented by graph.
- 2 The coefficient of variation of speed is kept as constant ($C_{nw}=0.1$) and the power is varied from 2.5 HP to 20 HP. The face widths obtained for these values are represented by graph.
- 3 The coefficient of variation of speed and the power are kept as constant ($C_{nw}=0.1$, $P = 10 \text{ HP}$) and the probability of failure is varied from 1×10^{-1} to 1×10^{-6} and the face widths obtained for these values are represented by graph.
- 4 Results obtained in deterministic design are tabulated in table 1 & table 2 for four speed and six speed gear boxes.

5.1 TABLES:

The face widths of gear pairs in a 4 speed gear box obtained by taking $C_{nw}= 0.1$ are shown in Table 1. .Input given: Pressure angle=20 degrees; Power=10HP; $Z=2.574899$.

Table: 1 –face width of gear pairs using deterministic design for four speed gear box

GEAR PAIR	DETERMINISTIC DESIGN(FS=2)
(G1,G3)	1.382
(G2,G4)	1.705
(G5,G7)	1.528
(G6,G8)	2.579

The face widths of gear pairs in a 6 speed gear box obtained by varying $c_{nw} = 0.1$ are shown in the table 2 .Input given: pressure angle=20 degrees; power=10hp; $z=2.574899$.

Table: 2 – face width of gear pairs using deterministic design for six speed gear box

GEAR PAIR	DETERMINISTIC DESIGN(FS=2)
(G1,G4)	1.397
(G2,G5)	1.9099
(G3,G6)	1.132
(G7,G9)	1.4109
(G8,G10)	3.456

5.2 GRAPHS:

5.2.1 4 speed gear box:

Face width is taken as Y-axis and C_{nw} is taken as X-axis. It is clear that the face widths of gear pairs increase with increase in C_{nw} .

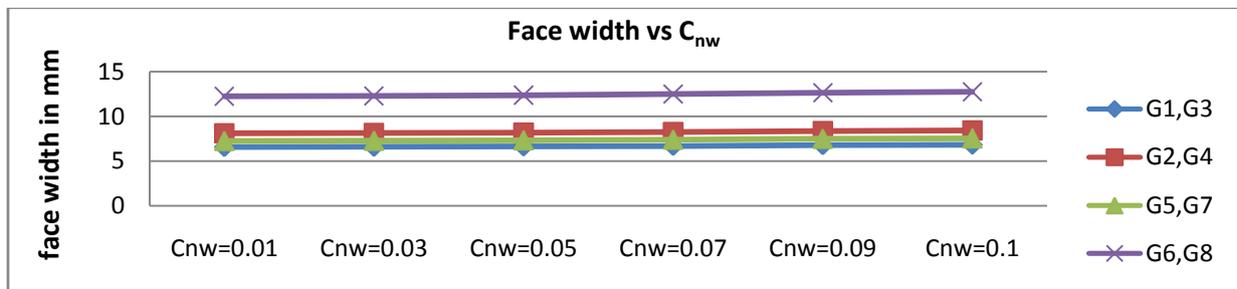


Fig3 Effect of C_{nw} on face width of gear pairs

Face width is taken as Y-axis and power is taken as X-axis. It is clear that from the graph, face widths of gear pairs increases with increase power.

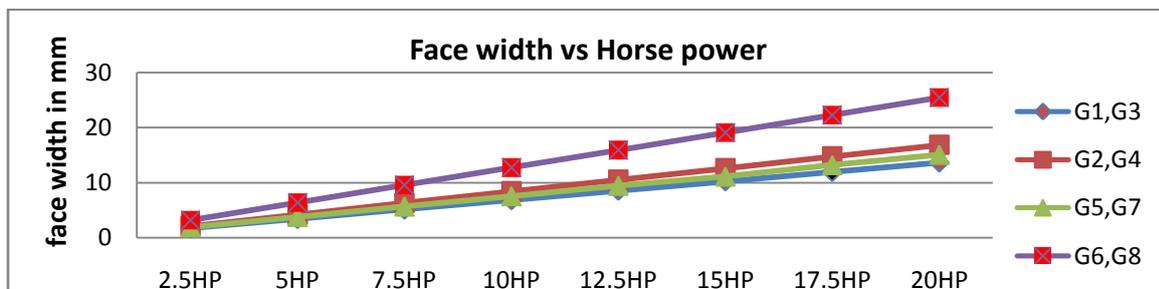


Fig4 Effect of horse power on face width of gear pairs

Face width is taken as Y-axis and Reliability is taken as X-axis. It is evident that the face widths of gear pairs increase with increase in reliability.

