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Analysis of Isotropic Perforated Stiffened Plate Using FEM

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ABSTRACT
In this paper, studied the structural instability caused by circular and square perforated plate having all round simply supported and clamped plate boundary conditions subjected to inplane uniaxial compression loading and also the strengthening effect of the stiffeners when they are reinforced to the unperforated and perforated plate in longitudinal and transverse directions. The area ratios, aspect ratios and types of stiffeners are the parameters considered. A general purpose finite element analysis software ANSYS is used to carry out the study. Results show that the presence of a central circular and square perforations causes reduction in buckling strength of plate and stiffeners can be used to compensate this reduction before it can be used to its best advantage. It is also observed that the plate strengthening effect of longitudinal stiffener is more than that of transverse stiffener for both unperforated and perforated plate. An economical design is possible by introducing stiffeners of optimum size.
Keywords – Buckling load factor, Cutout, Finite element method, Inplane loads, Stiffeners.

I. INTRODUCTION

Plates are used in civil, mechanical and aerospace industries. Generally, in plates cutouts are provided to decrease the self-weight, to provide access from across the plate. However, though the cutouts are provided to achieve certain structural advantages, it is worth to mention here that they may inadvertently affect the stability of the plate component in the form of buckling. The presence of cutouts results in a redistribution of the membrane stresses with change in mechanical behaviors of the plates. When the cutout is inevitable for the plates under high working stress, the reduced buckling strength of the perforated plate may be insufficient to meet the requirements of normal serviceability limits and structural safety. Hence the study of stability behavior is of paramount importance.

The stability of the plate always can be increased by increasing its thickness, but such a design will not be economical in respect to the weight of material used. An economical solution is obtained by keeping the thickness of the plate as small as possible and increasing the stability by introducing reinforcing stiffeners. Hence plates do need some additional flexural stiffness in the form of stiffeners.

The stability of plates under various compressive loadings and boundary conditions has been the subject and studied by Timoshenko.S.P and Gere .J.M. [1] and many others. Stability of plates using the finite element method carried out by Kanwar K, Kapur and Billy j. Hartz [2]. The paper compared the buckling loads obtained by the stability co-efficient matrix approach and the available exact solutions. M R Purohit [3] investigated the structural instability caused by a plain circular perforation for simply supported square plates under edge compression and also for those plates reinforced by two symmetric stiffeners in longitudinal and transverse manner based on the principle of minimum potential energy. A.K.L.Srivastava et.al [4] have used the finite element method to study the Elastic stability of square stiffened plates with cutout under biaxial loading by assuming that the forces to act in the plane of the undeformed middle surface of the plate and the characteristic equations for the natural frequencies, buckling loads and their corresponding mode shapes were obtained from the equation of motion. The ultimate strength behaviour of longitudinally stiffened plates with openings under axial compression was studied by M. Mahendran et.al [5] using second-order elastic and rigid-plastic analyses and laboratory experiments. Effects of the size of opening, the initial geometrical imperfections and the plate slenderness ratio on the strength of perforated stiffened plates were also studied. Lars Brubak et.al [6] investigated the applicability of various strength criteria that may be incorporated into semi-analytical methods for ultimate strength prediction of arbitrarily stiffened plates in local and global bending.

The present study is to exhibit the importance of providing stiffeners in enhancing the stability limits of structural plate like components under inplane uniaxial compression loading having simply supported and clamped plate boundary conditions with different aspect ratios and area ratios. To carry out the study, ANSYS software has been used with 8SHELL93 element [7]. The material of the plate is assumed to be homogeneous, isotropic and elastic with young's modulus E=210924 N/mm^2 and Poisson's ratio μ= 0.3. The model plate considered, 10X10 mesh, exhibits the accurate results [1]. The constraint equations are used to give contact between plate and stiffener.

II. PROBLEM DEFINITION

The problem of buckling of plate subjected to uniaxial compression loading along its ends is considered, Fig (1). The perforated plate having different shapes of cutouts such as circular and square with two longitudinal and transverse stiffeners with different boundary conditions such as all-round simply supported (SSSS) and clamped (CCCC) are considered. The plate has thickness t and dimensions a and b in x and y-directions, respectively. A circular cutout with radius R, square cutout of size A x B is considered in this study.

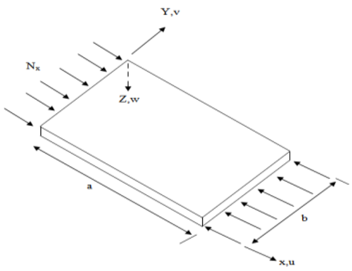

Fig. 1 Geometry and loading of the plate

2.1 Parameters considered in the study

The objective of the present study is to determine the buckling strength of isotropic plate with/without perforations of different shapes such as circular and square by providing stiffeners along longitudinal and transverse direction with various aspect ratios and area ratios when subjected to inplane uniaxial compression loading. The following parameters are considered.

