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Reliability Analysis of a 3-Machine Power Station Using State Space Approach

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ABSTRACT
With the advent of high-integrity fault-tolerant systems, the ability to account for repairs of partially failed (but still operational) systems become increasingly important. This paper presents a systemic method of determining the reliability of a 3-machine electric power station, taking into consideration the failure rates and repair rates of the individual component (machine) that make up the system. A state-space transition process for a 3-machine with 2^3 states was developed and consequently, steady state equations were generated based on Markov mathematical modeling of the power station. Important reliability components were deduced from this analysis. This research simulation was achieved with codes written in Excel®-VBA programming environment. System reliability using state space approach proofs to be a viable and efficient technique of reliability prediction as it is able to predict the state of the system under consideration. For the purpose of neatness and easy entry of data, Graphic User Interface (GUI) was designed.
***Keywords*-**Failure Probability, Markov model, Reliability, Repair Rate and State Space

I. Introduction

Energy is required for every aspect of socio-economic of modern life. It plays a vital role in the economic, social and political development of a nation [1]. It is also considered as one of the most important resources of any country. It is well known that high rate of industrial growth of any country is a function of the amount of energy available in that country and the extent to which this energy is utilized.

Nigeria as a country is reach in energy resources as the country is blessed with considerable fossil fuel-based energy reserves in the form of crude oil, natural gas, and coal. In addition, it has large undeveloped amount of renewable energy resources including solar, wind and biogas [2]. Presently there are newly built independent power plant and many are under construction. Integration of these power plants into the national grid may not result into technical challenges but the reliability of each power plant will definitely be. Prediction of the failure probability of the power station systems will aid maintenance and stability of the power station.

The power industry faces many problems, with one of the highest priority issues being reliability [3]. Providing reliable power delivery has always been an essential requirement in the design and maintenance of the power generation system [4]. Nevertheless, a useful technique is developed using Markov models of the generators. The assumptions of constant failure rate (λ) and repair rate (μ) are desirable in order to avoid extensive mathematical complications. One of the most important features of any Markov model is that the transition probability from state i to state j depends only on states i and j and is completely independent of all earlier states [5],[6],[7],[8],[9],[10].

Markov modeling is suitable to determine the system condition and the repair intervals needed to achieve a desired level of safety. Taking a look at power supply as the prime mover of technological and social development of any nation, the reliability analysis of electric power station is a necessity. A complete generator comes with its own reliability indices and when such generators are connected in parallel, the reliability analysis of the whole power station is obtainable.

A state space method of determining the reliability of a 3-machine electric power station was carried out, taking into consideration the failure rates and repair rates of the individual component (machine) that make up the system. A state-space transition process for a 3-machine with 2^3 states was developed and consequently, steady state equations were generated. Markov mathematical modeling of the power station will result into 2^3 by 2^3 transition matrix. Three machines were chosen for this research purpose, nevertheless the simulation be extended to more machines.

The research was simulated using Excel®-Visual Basic Application tools to generate the number of states, binary codes, steady state equations and then the state transition intensity matrix. Therefore, evaluating the reliability of power generating system using failure rates and repair rates in-turn determined the availability of a particular machine.

II. Development of a 3-mgenerator Markov model

The Events in the context of the power station can be of several types. These include the total failure of a machine, partial failure when a machine has to be de-rated in order to accommodate some degeneration of its performance and a repair when a failed machine is restored to a functioning status. The latter condition is determined by repair rate. Power plants only have two recognized states: working or failed. When more than one machine is involved, the system state can be represented as a binary word.

A 3-generator power station which admits only two possible states per machine can at any given time be represented in terms of the machine condition which for the sake of this work will associate a '1' with an "up" state and a '0' with a "failed/ down" state. The states of the three individual machines in the station may be concatenated to form a triplet of bits that is treated like a binary word. The full set of possible states that the station may have can be seen to be 2^3(8) in number and represents the set of binary numbers from 0 to 7. Associated with each state are the number of machines that are 'Up' and a number that are 'Down'. If state Ns then the numbers may be expressed as $N_{s,\ Up}$ and $N_{s,\ Down}$ and

$$N_S = N_{S,\ up} + N_{S,down} \quad (1)$$

WhereN_s is the total number of machines.

