BER Performance of a Vector Antenna Array versus a Uniform Linear In a Multipath Fading Channel

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ABSTRACT
In multipath environments, the transmitted signal polarization is affected by channel fading which results in a multipath polarized signal component on the receiver side. These receivers usually employ single polarized uniform linear antenna array (ULA) which measure one component of the received electromagnetic (EM) signal. In rich scattering environments, however, it may be possible to improve the performance by using “vector antennas”, as they can detect up to six independent components of the EM field. In this paper, we consider the advantage of vector antenna (VA) over uniform linear array (ULA) receiver for code-division multiple-access (CDMA) signals in a multiuser environment. We consider a closed loop power control (CLPC) for a beamformer-RAKE receiver for wireless CDMA multiuser system. The performance enhancement in Bit Error Rate (BER) is investigated for matched-filter receivers using various VA’s and ULA for a fading multipath channel. Analysis, theoretical curves, and simulations of VA and ULA receivers suggest that VA can better exploit multipath and therefore significantly improve the diversity gain and bit-error-rate performance of CDMA signals in the presence of a frequency selective rich multipath channel.

Keywords— code-division multiple-access, multipath channels, polarization diversity, receiver diversity, power control.

I. Introduction
In recent years, the number of users for wireless services such as internet and video have rapidly increased where providers of these services are required to deploy systems with higher data rates, combat fading and reduce interference thus provides higher quality of service. The space-time system, that employ multiple antennas at transmitter or receivers aim to exploits channel characteristics and provide diversity gain. Such systems employ uniform linear antenna array to respond to the vertical or horizontal polarization component of the EM field [1]. Extensions of these antenna array to dual-polarized antenna array have been considered in conjunction with multiuser detection in[2] where the diversity gain obtain is due to the effect of space and polarization systems used. Most of research have focused on developing an algorithms at both ends of the communication systems to enhance performance by using antenna arrays technology and multiuser receivers to combat fading and reduce correlation between users in order to lower error probability and therefore obtain a higher quality of received signal.

Since the signal consists of six field component (three electric and three magnetic), some potentially useful information may be neglected. Andrews et al [3] has suggested that dramatic gains in wireless capacity might be possible by exploiting these extra components of the received signal. In principle, a “Vector antenna” can independently detect all six EM field components enabling the com-the communication system to access an additional signaling dimensions, which may enhance performance in the same way as antenna arrays [1]. These extra dimensions can provide additional diversity to combat signal fading, allow the system to spatially leverage bandwidth by transmitting multiple separable signals in the same bandwidth, and improve the suppression of interference in multiuser environment.

A “tripole” antenna that consists of three mutually-orthogonal dipoles by Andrew have shown through simulation, that this antenna improves on the capacity of scalar and dual-polarized antennas for a simple propagation ex-ample involving a line-of-sight component and one reflected path. A similar study of a three-element antenna system consisting of a loop and two dipoles was presented in [4] where coupling between antenna elements and return loss was investigated. In [5], Konanur et al has shown an in-crease of capacity of vector antennas relative to a linear ar-ray antenna. Also, tripole antennas have
been investigated for a different number of receivers in frequency selective multipath Rayleigh fading channels for code division multiple access (CDMA) system [6].

In cellular communication system, it is known that the CDMA system is interference limited. In the uplink from mobile to base station, multiple access interference (MAI) due to multipath fading and cross correlation between users, results in power variation of the signal level between users located at different distances from base station. This is known as near-far effect [7]. As a result of this, MAI decreases the received signal level and therefore reduces signal quality. This in turn limits the capacity of the CDMA system. One way to overcome the effect of interference is by monitoring the signal power and maintaining an acceptable level of signal-to-interference plus noise ratio (SINR) by increasing or decreasing the transmitted power known as power control. In the last two decades, a lot of research has been devoted to different power control schemes. In the research study by Zander [8], Grandhi [9] and Foschini and Miljanic [10], power control has been applied in a centralized or distributed way. For systems that employ antenna array, a joint power control and beamforming was investigated for linear antenna system in a multiuser environment [11]. Similarly, to further reduce interference and increase capacity, multiuser detection was investigated in addition to the joint power control and beamforming [12].