- Aspect ratio of the plate (β = a/b=0.5 to 2.0)
- Area ratio of the stiffener and plate (δ = 0.05 to 0.2)
- Ratio of diameter of the circular cutout to the side of the square plate (η = 0.10 to 0.20)
- Ratio of the side of the square cutout to the side of square plate (ξ = 0.2 to 0.4)
- All round simply supported (SSSS) and clamped (CCCC) plate boundary conditions and
- Nature of the load is inplane uniaxial compression.

2.2 Boundary conditions of the plate

The critical buckling stress (σ_{cr}) of the uniaxially loaded plate are greatly affected by the plate's boundary conditions. When the four edges of the plate are simply supported, there will be no lateral edge displacements perpendicular to the plate's plane but rotations about the axis of each plate edge are allowed. When the four edges of the plate are clamped, there will be no lateral edge displacements and rotations. The degree of rotational restrain at the plate boundary for real plate may be somewhere between all CCCC and SSSS extremes. Hence the study is carried out for both simply supported and clamped plate boundary conditions.

III. FINITE ELEMENT FORMULATION

The effect of inplane deformations is taken into account in addition to the deformations due to bending. A eight-noded isoparametric element with six degrees of freedom (u, v, w, θ_x , θ_y and θ_z) per node is employed in the present analysis. The element matrices of the stiffened plate element consist of the contribution of the plate and that of the stiffener. The contribution of the stiffener to a particular node depends on the proximity of the stiffener to that node. For a given edge loading and boundary conditions, the static equation i.e., $[K]\ \{\Delta\} = \{F\}$ is solved to get the stresses. The geometric stiffness matrix is now constructed with the known stresses. The overall elastic stiffness matrix and geometric stiffness matrix are generated from the assembly of those element matrices and stored in a single array where the variable bandwidth profile storage scheme is used. The elastic stiffness matrix $[K_P]$ and geometric stiffness matrix $[K_{GP}]$ of the plate element may be expressed as follows

$$[K_P]=\int_0^1\int_0^1 [B_P]^T [D_P][B_P] |J_P| d\xi d\eta$$

$$[K_{GP}]=\int_0^1\int_0^1 [B_{GP}]^T [\sigma_P][B_{GP}] |J_P| d\xi d\eta$$

The elastic stiffness matrix $[K_s]$ and geometric stiffness matrix $[K_{Gs}]$ of a stiffener element placed anywhere within a plate element and oriented in the direction of x may be expressed, in a manner similar to that of the plate element as follows,

$$[K_S]=\int_0^1 [B_S]^T [D_S][B_S] |J_S| d\xi$$

$$[K_{GS}]=\int_0^1 [B_{GS}]^T [\sigma_S][B_{GS}] |J_S| d\xi$$

Where,

$[B_P] = [[B_P]_1\ [B_P]_2 \ldots [B_P]_r \ldots [B_P]_8]$;

$[B_{GP}] = [[B_{GP}]_1\ [B_{GP}]_2 \ldots [B_{GP}]_r \ldots [B_{GP}]_8]$

$[B_s] = [[B_s]_1\ [B_s]_2 \ldots [B_s]_r \ldots [B_s]_8]$;

$[B_{Gs}] = [[B_{Gs}]_1\ [B_{Gs}]_2 \ldots [B_{Gs}]_r \ldots [B_{Gs}]_8]$

and $|J_S|$ is the Jacobian of the stiffener, which is one-half of its actual length within an element.

The equation of equilibrium for the stiffened plate subjected to inplane loads can be written as,

$$[[K_b] - P\ [K_G]]\{q\} = 0$$

IV. RESULTS AND DISCUSSIONS

This section presents the results of buckling load factor k with respect to aspect ratios and area ratios of unperforated plate reinforced with central stiffener in longitudinal and transverse direction and also for perforated plate reinforced with two longitudinal and

transverse stiffeners subjected to inplane uniaxial compression loading having SSSS and CCCC plate boundary conditions.
The buckling load factor (k) is given by,

$$k = \frac{N_{cr} b^2}{\pi^2 D}$$

Where,

D = Plate flexural rigidity = $\frac{E t^3}{12\,(1-\mu^2)}$

N_{cr} = Critical buckling load.
t = Thickness of the plate.
b = Width of the plate

4.1 Comparative studies

The results obtained from the present work have been tabulated with the standard results in Table 1, Table 2 and Table 3.