For the given machine set, the universal set can be partitioned into four groups corresponding to the number of operating machines. This may be expressed as follows:

$$S = \{S_0, S_1, S_2, S_3\}$$

where

$S_0 = \{0\} = \{0\ 0\ 0\} =$ Set of states with 3 (all) machines failed

$S_1 = \{1, 2, 4\} = \{\{0\ 0\ 1\}, \{0\ 1\ 0\}, \{1\ 0\ 0\} =$ Set of states with 2 machines failed

$S_2 = \{3, 5, 6\} = \{\{0\ 1\ 1\}, \{1\ 0\ 1\}, \{1\ 1\ 0\}\} =$ Set of states with 1 machine failed

$S_3 = \{7\} = \{1\ 1\ 1\} =$ Set of states with no machine failed.

The Markov process describing the transitions of the power station is constrained to allow only one machine state change at a time - which may be either the repair of a failed machine or the failure of a functioning machine. Let the probability of being in any particular state be represented as p_k. Since the states represent the set of all possible conditions, then

$$\Sigma pk = 1 \quad (2)$$

In order to analyze a system with failure and repair events, the overall behaviour can be represented as in fig.1 which shows the state space diagram of three independent machines. The advantage of the state space diagram is that it neatly describes both the failure of a machine and its subsequent repair. It develops the probability of a machine being in a given state, as a function of the sequence through which the machine has travelled. For a given state in the transition, all paths to that state are summed and in a Markov random field, each state depends on its neighbours in any of the multiple directions.

Let the failure rates be denoted by the Greek letter λ (lambda) and the repair rates by μ, with the subscripts representing the appropriate machine as can be seen in figure.

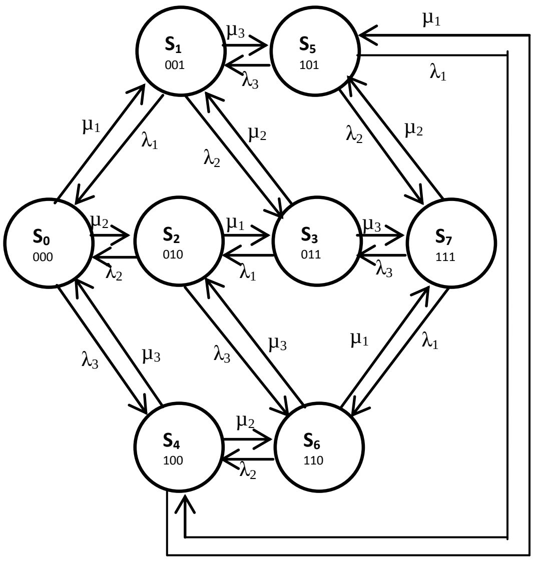

Fig. 1: State-Space diagram for three independent machines Generation System

Let the circles represent the states of the machines while the arrows indicate the direction of transition which may correspond to either a failure or the repair of a specified machine. The binary numbers under the state representation$(S_1, S_2, S_3, S_4, S_5, S_6, S_7)$are the binary representations of each machine state. Note that the convention chosen here is that the probability of transition from one state into another state assumes positive while probability of coming back to the same state assumes negative and in a homogenous Markov process like this, constant transition rate is ensured.

Since one of the assumptions made in performing the above Markov analysis is that only one transition can occur at a time, transitions involving intra group states are forbidden. It is therefore impossible for transitions which require the occurrence of two or more events within the small time interval (Δt). This is because as$(\Delta t) \to 0$, the probability of two events occurring within the small time interval(Δt) becomes negligible compared to the probabilities of the single occurrences, and hence transitions requiring several events are always omitted from this model (since the probability of an event occurring takes $\mu_x \Delta t or \lambda_x \Delta t$ which is a small time and $\mu_x(\Delta t)^2 or \lambda_x(\Delta t)^2$ would be very small for two events, etc.). Applying this constraint to the set of all possible transitions gives rise to the following forbidden transitions. From fig.1 the forbidden transitions are: $S_0 \leftrightarrow S_3$, $S_0 \leftrightarrow S_5$, S_0 and S_6, S_0 and S_7, S_1 and S_2, S_1 and S_4, S_1 and S_6, S_1 and S_7, S_2 and S_4, S_2 and S_5, S_2 and S_7, S_3 and S_4, S_3 and S_5, S_3 and S_6, S_4 and S_7, and then S_5 and S_6. A direct transfer from state S_5 to state S_6 could happen only if the failure of machine occurred at the same time with the repair of machine.