In this paper, we will study the performance and investigate diversity gain of antenna array receivers that employ CLPC to mitigate near-far effect in multiuser environment over Rayleigh fading channel. Two systems of antennas are studied: the ULA and the Vector antennas receivers. The antenna array is employed at the base station and the up-link situation is considered here. The CLPC is used for a different number of vector antenna systems in frequency selective multipath fading channel. We will evaluate the performance of the CDMA system with vector antenna as a function of Bit Error Rate (BER) for different propagation conditions.

The organization of this paper is as follows: in section II, we will propose the system model where the channel model and receiver using closed-loop power-control (CLPC) is presented. In section III, SINR calculation for RAKE receiver and theoretical results for SINR and BER is introduced. In section IV, simulation results are presented and finally a conclusion is drawn.

II. System Model

In this work, we will consider the application of antenna array system in uplink condition for n users and a single base station. We assume a BPSK modulation scheme for direct sequence synchronous code division multiple access (DS-CDMA) system to be used.

Consider a single vector antenna at the receiver. Each arriving multipath component is a two-dimensional vector \( \mathbf{R}(t) \), consisting of horizontally and vertically polarized components of the electric field. For narrowband sinusoidal signals, it is convenient to write the signal in terms of its complex envelope \( Z(t) \) where \( A(t) = \text{Re} \{e^{j\omega_0 t} Z(t) \} \), and \( \omega_0 \) is the carrier frequency. For any polarized signal, the complex envelope can be written as

\[
Z(t) = A(t) e^{j\psi(t)} \begin{bmatrix} \cos \alpha \sin \alpha & \cos \beta \\ -\sin \alpha \cos \alpha & j \sin \alpha \end{bmatrix}, \tag{1}
\]

where \( A, \psi, \alpha, \beta \) are the amplitude, phase, orientation angle, and ellipticity angle, respectively [13]. Since the signal is a function of the transmitted data, the amplitude, phase and polarization angles vary with time, and we write \( Z(t) = Z(t) \). If the multipath component is reflected or scattered by an object in the far-field, then the signal can be approximated as a plane wave at the receiver. Let \( E(t) \) and \( H(t) \) denote the three-dimensional complex envelopes of the electric and magnetic field vectors respectively at the receiver, and suppose that the multipath component arrives at the sensor from the direction \( \mathbf{u}_r \), where

\[
\mathbf{u}_r = \begin{bmatrix} \cos \theta \sin \varphi \\ \sin \theta \sin \varphi \\ \cos \varphi \end{bmatrix}, \tag{2}
\]

where \( \theta \) and \( \varphi \) are the azimuth and elevation, respectively, of the incoming signal in the receiver coordinates (see Fig. in [6]). For a narrowband plane wave propagating in a non-conductive, homogeneous, and isotropic medium, the received signal can be modeled by [13]

\[
\mathbf{Y}(t) = \begin{bmatrix} E(t) \\ H(t) \end{bmatrix} = B(\theta, \varphi) \mathbf{Z}(t) + \mathbf{n}(t) \tag{3}
\]

where \( \mathbf{n}(t) \) represents thermal noise, \( \eta \) is the intrinsic impedance of the propagation medium, and

\[
B(\theta, \varphi) = \begin{bmatrix} -\sin \theta & \cos \theta & \cos \varphi \\ \cos \theta & \sin \theta & \cos \varphi \\ 0 & -\sin \varphi & \end{bmatrix}. \tag{4}
\]

We are interested in using the antenna system described above to detect CDMA signals in the presence of rich multipath and multiuser system. The transmitted signal of user \( i \) is a complex baseband signal of the form

\[
S_i = \sum_{m=1}^{M} b_i(m) c_i(t - mT) \tag{5}
\]
where \( c_i(t) \) is a unit-energy spreading waveform that vanishes outside the interval \([0, T]\), and \( b_i \) is a sequence of transmitted information bits for user \( i \). When \( S_i(t) \) is transmitted, the horizontally and vertically polarized components of each multipath component consist of \( S_i(t) \) with some amplitude and phase shift, so that \( Z_i(t) = Z_i S_i(t) \) for some fixed complex vector \( Z_i \).