Table 1

Comparison of buckling load factor (k) for SSSS isotropic unperforated plate subjected to inplane uniaxial compression loading. (t = 6 mm; μ = 0.3)

a in mm	b in mm	Aspect ratio □ =a/b	Value of k [1]	Present study k
120	600	0.2	27.0	26.84
240	600	0.4	8.41	8.391
360	600	0.6	5.14	5.130
480	600	0.8	4.20	4.198
600	600	1.0	4.00	3.996
720	600	1.2	4.13	4.131
840	600	1.4	4.47	4.467
846	600	1.41	4.49	4.488

Table 2

Comparison of buckling load factor (k) for CCCC isotropic unperforated plate subjected to inplane uniaxial compression loading. (t=6mm;μ=0.3)

a in mm	b in mm	Aspect ratio □ =a/b	Value of k [1]	Present study k
600	600	1.0	10.07	10.05
750	600	1.25	9.25	9.26
900	600	1.5	8.33	8.36
1050	600	1.75	8.11	8.11
1200	600	2.0	7.88	7.91
1350	600	2.25	7.63	7.68
1500	600	2.5	7.57	7.64
1650	600	2.75	7.44	7.60
1800	600	3.0	7.37	7.54

Table 3

Comparison of buckling load factor (k) of SSSS isotropic square plate for uniaxial compression. (t = 6mm ; μ = 0.3)

Plate Type	Present Study Value	Reference Value
Solid Plate	3.996	4.0[1]
Plate having Circular cutout	3.9	3.896[3]
Solid plate with longitudinal stiffener	16.64	11.67[1]
Solid plate with transverse stiffener	6.28	6.5[1]
Plate having circular cutout with longitudinal stiffener	14.63	14.26[3]
Plate having circular cutout with transverse stiffener	10.84	10.55[3]

4.2 Analysis of isotropic unperforated SSSS plate having central stiffener with various aspect ratios and area ratios

In this section, the variation of buckling load factor k for simply supported unperforated plate having central longitudinal/transverse stiffeners with various aspect ratios(β) and area ratios(δ) subjected to inplane uniaxial compression loading are discussed, Fig 2 and 3.

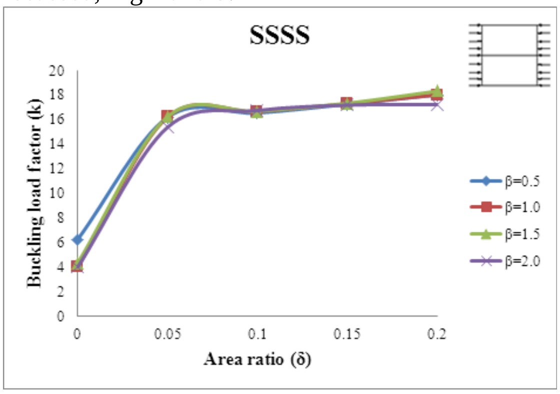

Fig.2: Variation of k with respect to β and δ for unperforated plate having central longitudinal stiffener.

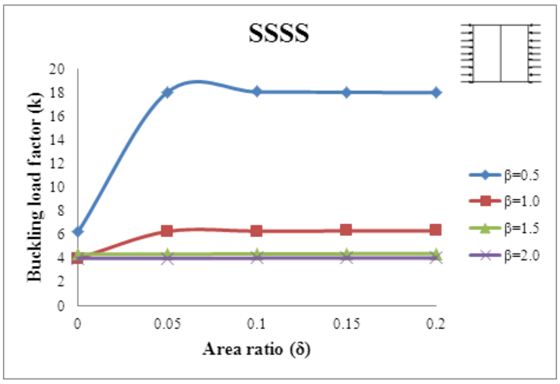

Fig.3: Variation of k with β and δ for unperforated plate having central transverse stiffener.

In Fig.2, it is clearly indicating that there is a linear increment of k and reaches maximum at δ = 0.05 and this increase is less in magnitude when δ>0.1. The increase in k is 1.64, 3.17, 2.84 and 3.19 times and it is 1.88, 3.51, 3.22 and 3.31times with central longitudinal stiffened plate when compared with unstiffened plate at β = 0.5 to 2 and δ = 0.1 and δ = 0.2 respectively. In Fig.3, the value of k increases linearly when β = 0.5 and at δ = 0.05 and it is 1.89 times higher than the unstiffened plate. For β =1.0, the value of k decreases about 65% for δ= 0.05 to 0.2 compared with β= 0.5. The increase in k is negligible when β ≥1.5 and δ ≥ 0.05.