The state transition equation (Markov process) of fig.1 is given in equation 3.

State Equations

$$\begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \end{bmatrix} = \begin{bmatrix} -(k_0) & \mu_1 & \mu_2 & 0 & \mu_3 & 0 & 0 & 0 \\ \lambda_1 & -(k_1) & 0 & \mu_2 & 0 & \mu_3 & 0 & 0 \\ \lambda_2 & 0 & -(k_2) & \mu_1 & 0 & 0 & \mu_3 & 0 \\ 0 & \lambda_2 & \lambda_1 & -(k_3) & 0 & 0 & 0 & \mu_3 \\ \lambda_3 & 0 & 0 & 0 & -(k_4) & \mu_1 & \mu_2 & 0 \\ 0 & \lambda_3 & 0 & 0 & \lambda_1 & -(k_5) & 0 & \mu_2 \\ 0 & 0 & \lambda_3 & 0 & \lambda_2 & 0 & -(k_6) & \mu_1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} . \quad (3)$$

For a particular scenario, if $\lambda_1 = 0.001$, $\mu_1 = 0.1$, $\lambda_2 = 0.001$, $\mu_2 = 0.1$, $\lambda_3 = 0.001$, $\mu_3 = 0.1$, such that the three machines are identical, then the state transition table is as shown in Table 1.Equation 3 can be written in matrix form,

$AP = b$ (4)

Where matrix A is called the state transition intensity matrix and the vector $b = 0$

Table 1 shows the Markov process for three independent machines which contains all the information about the transitions between different states of the system (i.e. from state 0 to state 7). The state equations representing the dynamic of transitions can be determined by considering the probability of being in a particular state. It is assumed that the transition intensity matrix is A. The target here is to seek for the long-run (steady state) solutions.

$$\begin{bmatrix} -(k_0) & \mu_1 & \mu_2 & 0 & \mu_3 & 0 & 0 & 0 \\ \lambda_1 & -(k_1) & 0 & \mu_2 & 0 & \mu_3 & 0 & 0 \\ \lambda_2 & 0 & -(k_2) & \mu_1 & 0 & 0 & \mu_3 & 0 \\ 0 & \lambda_2 & \lambda_1 & -(k_3) & 0 & 0 & 0 & \mu_3 \\ \lambda_3 & 0 & 0 & 0 & -(k_4) & \mu_1 & \mu_2 & 0 \\ 0 & \lambda_3 & 0 & 0 & \lambda_1 & -(k_5) & 0 & \mu_2 \\ 0 & 0 & \lambda_3 & 0 & \lambda_2 & 0 & -(k_6) & \mu_1 \\ 0 & 0 & 0 & \lambda_3 & 0 & \lambda_2 & \lambda_1 & -(k_7) \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \end{bmatrix} = 0 \quad (5)$$

Where; $k_0 = (\lambda_1+\lambda_2+\lambda_3)$, $k_4 = (\lambda_1+\lambda_2+\mu_3)$,
$k_1 = (\lambda_2+\lambda_3+\mu_1)$, $k_5 = (\lambda_2+\mu_1+\mu_3)$,
$k_2 = (\lambda_1+\lambda_3+\mu_2)$, $k_6 = (\lambda_1+\mu_2+\mu_3)$
$k_3 = (\lambda_3+\mu_1+\mu_2)$, $k_7 = (\mu_1+\mu_2+\mu 3)$

Equation 4 can be made non-homogeneous by changing one of the states in equation 3(e. g. state 7) with equation 6. This will not affect our solution since the eight equations above are not independent one can be omitted (the last equation), and for all the p_k's sum up to 1, then it is replaced by:

$$P_0+P_1+P_2+P_3+P_4+P_5+P_6+P_7=1 \quad (6)$$

Then vector $b = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1]^T$

Hence $P = A^{-1}b$ (7)

