

Quality Costs (IRR) Impact on Lot Size Considering Work in Process Inventory

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Abstract

Economic order quantity model and production quantity model assume that production processes are error free. However, variations exist in processes which result in imperfection particularly in high machining environments. Processes variations result in nonconformities that increase quality costs in the form of rework, rejects and quality control techniques implementations to ensure quality product delivery. This paper is an attempt towards development of inventory model which incorporate inspection, rework, and rejection (IRR) quality costs in optimum lot size calculation focusing work in process inventory. Mathematical model is derived for optimum lot size based on minimum average cost function using analytical approach. This new developed model (GTOQIRR) assume an imperfect production environment. Numerical examples are used to visualize the significant effect of quality cost in the proposed model in comparison to the previously developed models. The proposed model is highly recommendable for quality based high machining manufacturing environments considering work in process inventories.

Keywords: WIP inventory; EOQ model; quality cost; lot size; GTOQ model.

I. Introduction

During past few decades, significant attention has been given to the area of inventory management because of its prime importance for most manufacturing organizations. Organizations use economic order quantity (EOQ) and production order quantity (POQ) models since 1913 to calculate optimum inventory lot size considering holding and setup costs. Researchers extended these models to more generic and practical situations of everyday life. However these two basic models are based on an unrealistic assumption that perfect products are produced every time. A number of factors affect manufacturing processes which include machine failure, changes in raw material supplier, tool wear and tear, outside conditions etc. especially where machining time is large relatively. Therefore the phenomena of rework, reject and inspection is common in industries like machine tool industries, aerospace industries and glass industries etc. High machining environments, rework, rejects and inspection compel to introduce quality costs incurred during these process and its ultimate impact on inventory lot size.

Rosenblatt and Lee [1] developed EOQ model for imperfect manufacturing environments with conclusion that lot size reduces as imperfection is increased. Later Sarker et al. [2] extended EOQ model to a multistage manufacturing environment with imperfection in processes. The developed model assumed that rework operation is performed at the end of each cycle and at the end of N cycle. Further

extension of such models can be realized in research articles published in the last decade [3-12]. They realized that the impact of imperfection on inventory lot size in addition to the importance of inspection in real world manufacturing environments. Combined effect of inspection with imperfections was emphasized in few papers such as ([13] and [18]). Furthermore, ([14], [15], [16], [17] and [19]) developed models for optimum lot size by considering other important aspects of the real world manufacturing systems. Researchers mainly focused raw materials and finish goods inventories in calculation of optimum lot size calculation. However, the third type of inventory, work in process, in combination with quality cost has been relatively ignored in previous literature.

Today, industries put their efforts to reduce their inventories cost either in the form of raw materials, finish goods or work in process. Boucher [20] introduced the concept of work in process inventory in group technology based manufacturing environment for calculation of optimum lot size. The model was named as group technology order quantity (GTOQ) model. Barzoki et al. [21] extended GTOQ model for imperfect production environments under the title of group technology order quantity model with rework (GTOQR). Their model considered 100% qualification of rework products. Numerical examples were used for model comparison against previous models. These models are recommended for group technology work environment where machining time and lot size are relatively large. However, increase

machining time and higher demand rate significantly contribute towards quality costs in processes. Higher machining rate and large machining time results in increased quality costs due to high tool wear and tear, process failure etc. Increased quality costs are in the form of rework, rejects and inspection that affect total cost for optimum lot size calculation. This paper is an attempt towards calculation of quality costs impact on the optimal lot size considering imperfection in processes. This paper incorporate quality costs due to inspection, rework and rejection (IRR) in total cost calculation. Inclusion of quality costs in inventory models and its impact on lot size is lacking in previous papers incorporating work in process inventories. The remaining sections are listed as follow;

Section 2 explains problem statement with assumptions and notations. In section 3, the problem is modeled based on modeling for relevant costs associated with this problem. Section 4 comprises comparison of models based on numerical computations. Conclusion is presented in the last section.

II. Problem statement

We consider a single discrete manufacturing setup. A machining station is followed by an inspection process with three buffer stations in the manufacturing workshop. Buffer stations are used for work in process inventories i.e. raw material (unprocessed items, waiting for operations), good

quality products and poor quality products (rejected items).