4.3 Analysis of isotropic perforated SSSS plate having stiffeners with various aspect ratios and area ratios

In this section, the variation of buckling load factor k for simply supported plate with circular/square perforations having two symmetric longitudinal/transverse stiffeners with various aspect ratios(β) and area ratios(δ) subjected to inplane uniaxial compression loading are discussed, Fig.4 to Fig.7.

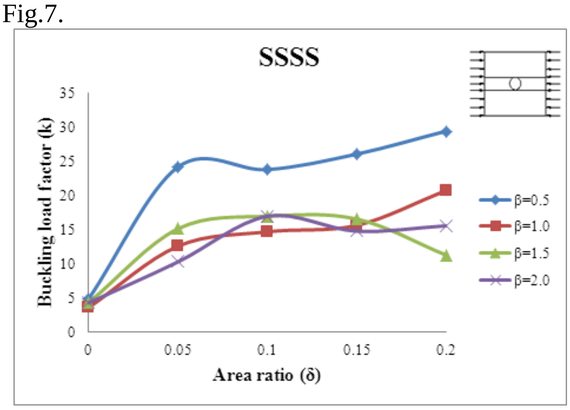

Fig.4: Variation of k for plate with circular cutout reinforced by two longitudinal stiffeners with respect to β and δ.

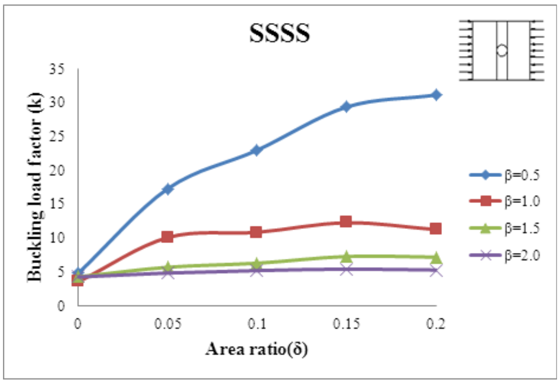

Fig.5: Variation of k for plate with circular cutout reinforced by two transverse stiffeners with respect to β and δ.

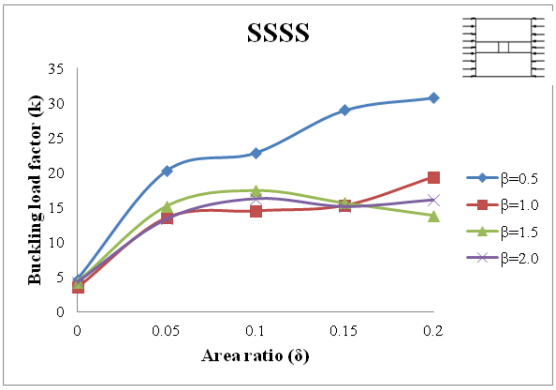

Fig.6: Variation of k for plate with square cutout reinforced by two longitudinal stiffeners with respect to β and δ.

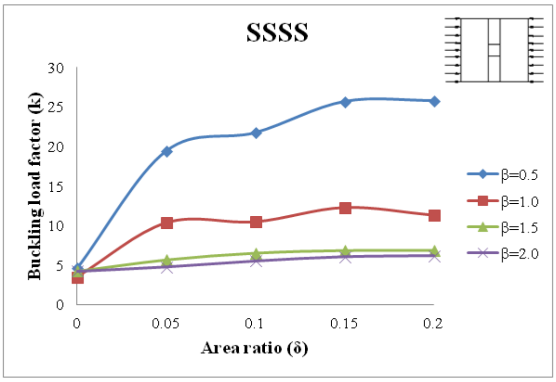

Fig.7: Variation of k for plate with square cutout reinforced by two transverse stiffeners with respect to β and δ.

In Fig.4 and Fig.6, the variation of k for β = 0.5 is linear upto δ =0.05 and it increases about 4 to 5 times with respect to δ= 0.05 to 0.2 when compared with the perforated plate without stiffener. For β = 1.0, the value of k decreases to 25 to 45% with respect to δ=0.05 to 0.2 compared with β = 0.5. For β = 1.5, k increases about 15 to 21% upto δ=0.05. The plate with circular/square perforation having two longitudinal stiffeners have the same variation of k.

In Fig.5 and Fig.7, the variation of k for β = 0.5 is linear upto δ =0.05 and it increases to 2 to 5 times with respect to δ= 0.05 to 0.2 when compared with the perforated plate without stiffener. For β ≥ 1 the variation of k is negligible. The plate having circular cutout with two transverse, the value of k is more than that of the plate having square cutout with two transverse stiffener.