$$\begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \end{bmatrix} = \begin{bmatrix} -(k_0) & \mu_1 & \mu_2 & 0 & \mu_3 & 0 & 0 & 0 \\ \lambda_1 & -(k_1) & 0 & \mu_2 & 0 & \mu_3 & 0 & 0 \\ \lambda_2 & 0 & -(k_2) & \mu_1 & 0 & 0 & \mu_3 & 0 \\ 0 & \lambda_2 & \lambda_1 & -(k_3) & 0 & 0 & 0 & \mu_3 \\ \lambda_3 & 0 & 0 & 0 & -(k_4) & \mu_1 & \mu_2 & 0 \\ 0 & \lambda_3 & 0 & 0 & \lambda_1 & -(k_5) & 0 & \mu_2 \\ 0 & 0 & \lambda_3 & 0 & \lambda_2 & 0 & -(k_6) & \mu_1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} . \quad (8)$$

The expected power output at any time is equal to the total power supplied by the available system(s).

III. Model Simulations

Systems are usually composed of a number of individual machines that manifest their own specific failure and repair characteristics. While each machine's performance may be studied using some relatively basic concepts of probability, systems employing several machines require the techniques of stochastic process to extract their combined performance. Applying Markov model, it is possible to determine the steady state probabilities for a given system and subsequently study the effect of various changes on performance. In this research work, scenarios are developed for a system comprising of three machines. This scenario involved a set of identical machines for situations where the machines have differing failure rates and progressively increasing repair rates characteristic of aging machines. The results are fully discussed below.

The system studied consists of three independent machines that have identical performance - i.e. failure and repair rates. As stated earlier above, the mechanism of deriving their state probabilities of this three machine system is a transition state space that admits two states only and hence generates a 2^3 or 8 state spaces. An EXCEL – VBA tool box was developed for this work, it can automatically generate the system state space, then the transition matrix for the Markov state space, compute the state probabilities and the expected power output. For the purpose of neatness and easy entry of data, Graphic User Interface (GUI) was designed.

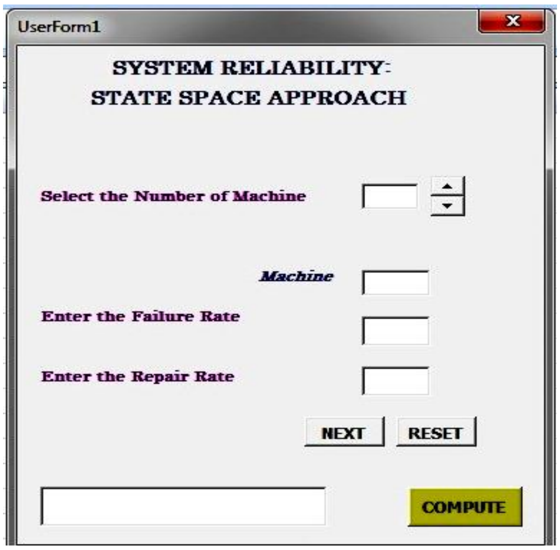

Fig. 2: GUI for simulations of system reliability using State-Space approach

Simulations was carried out for different cases, hence deductions and inferences were made from the results obtained from the simulations. The generate system state space; the state probabilitiesand the expected power output are tabulated in each case.

Case 1: □ = 0.001 and □ = 0.1

Table 1: System States and Binary Codes for 3-Machine Generation System with λ=0.001 and μ = 0.1

Number of Machines	3		
Number of States	8		
Failure Rates	0.001		
Repair Rates	0.1		
States		Binary	
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

Table 2: State Probability and Average Expected Power Outputfor 3- Machine Generation System with λ=0.001 and μ = 0.1

No of Working Machine(s)	State Probability	Expected Power Output(MW)
0	9.71 x10^{-7}	0
1	0.000291	60
2	0.029118	120
3	0.97059	180
Average Expected Power Output178.2178		

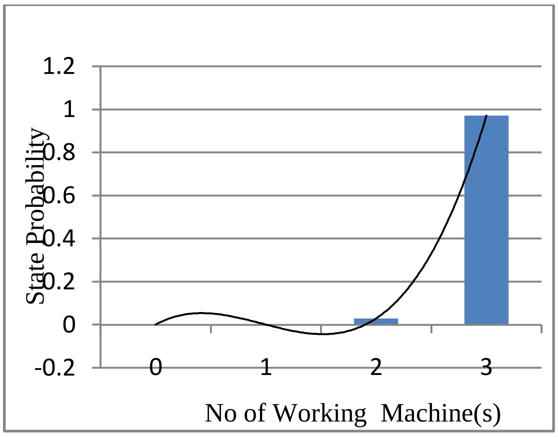

Fig.3: State probability against system performance for a 3- machine generation system with λ=0.001 and μ = 0.1

Case 2: □ = 0.01 and □ = 0.1

Now consider the situation wherethe failure rate has increased from 0.001 to 0.01 while the repair rate still constant and this process continued in order to observe the behaviour of the system.