A lot of material (Q) arrives in the workshop processed on machining station in each cycle. Lot (Q) is released for inspection to declare products as either good quality products or poor quality products. As the process is imperfect so three kinds of products are produced during the whole cycle time. The process produces, Qp_{10} , percent of parts as poor quality products, Qp_{1r} , percent of parts as reworked and the remaining $Q(1 - (p_{10} + p_{1r}))$ as good quality products in the first phase of the lot Q as shown in Figure 1.

During the next phase 2, the rework items $Qp_{1r} = Q(p_{20} + p_{21})$ are reprocessed and re-inspected. Qp_{21} percent of products are good quality products and, Qp_{20} , percent of products are poor quality in the phase 2 as shown in Figure 2. We assume that no rework is needed after that. At the end of the cycle, the following conclusion is drawn regarding the lot size Q .

Good quality products produced, $Qp_1 = Q(1 - (p_{10} + p_{20}))$

Poor quality products produced, $Qp_0 = Q(p_{10} + p_{20})$

Products that are rework able after phase 1,

$Qp_{1r} = Q(p_{20} + p_{21})$

Therefore, our objective is to develop an economic order quantity model for an imperfect manufacturing environment taking quality costs into consideration with work in process inventory.

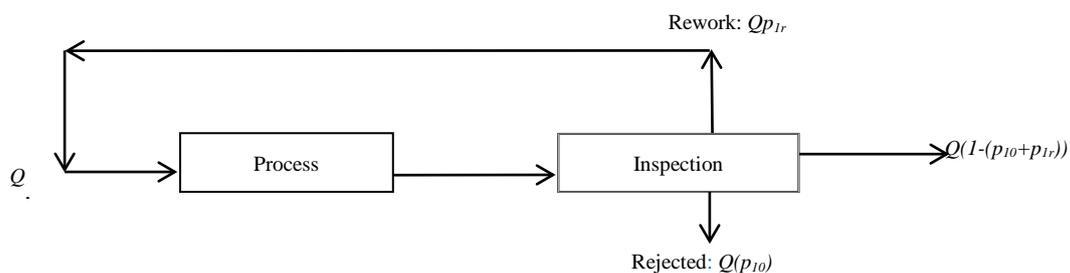


Figure 1. Processing of lot size Q during phase 1

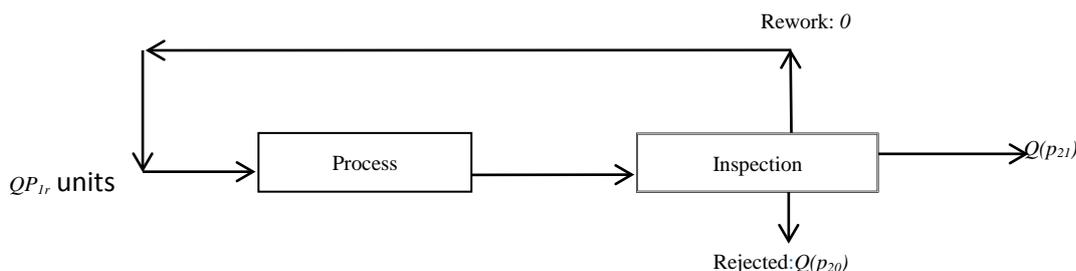


Figure 2. Processing of rework products in phase 2.

2.1. Notations

d customer demand

Q lot size processed per unit cycle

M_c raw material cost per unit

P_c purchasing cost per unit of time

S_c setup cost per unit of time (\$/unit of time)

Q_c cost of quality per unit of time (\$/unit of time)
 C_i inspection cost
 RW_c rework cost
 RJ_c rejection cost
 WIP_c work in process holding cost
 IH_c inventory holding cost
 T_c total cost
 t_s setup time
 t_{m1} machining time in phase 1. (unit time per unit product)
 t_{mr} machining time per unit for rework products
 p_{10} proportion of poor quality products in phase 1
 p_{11} proportion of good quality products in phase 1
 p_{21} proportion of good quality products in phase 2
 p_{20} proportion of poor quality products in phase 2
 p_{1r} proportion of rework able products in phase 1
 p_0 proportion of poor quality products at the end of cycle
 p_1 proportion of good quality products at the end of cycle
 t_c cycle time
 t_p total processing time
 \bar{t} average manufacturing time for each product item
 G average storage inventory
 \bar{W} average monetary value of the WIP inventory (\$)
 i inventory holding cost per unit of time (\$/unit of time)
 c average unit value of each product (unit of money (\$)) per unit
 k rate charged per unit of cell production time including all overheads, moving cost, loading/unloading cost etc. (unit of money (\$)) per unit of time)