We assume that each user’s transmitted signal propagates to the receiver by multiple paths produced by \( L \) different scatterers. If the \( \ell \)-th path for user \( i \) arrives at the receiver from direction \((\theta_{i\ell}, \phi_{i\ell})\), the superposition of the multipaths of all \( n \) users and noise at the receiver is given by

\[
Y(t) = \sum_{i=1}^{n} \sum_{\ell=1}^{L} \sqrt{P_i} B(\theta_{i\ell}, \phi_{i\ell}) Z_{i\ell}(t-\tau_{i\ell}) + n(t) \tag{6}
\]

here \( P_i \) is the signal power of the \( i \)-th user, \( B(\theta, \phi) \) is the \( 6 \times 2 \) antenna response given in eq. 4, \( \tau_{i\ell} \) is the propagation delay of the \( \ell \)-th path of user \( i \), and \( Z_{i\ell} \) reflects the change in amplitude, phase, and polarization experienced by the \( \ell \)-th multipath component as it propagates from the transmitter of user \( i \) to the receiver. The noise \( n(t) \) is a zero-mean complex Gaussian vector with covariance \( \sigma_n^2 \).

A \( t_{\text{sw}} \)-element vector antenna with \( t_{\text{sw}} \leq 6 \) can sense only \( t_{\text{sw}} \) components of \( Y(t) \). The received signal in eq. 2 above can be re-written as

\[
Y(t) = \sum_{i=1}^{n} \sum_{\ell=1}^{L} \sqrt{P_i} G_{i\ell} S_i(t-\tau_{i\ell}) + n(t). \tag{7}
\]

where \( G_{i\ell} \) consists of the appropriate term components of \( B(\theta_{i\ell}, \phi_{i\ell}) Z_{i\ell} \). The components of the vector \( Z_{i\ell} \) are modeled as i.i.d. zero-mean complex Gaussian random variables and therefore the elements of \( G_{i\ell} \) are also zero-mean complex Gaussian random variables.

Assuming we know the channel coefficients, delay spread and user’s spreading waveforms at the receiver, we can write the post-correlation signal of the first user for the \( m \)-th bit and \( \ell \)-th multipath as

\[
Y_{1\ell}(m) = \int_{(m-1)T+\tau_{1\ell}}^{mT+\tau_{1\ell}} c_1^*(t-mT+\tau_{1\ell}) Y(t) dt \tag{8}
\]

where \( c_1(t-\tau_{1\ell}) \) is the spreading code of the first user delayed by \( \tau_{1\ell} \), \( b_1(m) \) is the \( m \)-th data bit for the first user, and \( n_{1\ell} \) is the thermal noise.

This model is also easily generalized to arrays that consist of multiple spatially-separated, non-interacting vector antennas. Suppose that the receive array consists of \( r \) spatially-separated vector antennas located at positions \( x_1, \ldots, x_r \) in receiver coordinates, that \( Y(t) \) is the signal detected at the \( i \)-th receiver antenna, and that

\[
Y(t) = \begin{bmatrix} Y_1(t) \\ \vdots \\ Y_r(t) \end{bmatrix} \tag{9}
\]

A simple model for the resulting space-polarimetric channel is given with

\[
G_{i\ell} = [A_r(\theta, \phi) \otimes B(\theta, \phi)]Z_{i\ell} \tag{10}
\]

\( \otimes \) is the Kronecker product, \( A_r(\theta, \phi) \) is the classical narrow band array response

\[
A_r(\theta, \phi) = \begin{bmatrix} \exp(-j\omega_c d_1) & \cdots & \exp(-j\omega_c d_r) \end{bmatrix}, \quad d_i = -\frac{w_i - x_i}{c} \tag{11}
\]

and \( c \) denotes the speed of light.