Table 3: System States and for 3- Machine Generation System with λ = 0.01 and μ = 0.1

Number of Machines	3		
Number of States	8		
Failure Rates	0.01	0.01	0.01
Repair Rates	0.1	0.1	0.1

Table 4: State Probability and Average Expected Power Output for 3- Machine Generation System with λ = 0.01 and μ = 0.1

No of Working Machine(s)	State Probability	Expected Power Output(MW)
0	0.000751	0
1	0.022539	60
2	0.225394	120
3	0.751315	180
Average Expected Power Output163.6364		

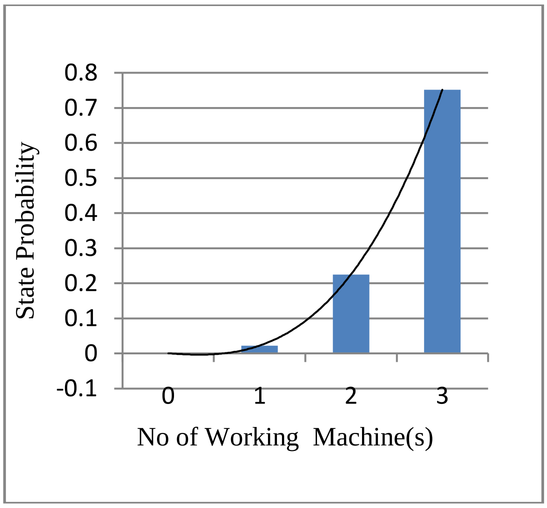

Fig.4: State Probability against System Performance for 3- Machine Generation System with λ=0.01 and μ= 0.1

Case 3: □ = 0.05 and □ = 0.1

The failure rate was increased from 0.01 to 0.05 while the repair rate still constant

Table 5: Number of System State and Binary Codesfor 3- Machine Generation System with λ = 0.05 and μ = 0.1

Number of Machines	3		
Number of States	8		
Failure Rates	0.05	0.05	0.05
Repair Rates	0.1	0.1	0.1

Table 6: State Probability and Average Expected Power Output for 3- Machine Generation System with λ = 0.05 and μ = 0.1

No of Working Machine(s)	State Probability	Expected Power Output(MW)
0	0.037037	0
1	0.222222	60
2	0.444444	120
3	0.296296	180
Average Expected Power Output120		

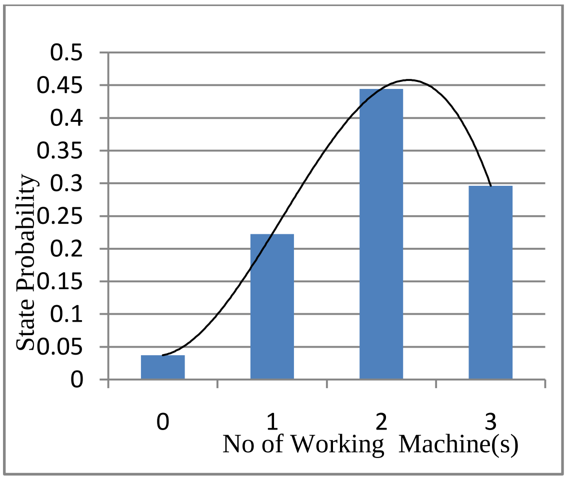

Fig.5: State Probability against System Performance for 3- Machine Generation System with λ = 0.05 and μ = 0.1

Case 4: □ = 0.1 and □ = 0.1

Table 7: Number of System State and Binary Codesfor 3- Machine Generation System with λ = 0.1 and μ = 0.1

Number of Machines	3		
Number of States	8		
Failure Rates	0.1	0.1	0.1
Repair Rates	0.1	0.1	0.1

Table 8: State Probability and Average Expected PowerOutput for 3- Machine Generation System with λ = 0.1 and μ = 0.1