4.4 Analysis of isotropic unperforated CCCC plate with central stiffener with various aspect ratios and area ratios

In this section, the variation of buckling load factor k for clamped unperforated plate having central longitudinal and transverse stiffeners with various aspect ratios(β) and area ratios(δ) subjected to inplane uniaxial compression loading are discussed, Fig.8 and Fig.9.

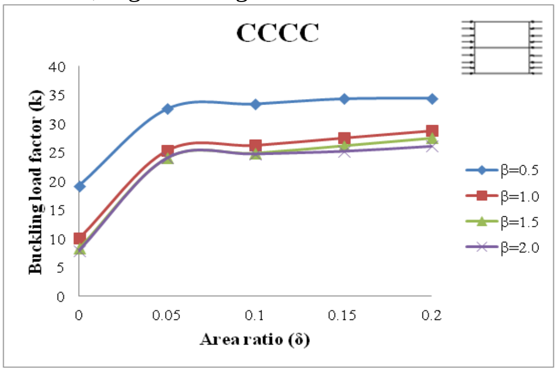

Fig.8: Variation of k with respect to β and δ for unperforated plate having central longitudinal stiffener.

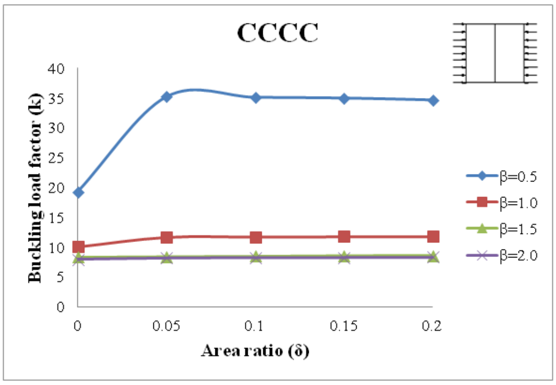

Fig.9: Variation of k with respect to β and δ for unperforated plate having central transverse stiffener.

In Fig.8, the value of k increases linearly about 0.69 times for β = 0.5 and δ=0.05 and further increment in k is about 0.76 times for δ= 0.1 to 0.2 compared with the unstiffened plate. For β ≥1.0, the value of k decreases about 20 to 48% for δ=0.05 to 0.2 compared with β=0.5. In Fig.9, the value of k increases linearly upto δ=0.05 and negligible variation is observed afterwards. When β > 0.5, k decreases upto 67% for δ=0.05 to 0.2.

4.5 Analysis of isotropic perforated CCCC plate having stiffeners with various aspect ratios and area ratios

In this section, the variation of buckling load factor k for clamped plate with circular/square perforation having two symmetric longitudinal/transverse stiffeners with various aspect ratios(β) and area ratios(δ) subjected to inplane uniaxial compression loading are discussed, Fig.10-Fig.13.

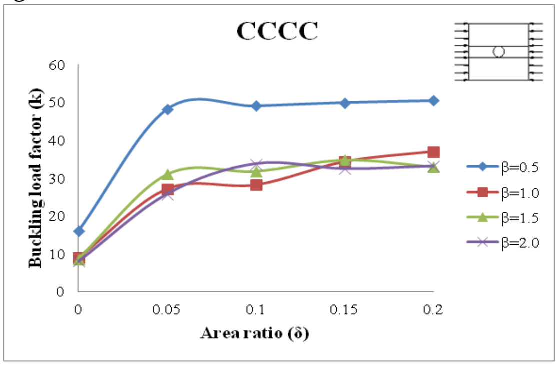

Fig.10: Variation of k for plate with circular cutout reinforced by two longitudinal stiffeners with respect to β and δ.

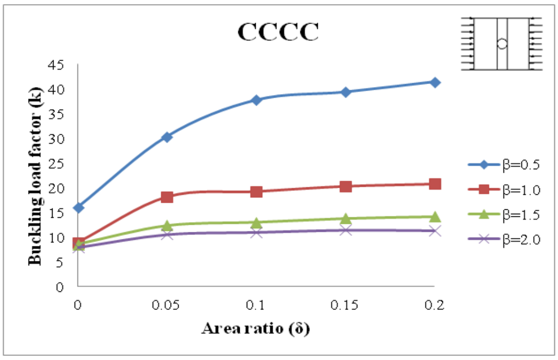

Fig.11: Variation of k for plate with circular cutout reinforced by two transverse stiffeners with respect to β and δ.