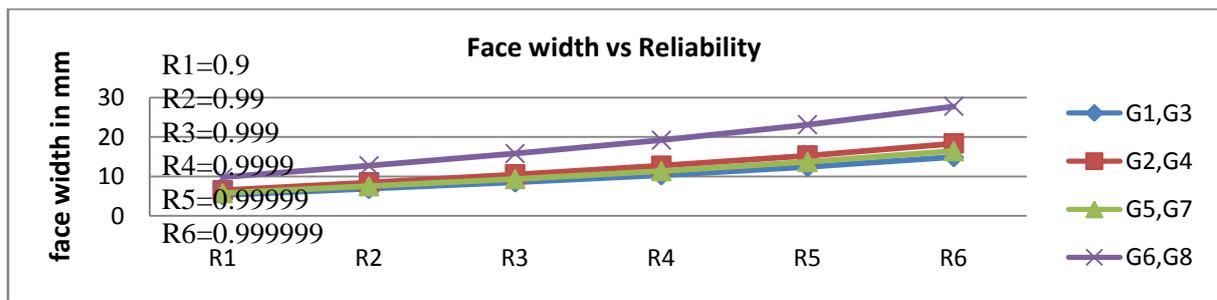


Fig5 Effect of reliability on face width of gear pairs

5.2.2 6 SPEED GEAR BOX

Face width is taken as Y-axis and C_{nw} is taken as X-axis. It is shown that the face widths of gear pairs increase with increase in C_{nw} .

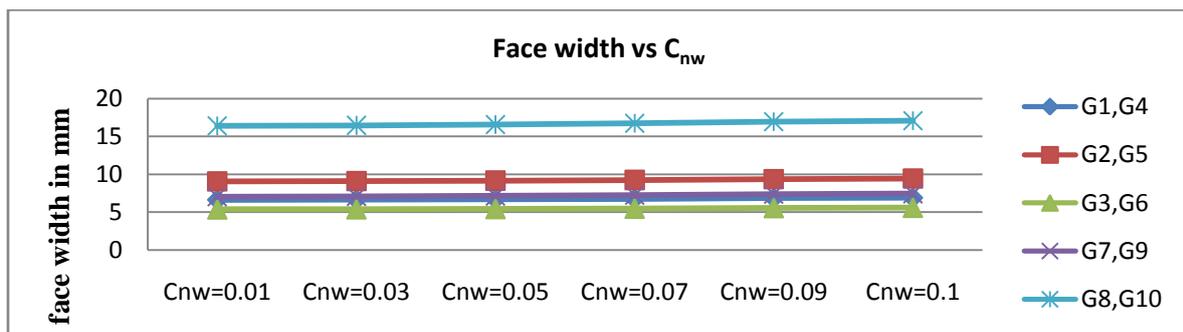


Fig6 Effect of C_{nw} on face width of gear pairs

Face width is taken as Y-axis and power is taken as X-axis. It is clear that from the graph, face widths of gear pairs increases with increase in power.

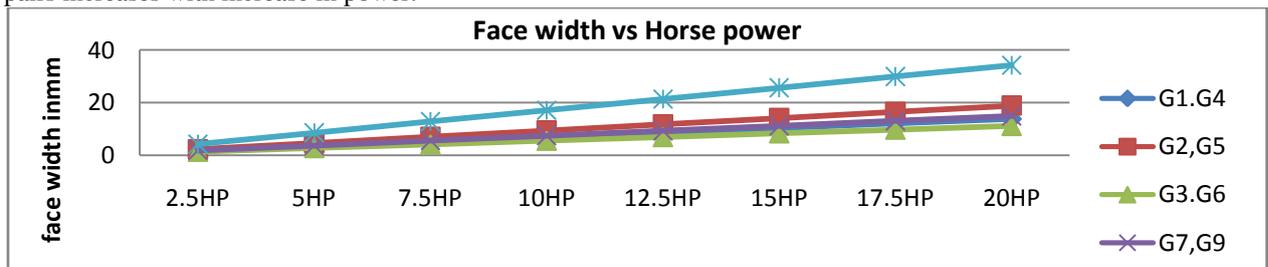


Fig7 Effect of horse power on face width of gear pairs

Face width is taken as Y-axis and Reliability is taken as X-axis. It is evident that the face widths of gear pairs increase with increase in reliability.