No of Working Machine(s)	State Probability	Expected Power Output(MW)
0	0.125	0
1	0.375	60
2	0.375	120
3	0.125	180
Average Expected Power Output90		

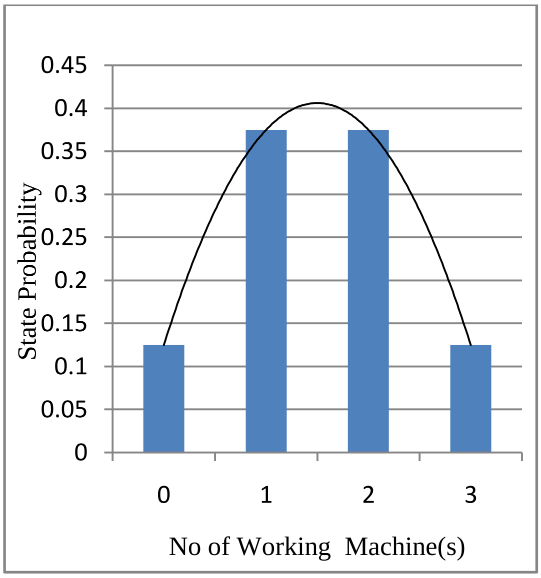

Fig.6: State Probability against System Performance for 3- Machine Generation System with λ = 0.1 and μ= 0.1

Case 5: □=0.25 and □ = 0.1

Table 9: Number of System State and Binary Codes for 3- Machine Generation System with λ = 0.25 and μ = 0.1

Number of Machines	3		
Number of States	8		
Failure Rates	0.25	0.25	0.25
Repair Rates	0.1	0.1	0.1

Table 10: State Probability and Average Expected Power Output for 3- Machine Generation System with λ = 0.25 and μ = 0.1

No of Working Machine(s)	State Probability	Expected Power Output(MW)
0	0.364431	0
1	0.437318	60
2	0.174927	120
3	0.023324	180

Average Expected Power Output51.42857

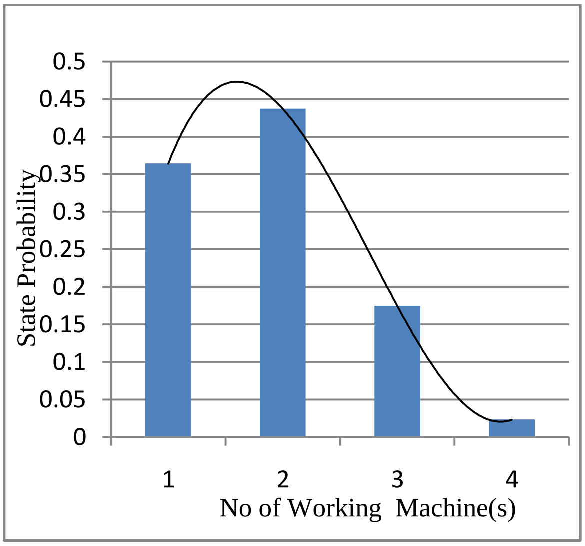

Fig.7: State Probability against System Performance for a 3- Machine Generation System

Case 6: □=0.5 and □ = 0.1

Table 11: Number of System State and Binary Codesfor 3- Machine Generation System with λ = 0.5 and μ = 0.1

Number of Machines	3		
Number of States	8		
Failure Rates	0.5	0.5	0.5
Repair Rates	0.1	0.1	0.1

Table 12: State Probability and Average Expected Power Outputfor 3- Machine Generation System with λ = 0.5 and μ = 0.1

No of Working Machine(s)	State Probability	Expected Power Output(MW)
0	0.578704	0
1	0.347222	60
2	0.069444	120
3	0.00463	180