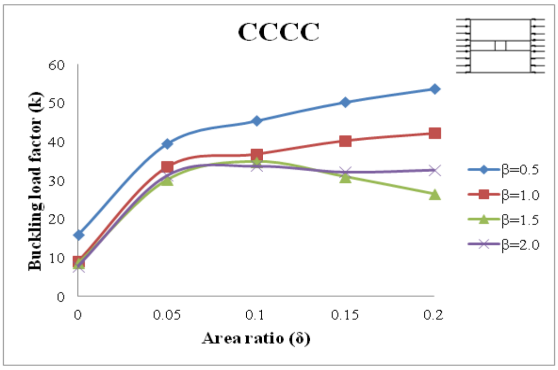

Fig.12: Variation of k for plate with square cutout reinforced by two longitudinal stiffeners with respect to β and δ.

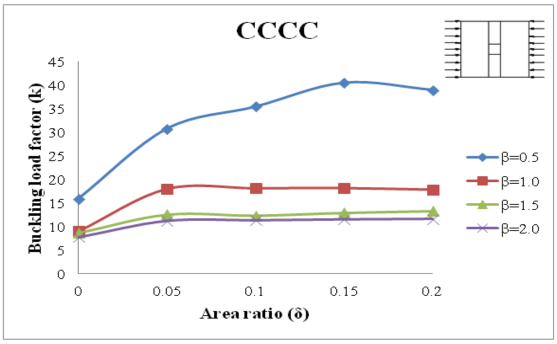

Fig.13: Variation of k for plate with square cutout reinforced by two transverse stiffeners with respect to β and δ.

In Fig.10, the variation of k for β = 0.5 is linear upto δ =0.05 and it increases about 2 times when compared with the unstiffened perforated plate. For β = 1.0 and δ=0.05 to 0.2, the value of k decreases about 28% to 44% compared to β = 0.5. For β = 1.5 and δ upto 0.15, the value of k increases about 2 to 3 times compared with unstiffened perforated plate. In Fig.11 and Fig.13, for β= 0.5 increase in k value is about 1.5 times for δ = 0.05 to 0.2 when compared with the perforated plate without stiffener. For β = 1.0, the variation in k is 40% to 55% less than that for β = 0.5 and δ = 0.05 to 0.2. In Fig.12, the variation of k for β = 0.5 is linear upto δ =0.05 and it increases about 1 to 2 times when compared with the unstiffened perforated plate. For β=1.0, the variation of k is 15% to 45% for δ= 0.05 to 0.2 compared with β= 0.5. For β≥1.5, the value of k increases about 2 times upto δ = 0.1, for δ >0.1 it decreases when compared with the unstiffened perforated plate.

4.6 Analysis of isotropic perforated SSSS square plate having stiffeners with various area ratios and cutout ratios

In this section, the variation of buckling load factor k for simply supported plate with circular/square perforation having two symmetric longitudinal/transverse stiffeners with various area ratios(δ) and cutout ratios(η and ξ) subjected to inplane uniaxial compression loading are discussed Fig.14 to Fig.17.

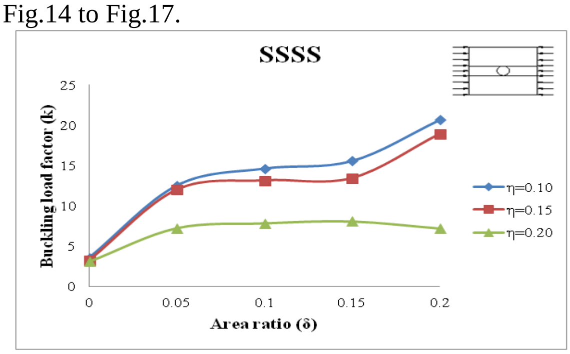

Fig.14: Variation of k for perforated square plate reinforced by two longitudinal stiffeners with respect to δ and η.

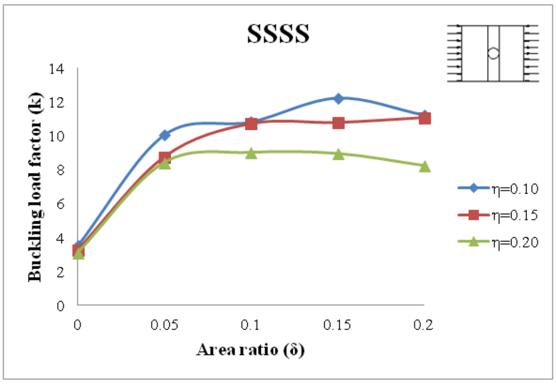

Fig.15: Variation of k for perforated square plate reinforced by two transverse stiffeners with respect to δ and η.