2.2 Assumptions

- Shortages are not allowed
- Demand is known and pre-determined.
- Inspection is performed of all lot and rework able products.
- Poor quality products are produced during the rework operation.
- Rework operation is done one time only.
- No stoppage is allowed during the manufacturing of one lot.

All parameters including demand and production rate, setup times etc. are constants and deterministic.

III. Modeling

Modeling of the problem is based on cycle time calculation in terms of demand rate and lot size Q in addition to the imperfection in the production processes.

As the objective of almost every manufacturing setup is to meet customer demand rate (d) therefore lot size (Q) is defined keeping customer demand in mind. Processes undergoing imperfection results in poor quality products at the end of each cycle.

Therefore we can have following relation for the above mentioned scenario

$$Q(1 - p_0) = dt_c$$

$$t_c = \frac{Q(1-p_0)}{d} \quad (1)$$

The total processing time (t_p) of a product having lot size Q comprises of setup time, manufacturing time, inspection time per unit product (t_i), rework time and re-inspection time.

$$t_p = t_s + Qt_{m1} + t_iQ + Qt_{m1}p_{1r} + t_iQp_{1r} \quad (2)$$

Therefore average processing time is given by

$$\bar{t} = \frac{t_p}{Q}$$

$$\bar{t} = \frac{t_s + Qt_{m1} + t_iQ + Qt_{m1}p_{1r} + t_iQp_{1r}}{Q}$$

If k is the average cost of production per unit of time then the average cost added to each product unit during its manufacturing is v given by

$$v = k\bar{t} = k \left(\frac{t_s + Qt_{m1} + t_iQ + Qt_{m1}p_{1r} + t_iQp_{1r}}{Q} \right)$$

We assume that M_c be the material cost per unit of product, then the average total cost added to each product unit is given by

$$c = M_c + k \left(\frac{t_s + Qt_{m1} + t_iQ + Qt_{m1}p_{1r} + t_iQp_{1r}}{Q} \right) \quad (3)$$

3.1 Cost calculation

As we know that most of the inventory models are based on the cost minimization, therefore we are interested to calculate associated costs with this model. Quality costs, inventory holding cost, setup cost and material purchasing cost are the main components of total cost calculation. All these costs are calculated keeping imperfection in processes under consideration.

3.1.1 Quality cost (IRR)

Quality costs considered in this paper are mainly associated with imperfection in processes. These imperfection results in products failure either in the form of rework or rejected products. Similarly, quality control techniques including inspection exists within the manufacturing setup which also results in quality cost. Other kinds of quality cost related to appraisal and prevention has been neglected. Therefore, quality costs considered in this model are (a) Quality costs due to products inspection (C_i), (b) Quality costs due to rework (C_{rw}), (c) Quality costs due to rejection (C_{rj}).

Therefore quality cost (C_q) is given by

$$Q_c = C_i + RW_c + RJ_c \quad (4)$$

These costs are modeled as follow

a) Quality costs due to inspection

Inspection is performed for all manufactured and reworked products. 100% inspection of lot is carried out when it is manufactured. Similarly reworked products are also passed through the inspection process. The inspection cost Ci per unit of cycle time is given by

$$Ci = \frac{I_c Q}{t_c} + \frac{(p_{1r}) Q I_c}{t_c}$$

$$Ci = \frac{I_c d}{(1-p_0)} + \frac{I_c d (p_{1r})}{(1-p_0)}$$

$$Ci = \left(\frac{I_c d}{1-p_0} \right) (1 + p_{1r}) \quad (5)$$

b) Quality costs due to rework

Let p_{1r} is the percentage of rework product in a lot size Q in a complete cycle and c be the value added to the products in the manufacturing process. Then the quality cost per unit time for rework items is given by

$$RW_c = \left(\frac{np_{1r}Q}{t_c} \right) c \quad (6)$$

where n is the no. of times rework operation is performed.