Average Expected Power Output30

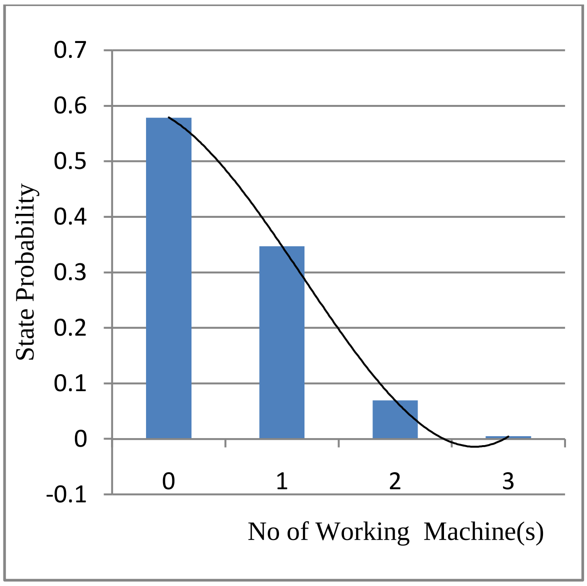

Fig.8: State Probability against System Performance for 3- Machine Generation System with λ = 0.5 and μ= 0.1

Case 6: □=0.75 and □ = 0.1

Table 13: Number of System State and Binary Codesfor 3- Machine Generation System with λ = 0.75 and μ = 0.1

Number of Machines	3		
Number of States	8		
Failure Rates	0.75	0.75	0.75
Repair Rates	0.1	0.1	0.1

Table 14: State Probability and Average Expected Power Output for 3- Machine Generation System with λ = 0.75 and μ = 0.1

No of Working Machine(s)	State Probability	Expected Power Output(MW)
0	0.686953	0
1	0.274781	60
2	0.036637	120
3	0.001628	180
Average Expected Power Output21.17647		

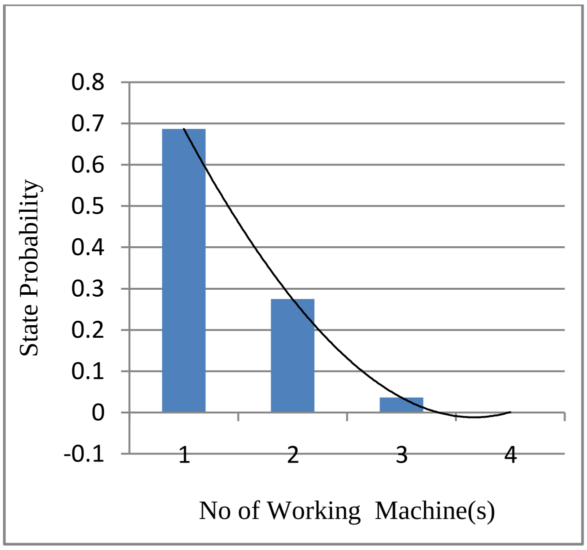

Fig.9: State Probability against System Performance for a 3- Machine Generation Systemfor 3- Machine Generation System with λ = 0.5 and μ = 0.1.

Table 15: Summary of the Average Expected Power Output and Failure rate of the Machine(s) at μ= 0.1 but increasing repair rate

Repair rates(μ) (hr)	Failure rates(λ) (hr)	Average Expected Power Output(MW)
0.1	0.001	178.21
0.1	0.01	163.63
0.1	0.05	120
0.1	0.1	90
0.1	0.25	51.42
0.1	0.5	30
0.1	0.75	21.17

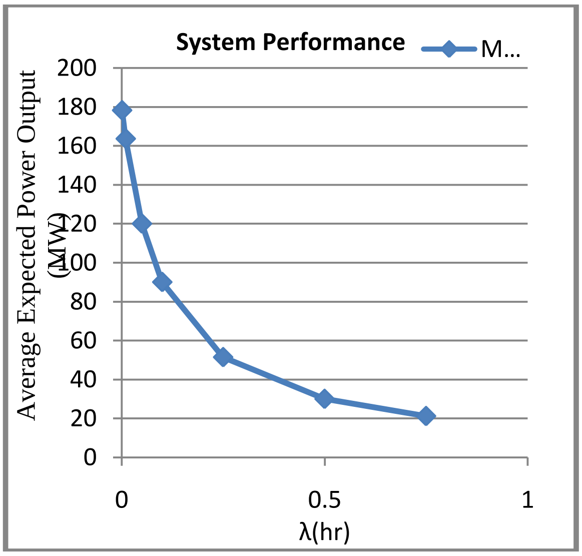

Fig. 10: Average Expected Power Output against MTBF at μ=0.1)

Case 7: Having solved the model for different values of failure rate at a fixed repair rate, the experiment was extended by repeating the experiment for varying repair rates.