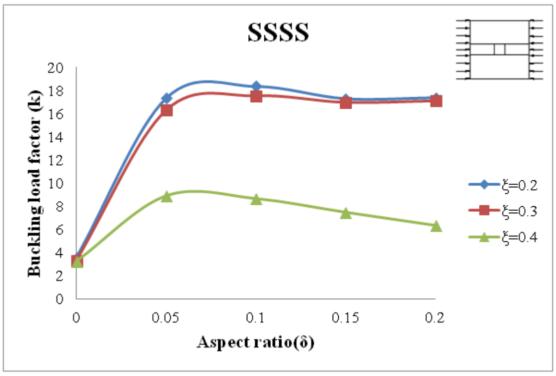

Fig.16: Variation of k for perforated square plate reinforced by two longitudinal stiffeners with respect to δ and ξ.

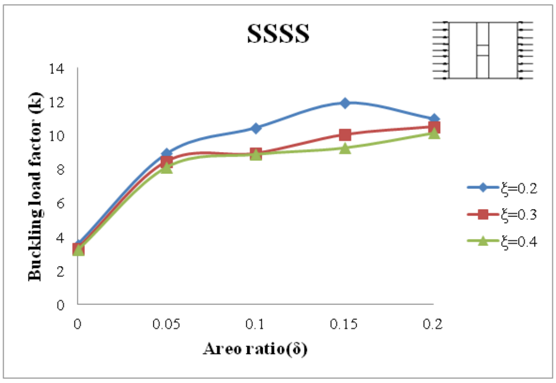

Fig.17: Variation of k for perforated square plate reinforced by two transverse stiffeners with respect to δ and ξ.

In Fig.14, the variation of k for η=0.10 is linear upto δ = 0.05 and it increases about 2 to 4 times when compared with the perforated plate without stiffener. For η =0.15, the value of k decreases about 4% to 15% compared with η = 0.10. For η=0.2, the value of k is less. In Fig.15, the variation of k for η =0.10 is linear upto δ = 0.05 and it increases about 1.8 to 2.4 times when compared with the perforated plate without stiffener. For η = 0.15, the value of k decreases about 1% to 13% compared with η = 0.10. Further increasing the aspect ratio, the value of k decreases. In Fig.16, the variation of k for ξ =0.2 and ξ =0.3 is linear upto δ = 0.05 and it increases about 3 to 4 times when compared with the perforated plate without stiffener. For ξ = 0.4, the value of k decreases about 1 to 2 times when compared with the perforated plate without stiffener. In Fig.17, the variation of k for ξ =0.2 is linear upto δ = 0.05 and it increases about 1 to 2.5 times when compared with the perforated plate without stiffener. For ξ =0.3 to 0.4, the value of k increases linearly about 1.5 times upto δ = 0.05 and further slight increase in k is observed.

4.7 Analysis of isotropic perforated CCCC square plate having stiffeners with various area ratios and cutout ratios

In this section, the variation of buckling load factor k for clamped plate with circular/square perforation having two symmetric longitudinal/transverse stiffeners with various area ratios(δ) and cutout ratios(η and ξ) subjected to inplane uniaxial compression loading are discussed, Fig.18 to Fig.21.

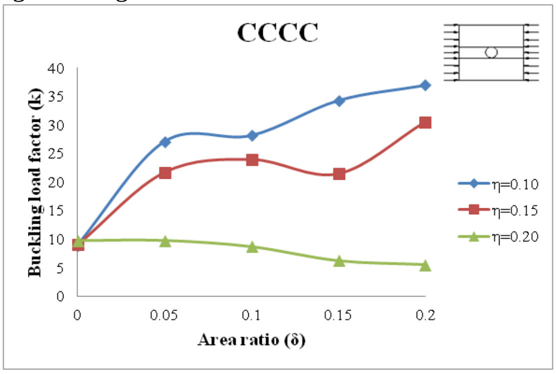

Fig.18: Variation of k for perforated square plate reinforced by two longitudinal stiffeners with respect to δ and η.

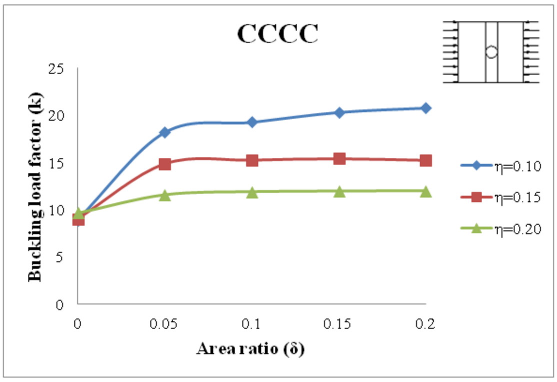

Fig.19: Variation of k for perforated square plate reinforced by two transverse stiffeners with respect to δ and η.