### III. MAI and SINR Calculation

The multiple-access interference (MAI) contributed from other users and self-interference is given by

\[
i_{1\ell} = \sum_{k=1, k \neq \ell}^{n} \sqrt{P_k} G_{1k} I_{1\ell, 1k} + \sum_{i, j=2}^{n} \sum_{k=1, k \neq \ell}^{L} \sqrt{P_i} G_{ik} I_{1\ell, 1k} \tag{12}
\]

where the first term corresponds to self-interference due to multipath, and the second term is due to multiple-access interference (MAI) and

\[
I_{1\ell, 1k} = \int_{(m-1)T+\tau_{1\ell}}^{mT+\tau_{1\ell}} b_{1m}^* c_i^*(t-mT+\tau_{1k}) dt \tag{13}
\]

If we denote the total undesired interference as \( \mathbf{R}_{1\ell} = i_{1\ell} + n_{1\ell} \), then the correlation matrix can be written as

\[
\mathbf{R}_{1\ell} = \sum_{k=1, k \neq \ell}^{n} P_k G_{1k} G_{1k}^* + \sum_{i, j=2}^{n} \sum_{k=1, k \neq \ell}^{L} P_i G_{ik} G_{ik}^* + \sigma_n^2 I \tag{14}
\]

where \( \rho_{1,j} \) is the correlation factor of spreading sequence between first user and user \( j \).

If the weight vector of the desired user for path is given by \( w_{1\ell} \), then the output of the beamformer of the space-time RAKE receiver for path is given as
\[ z_{1,\ell}(m) = w_{1,\ell}^* Y_{1,\ell}(m) = \sqrt{P_1} w_{1,\ell}^* G_{1,\ell} b_m + w_{1,\ell}^* \tilde{n}_{1,\ell}. \]  
(15)

Where the optimum weight of the beamformer that maximizes the signal-to-interference-plus-noise ratio (SINR) is given by [14]

\[ w_{1,\ell}^* = \frac{G_{1,\ell}}{\sum_{i=1}^{N} |G_{i,\ell}|^2 + \sigma_n^2}. \]  
(16)

For maximal ratio combining (MRC), the decision statistic is given as scalar output

\[ \hat{b} = \text{sgn} \left( \sum_{\ell=1}^{L} z_{1,\ell}(m) \right). \]  
(17)

For closed loop power control, we compare \( \gamma \) for the desired user to a threshold \( \gamma_{th} \) and the performance is determined by SINR. The SINR after RAKE combining defined as Post-RAKE, the SINR for \( \ell \) path is given by

\[ Y_{1,\ell} = \frac{P_1 |w_{1,\ell}^* G_{1,\ell}|^2}{\sum_{i=1}^{N} \sum_{\ell=1}^{L} P_1 |w_{i,\ell}^* G_{i,\ell} G_{1,\ell} w_{1,\ell}| + \sigma_n^2} = \frac{P_1 w_{1,\ell}^* G_{1,\ell} w_{1,\ell}}{w_{1,\ell}^* \tilde{n}_{1,\ell}}, \]  
(18)

And the total SINR after RAKE combining is given by

\[ \gamma_b = \sum_{\ell=1}^{L} \gamma_{\ell} = \sum_{\ell=1}^{L} \frac{1}{\sum_{i=1}^{N} \sum_{\ell=1}^{L} |G_{i,\ell}|^2 + \sigma_n^2} \]  
(19)

If the noise is white (\( \mathbf{R}_n = \sigma_n^2 \mathbf{I} \)) and \( G_{i,\ell} \) is a zero-mean complex Gaussian random variable, then \( \gamma_{\ell} \) has \( \chi^2 \) probability density function(pdf) with \( 4L \) degrees of freedom [16]:

\[ f_{\gamma_b}(\gamma) = \frac{\gamma^{2L-1}}{(2L-1)!} e^{-\gamma/\bar{\gamma}} \]  
(20)

where \( \bar{\gamma} = \frac{\rho}{\sigma_n^2} \) is the average SNR-per-path-per-antenna. The average BER of BPSK modulation is therefore given by proakis [16]

\[ P_b = \int_0^\infty Q \left( \sqrt{2\gamma} \right) f_{\gamma_b}(\gamma) d\gamma = \left( \frac{1+u}{2} \right)^{2L} \sum_{\ell=0}^{2L} \left( \frac{2L-1+\ell}{2L-1} \right) \left( \frac{1+u}{2} \right) ^\ell \]  
(21)

where for the tripoole antenna

\[ u = \sqrt{\frac{\bar{\gamma}}{1 + \bar{\gamma}}} \]  