Table 16: The Average Expected Power Output and Failure rate of the Machine(s) at different repair rate

μ/hr	λ/hr	0.001	0.01	0.05	0.1	0.25	0.5	0.75
0.1	Average Expected Power Output (MW)	178.21	163.63	120	90	51.42	30	21.17
0.2		179.1	171.42	144	120	80	51.42	37.89
0.3		179.4	174.19	154.28	135	98.18	67.5	51.42
0.5		179.64	176.47	163.63	150	120	90	72

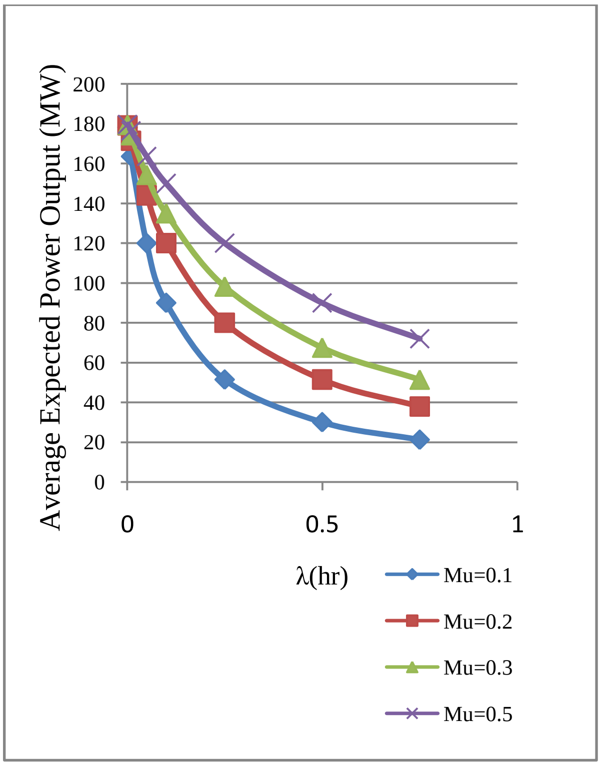

Fig.11: Average Expected Power Output against λ (hr) at different repair rate

IV. DISCUSSION OF RESULTS

The results of the fixed repair rate study indicate a very interesting picture of what was happening. It can be seen from fig. 3 to fig.9 that as the failure rate increased from very low to very high, the shape of the probability curve evolved from a monotonically increasing shape through a symmetric "normal curve" to a negative exponential curve. This is consistent with expectations because at very low failure rates, most of the machines would be operating whilst for high failure rates, most of the machines would be in a failed state. It is interesting to note that when the failure and repair rates are equal, the curve is symmetric. The combined performance of the system resulted in fig. 10. It can be seen that the expected station output monotonically decreases as the failure rate increases. Towards the high end, the rate seems to slow down.

The experiment was repeated using increased values of repair rate; the curves are as shown in fig. 10. The curve reveals a family of curves that have similar shape but with the terminal values increasing with the repair rate. The implication of this is that the repair rate of a power station has very serious implications to the operators. Clearly it would be better to invest in very reliable machines coupled with skillful repair crew. In that instance the investment will yield near the optimum, although a global optimum can only be achieved when other factors such as fuel consumption etc. are included in the study.

V. Conclusion

The potential of a Markov model for solving the reliability prediction of a 3-machine electric power station has been studied and programmed in an Excel®- Visual Basic Application environment. Although certain assumptions are required in carrying out the study, Markov model using the state space approach has been able to determine the reliability of electric power station. It is a viable (potential) alternative for studying reliability analysis of a repairable system.

System reliability prediction using the state space approach is an efficient technique for forecasting the reliability of repairable systems with constant transition rates (mainly systems of independent components). This technique could be useful for analyzing newly designed systems. It is recommended that future research should explore other methods that use iterative and partitioning techniques or combinations thereof to address the burden of computation as the number of units increase.

Excel®-VBA was chosen because it is easily available and can be used in handling and processing of large amount of data, through the manipulation and development of simple codes. From the analysis, it is seen that the state probabilities of the systems is successfully calculated. Experiments with different values of failure rate resulted in families of curves that indicate relationship between performance and machine characterization. The results obtained indicate that as the failure rate of the machines in the system increased, the effective system output declined.

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