c) Quality costs due to rejection

Let p_0 is the percentage of rejected product in a lot size Q in a complete cycle and c be the value added to the products in the manufacturing process. Then the quality cost per unit time for rejected products is given by

$$RJ_c = \left(\frac{p_0 Q}{t_c} \right) c \quad (7)$$

Therefore, Equation (4) becomes,

$$Q_c = \left(\frac{I_c d}{1-p_0} \right) (1 + p_{1r}) + \left(\frac{np_{1r}Q}{t_c} \right) c + \left(\frac{p_0 Q}{t_c} \right) c$$

$$Q_c = \left(\frac{I_c d}{1-p_0} \right) (1 + p_{1r}) + \left(\frac{cd}{1-p_0} \right) (np_{1r} + p_0) \quad (8)$$

3.1.2 Inventory holding cost

Silver *et al.* [22] introduced following relationship for calculation of inventory holding cost per unit of time.

$$\bar{W} = \frac{1}{2} \frac{Q t_p}{t_c} (M_c + (1 - p_0)c + p_0 c)$$

$$\bar{W} = \frac{1}{2} \left(\frac{d}{1-p_0} \right) (t_s + Q t_{m1} + t_i Q + Q t_{m1} p_{1r} + t_i Q p_{1r}) (M_c + c)$$

Given Equation (3), the average value of total work in process inventory will be

$$\bar{W} = \frac{1}{2} \left(\frac{d}{1-p_0} \right) \left(2M_c + \frac{kt_s}{Q} + kt_{m1} + kt_{m1} p_{1r} + t_i k + t_i k p_{1r} \right) (t_s + Q t_{m1} + t_i Q + Q t_{m1} p_{1r} + t_i Q p_{1r}) \quad (13)$$

Therefore given Equations (12) and (13), the average carrying charge of the work in process inventory will be

$$WIP_c = \frac{1}{2} \left(\frac{di}{1-p_0} \right) \left(2M_c + \frac{kt_s}{Q} + kt_{m1} + kt_{m1} p_{1r} + t_i k + t_i k p_{1r} \right) (t_s + Q t_{m1} + t_i Q + Q t_{m1} p_{1r} + t_i Q p_{1r}) \quad (14)$$

3.1.4 Material Cost

Material cost per unit of cycle time (P_c) is given by the following equation

$$P_c = M_c Q / t_c$$

We can write in terms of demand (d) is

$$P_c = \frac{M_c d}{(1-p_0)} \quad (15)$$

$$IH_c = icG, \quad (9)$$

Here G = average inventory over each cycle by the cycle period.

c = average unit monetary value for each item.

i = carrying charge.

The average holding inventory (G) per unit cycle time is given as

$$G = \frac{\frac{1}{2} Q (1-p_0) t_c}{t_c} = \frac{1}{2} Q (1-p_0) \quad (10)$$

Given Equations (3) and (10), the inventory holding cost per unit of time will be

$$IH_c = \frac{1}{2} i \left(\left(M_c + k \left(\frac{t_s + Q t_{m1} + t_i Q + Q t_{m1} p_{1r} + t_i Q p_{1r}}{Q} \right) \right) Q (1-p_0) \right) \quad (11)$$

3.1.3 Work in process cost

Based on the same concepts as [21] and [22], the work in process inventory holding cost is calculated by:

$$WIP_c = i \bar{W}, \quad (12)$$

where i = it is the carrying charge per unit of time

\bar{W} = average work in process inventory

\bar{W} can be computed by summation of following types of inventories in the workshop.

(a). The unprocessed product item waiting for operation on machine versus manufacturing time t_p during each cycle. (b). Good quality products inventory over time. (c). Rejected parts inventory over time.

Hence the average value of the total work in process inventory will be equal to

$$\bar{W} = \left(\frac{1/2 Q t_p}{t_c} \right) M_c + \left(\frac{\frac{1}{2} Q (1-p_0) t_p}{t_c} \right) c + \left(\frac{\frac{1}{2} Q p_0 t_p}{t_c} \right) c$$

3.1.5 Setup cost

Machines and inspection setup take place only once during whole lot manufacturing. We assume that no extra setup is required for rework operations.