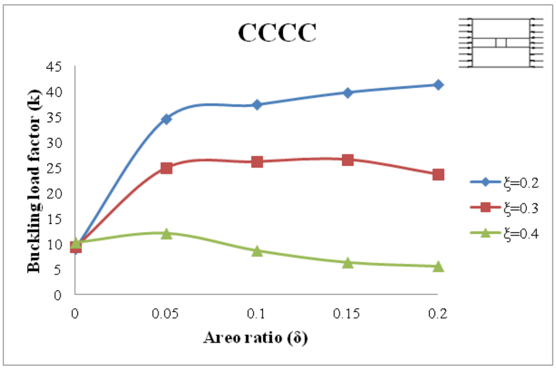

Fig.20: Variation of k for perforated square plate reinforced by two longitudinal stiffeners with respect to δ and ξ.

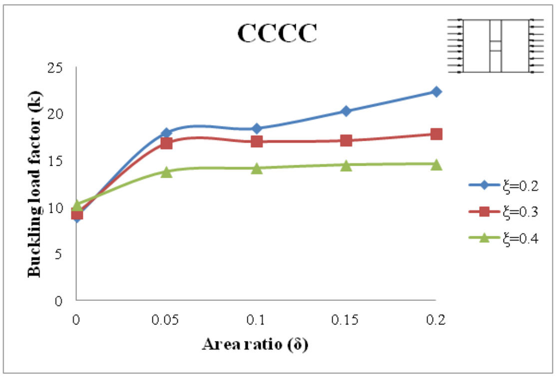

Fig.21: Variation of k for perforated square plate reinforced by two transverse stiffeners with respect to δ and ξ.

In Fig.18, the variation of k for η =0.10 is linear upto δ = 0.05 and it increases about 2 to 3 times when compared with the perforated plate without stiffener. For η =0.15, the value of k increases upto δ = 0.1 it is 1.6 times and at δ = 0.15 it decrease to 1.38 times and further increases about 2.38 times compared with unstiffened perforated plate. For η=0.2 and δ =0.05 to 0.2, the value of k decreases about 55% to 81% compared to η = 0.15. In Fig.19, the variation of k for η =0.10 is linear upto δ = 0.05 and it increases about 1 time when compared with unstiffened perforated plate. For η= 0.15, the value of k decreases compared to η = 0.1. In Fig.20, the variation of k for ξ = 0.2 is linear upto δ = 0.05 and it increases about 2 to 3 times when compared with the perforated plate without stiffener. For ξ = 0.3 and ξ = 0.4, the value of k decreases about 20% to 42% and 51% to 76% for δ =0.05 to 0.2 compared with ξ =0.2 and ξ = 0.3 respectively. In Fig.21, the variation of k for ξ = 0.2 is linear upto δ = 0.05 and it increases about 1 time when compared with the perforated plate without stiffener. For ξ = 0.3, the value of k decreases when compared with ξ = 0.2 and the variation in k is negligible for δ >0.05.

V. CONCLUSION

Based on the results obtained in the present study, the following conclusions are drawn:

1. The value of k in CCCC unperforated plate with central longitudinal and transverse stiffeners is approximately 48 – 50% greater than the SSSS plate for β=0.5 and δ = 0.05 to 0.2.
2. The influence of central transverse stiffener on unperforated plate is less when compared to central longitudinal stiffener.
3. When the cutout ratio increases the buckling load decreases. The conventional wisdom is that, as the cutout ratio increases, the plate losses more material and consequently loss more buckling strength.
4. The two longitudinal stiffeners on either side of the circular perforation of the plate the buckling load factor k increases.
5. The buckling load factor k in CCCC plate with circular perforation having two longitudinal and transverse stiffeners is approximately 41-50% and 24-43% greater than the SSSS condition for β=0.5 and δ = 0.05 to 0.2.
6. The buckling load factor k in CCCC plate with square perforation having two longitudinal and transverse stiffeners is approximately 42-48% and 33 - 36% greater than the SSSS condition for β=0.5 and δ = 0.05 to 0.2.
7. In case of CCCC plate with circular perforation reinforced by two longitudinal and transverse stiffeners, the value of k increases upto 54% and 44% for cutout ratio 0.1 and δ = 0.05 to 0.2 compared to SSSS boundary condition.

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