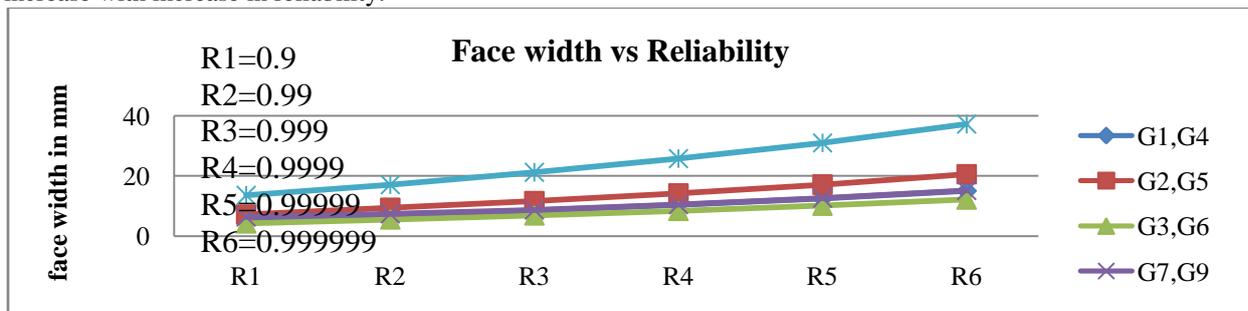


Fig 8 Effect of reliability on face width of gear pairs

5.3 CONCLUSIONS:

1. Reliability based design procedure is more realistic and useful.
2. In the present work, normal distribution for the variables is assumed. Even, if the variables follow other than normal distribution, the basic approach applied here will not change.
3. The probabilistic design procedure is quite general and can be applied to any machine element.
4. Over designing of the elements can be avoided with the help of probabilistic design.
5. It is evident from the above results that as the reliability increases face width of the gear pair also increases.
6. As the coefficient of variation of speed increases, the face width of all gears increases.
7. As the input power increases face widths of the gear pairs increase linearly.

REFERENCES

- [1.] E.Balaguruswamy: *Reliability engineering*, Tata McGraw Hill Publishers, 2002.
- [2.] O.C.Smith: *Introduction to Reliability in Design*, Tata McGraw Hill publishers, Inc., 1976.
- [3.] Reshtov.D. Ivanov.A and Fadeev.V: *Reliability of Machines*, Mir publishers, Moscow, 1990.
- [4.] K.C.Kapur and L.R.Lamberson: *Reliability in Engineering Design*, John Wiley & sons,
- [5.] E.B.Haugen: *Probabilistic Mechanical Design*, Wiley inter-science, Newyork, 1980..
- [6.] A.Mubeen: *Machine Design*, Khanna publishers, Delhi, 1995.
- [7.] R.K.Jain: *Machine Design*, Khanna publishers, New Delhi, 2005.
- [8.] G.C.Sen and A. Bhattacharya: *Principles of Machine tools*, New Central book agency, 1999.
- [9.] K.Lingaiah: *Machine Design Data Hand Book*, vol II, Suma publishers, Bangalore, 1984.
- [10.] G.M.Maitra: *Hand book of Gear Design*, Tata McGraw Hill book co, Newyork, second edition, 2005.
- [11.] George E.Dieter: *Engineering Design*, McGraw Hill International editions, second edition, 2005..
- [12.] Mahadevan and Balaveera Reddy: *Machine Design Data Hand Book*, CBS Publishers, New Delhi, 1996.

NOTATIONS

P=Power in H.P	n_w = wheel speed
F.S=factor of safety	M_t =Torque acting on the gear teeth (kg/cm)
s_b =Induced bending stress (kg/cm ²)	S_b =Bending strength (kg/cm ²)
A=center distance between the gears (cm)	t=face width (cm)
K_c =stress concentration factor	K_d =dynamic load factor
i=transmission ratio	T_w =no. of teeth on the wheel
T_p =no. of teeth on the pinion	α =pressure angle
Y=lewis factor	s_w =induced wear stress (kg/cm ²)
E_g =young's modulus of the material of the gear (kg/cm ²)	S_w =wear strength (kg/cm ²)
E_p =young's modulus of the material of the Pinion (kg/cm ²)	Z=standard normal variable
t_1 =face width obtained for bending (cm)	t_2 =face width obtained for wear (cm)
X=random variable	\bar{x} =mean value of x
σ_x =standard deviation of x	C_x =coefficient of variation of x
Pf=probability of failure	R=reliability= (1-pf)