It can be modeled as follow

$$S_c = \frac{A}{t_c} = \frac{Ad}{Q(1-p_0)} \quad (16)$$

Where A is the setup cost. It is a product of set up time per cycle (t_s) and the rate charged by the production cycle (k).

3.2 Total cost per unit of time

The total cost per unit of time are given by the following equation:

$$T_c = Q_c + IH_c + WIP_c + P_c + S_c \quad (17)$$

Given equations (4), (5), (10), (13), and (16) in Eq. (17)

$$T_c = \left(\frac{I_c d}{1-p_b}\right)(1+p_{1r}) + \left(\frac{cd}{1-p_0}\right)(np_{1r} + p_0) + \frac{1}{2}i \left(\left(M_c + k \left(\frac{t_s + Qt_{m1} + t_i Q + Qt_{m1} p_{1r} + t_i Q p_{1r}}{Q} \right) \right) Q(1-p_0) \right) + \frac{1}{2} \left(\frac{di}{1-p_0} \right) \left(2M_c + \frac{kt_s}{Q} + kt_{m1} + kt_{m1} p_{1r} + t_i k + t_i k p_{1r} \right) (t_s + Qt_{m1} + t_i Q + Qt_{m1} p_{1r} + t_i Q p_{1r}) + \frac{M_c d}{(1-p_0)} + \frac{Ad}{Q(1-p_0)} \quad (18)$$

3.3 Optimum lot size

Equation (18) gives average cost function considering quality cost. Optimum lot size can be calculated based on average cost minimization. Therefore by taking 1st derivative of the function w.r.t. Q and equating to 0 i.e.

$$\frac{dT_c}{dQ} = 0$$

and sufficient condition that the equation must hold and satisfy for optimum lot size calculation is given by

$$\frac{d^2 T_c}{dQ^2} > 0$$

Hence the optimum lot size is given by

$$GTOQIRR = \sqrt{\frac{2Ad + idk(t_s)^2 + 2kt_s d(np_{1r} + p_0)}{i(1-p_0)^2(M_c + kt_{m1} + t_i k) + (kt_{mr} + t_i k)p_{1r}} + di \left((t_{m1} + t_{mr} p_{1r})(2M_c + kt_{m1} + kt_{mr} p_{1r}) + (t_i(1+p_{1r})(2M_c + t_i k + 2kt_{m1} + 2kt_{mr} p_{1r} + t_i k p_{1r})) \right)} \quad (19)$$

The addition of the term “ $2kt_s d(np_{1r} + p_0)$ ” represents the quality cost (IRR) impact on the optimum lot size. Where ‘n’ represents the no. of times rework operation is performed. p_{1r} represents the proportion of rework components produced during the process and p_0 represent the rejected products during the production process. It can be observed that increased in imperfection, number of rework operations increased quality costs and optimal lot size.

IV. Results

The proposed model is compared with the most well-known EOQ model and GTOQ model developed by Boucher [20]. Five different cases data is taken from a US tool manufacturing company, initially introduced by Boucher [20]. Few assumptions were made regarding data calculation e.g. percentage of rejects $p_0 = 20\%$, the percentage of rework $p_{1r} = 5\%$, $t_{mr} = 5\%$ of actual machining time t_{m1} and $k = 3000$ (\$/year) in all five cases. It is assumed that rework operation is performed only once.

Table 1. Optimum lot size using GTOQ, EOQ and proposed model. (assume, $i = 35\%$)

Case No.	t_i (min/unit)	A (\$/unit)	d (units/year)	M_c (USD/unit)	t_s^* (min/setup)	t_{m1}^* (min/unit)	EOQ	GTOQ	GTOQ IRR
1	20	14.349	77	5.63	574	100	28	26	34
2	20	12.75	233	1.57	510	32	85	81	96
3	20	12.951	580	1.42	518	87	109	87	98
4	20	14.349	1877	1.64	574	67	216	135	139
5	20	17.274	5361	1.12	691	41	497	255	233

It can be observed that the response of all three models is different to all cases. Response of the proposed model is significant in comparison to the previous developed model (EOQ and GTOQ) as imperfection and quality costs are taken into consideration.

The impact of quality cost (IRR) has also been highlighted in the table 2 below. We assume $i = 35\%$, $M_c = 1$ (\$/unit), $d = 14000$ (units/year), $k = 7000$ (\$/year), $t_s = 0.0017$ (years/unit), $t_{m1} = 0.12$ (mints/unit), $t_{mr} = 5\%$ (m_1), $A = 11.9$ (\$/unit).It can be observed that as imperfection in processes increase, quality costs shoots up and optimal lot size is increased. This impact of imperfection on quality cost and optimum lot size has also been shown in Fig.3. Quality costs are almost negligible when there processes are perfect and goes on increasing as imperfection in the form of rework and rejected products increases.

Table 2.Impact of imperfection on quality cost in inventory model.

Case No.	$p_b(\%)$	$p_{1r}(\%)$	GTOQIRR	Setup cost	WIP cost	Finish goods cost	Quality cost (IRR)
1	0	0	943	176.63	17.81	471.61	0.01
2	5	5	1037	169.05	20.01	492.76	1511.78
3	10	10	1139	162.57	22.59	512.40	3189.48
4	15	15	1248	157.00	25.64	530.57	5062.92
5	20	20	1368	152.22	29.28	547.23	7169.20
6	25	25	1499	148.14	33.67	562.29	9555.25

Optimal lot size GTOQIRR and quality cost (IRR) variation with change in imperfection in process)

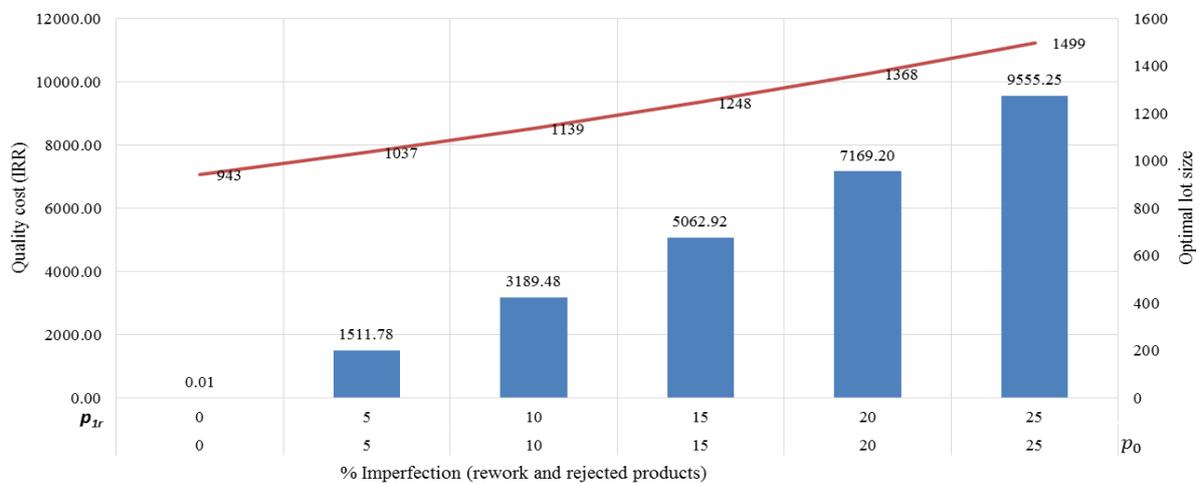


Fig. 3 Change in optimal lot size and quality cost (IRR) with change in Imperfection

V. Conclusion

The importance of quality in manufacturing setups cannot be neglected in general and particularly in environments where machining work is large relatively. Boucher [20] realized in his model that GTOQ is good for high machining environments. This increased machining ultimately impact quality costs as well. Impact of quality costs on lot size has not been considered till now while developing such models for work in process inventories. Different inventory models are developed during last few decades to solve the real world problems and optimize the lot size as per actual scenarios. However work in process based inventory model are lacking the component of quality costs in their previously developed models especially when the imperfection

and high machining rate of processes are considered. This paper is an attempt towards the development of such model for imperfect process considering IRR quality cost component. The proposed model is of significance for manufacturing environments having concentration on quality costs. It is more generalized and incorporate all aspects of the previously developed models. This research can be further extended by estimating the effect of shortages and machine breakdowns on lot size focusing work in process inventory.

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