And for six element vector antennas

\[ u = \frac{2\gamma}{1 + 2\gamma} \]  

The difference between the \( \gamma \) and \( \gamma_{th} \) is compared and the error command \( e = \gamma - \gamma_{th} \) is evaluated. Then a power control command bit is sent to increase or decrease transmitted power for next sampling period \( T_p \) bits by \( \Delta p \) increment. We can summarize the CLPC as follows

\[ P(i + 1) = P(i) - e(i)\Delta p \]  
(22)

Where,

\[ PC \text{ bit} = \text{sgn}[e(i)] = \begin{cases} -1, & e(i) < 0 \\ +1, & e(i) > 0 \end{cases} \]  
(23)

To compare the performance of the vector-antenna array to a uniform linear array (ULA), we present the equivalent ULA model. The ULA [15] is modeled as a collection of dipole antenna elements that are spatially and uniformly distributed in a plane containing array elements. The antenna response for a uniform linear array parallel to the \( z \)-axis and centered on the \( x \)-axis can be modeled as

\[ B(\theta, \varphi) = \sin \varphi \begin{bmatrix} e^{-j2\pi x_1 \cos \theta / \lambda} & 0 \\ \vdots & \vdots \\ e^{-j2\pi x_M \cos \theta / \lambda} & 0 \end{bmatrix} \]  
(24)

where \( x_i, \ i = 1, 2, \ldots, M \) corresponds to the \( x \)-coordinates of the antennas. This uniform antenna system can respond to one polarization component, horizontal or vertical.

### IV. Simulation

In this section we numerically plot the BER of the matched filter (MF) receiver for vector and ULA antennas. We consider a single base station and the uplink from mobile users to the base station, where all users transmit to the same base station. In these simulations, we consider uniform linear arrays and vector antennas that consist of \( t_m = 2, 3, 6 \) elements implemented at the base station which applies CLPC scheme in a multipath environment.

We assume that the multipath components of the desired and interference signals are uniformly distributed on a sphere where the azimuth angle \( \theta_{i,\ell} \), are i.i.d. and uniform on \( \in [0, 2\pi] \), and the elevation angles \( \varphi_{i,\ell} \), are i.i.d. with probability density function

\[ P(\varphi_{i,\ell}) = \begin{cases} \\frac{1}{2} \sin(\varphi_{i,\ell}), & 0 < \varphi_{i,\ell} < \pi \\ 0, & \text{otherwise} \end{cases} \]  
(25)
and the polarizations \(Z_i\) are i.i.d. \(CN(0, 1)\). The noise \(n(t)\) is spatially and temporally white and Gaussian (i.e. \(E[n(t)n^*(t+\tau)] = \delta(\tau)\)). The channel is assumed to be a frequency-selective, slowly Rayleigh fading, and the multipath delays \(\tau\) are set at discrete intervals, which is equivalent to one chip duration \((T_c)\). The scenario of synchronous users is assumed here. The number and values of the multipath delays \(L\) are equal for all users at the receiver and we assume the detection of the first user. 

In all of our simulation we consider \(L = 2\) multipath components with delays \(\tau_1\) and \(\tau_2\), respectively, such that \(\tau_2\) is delayed by one chip relative to \(\tau_1\). At the transmitter we assume BPSK modulation and Gold spreading sequences of length 31 [17] and a unit-energy spreading waveform. The SINR threshold \(\gamma_0\) for the CLPC is considered for the desired user where other users are set to a fixed \(\gamma\) value that is equal to target \(\gamma_0\). The 3-element tripole antenna proposed by Andrews et al consists of three mutually orthogonal dipoles, where \(B(\theta, \varphi)\) are given by

\[
B(\theta, \varphi) = \begin{bmatrix}
-\sin \theta & \cos \theta & \cos \varphi \\
\cos \theta & \sin \theta & \cos \varphi \\
0 & -\sin \varphi & \cos \varphi
\end{bmatrix}
\]  

(26)

where the first two component in above matrix correspond to the dual antenna case. The 3-element vector antenna proposed by Konanur et al [4] which consists of two dipoles and a loop, where the elements of \(B(\theta, \varphi)\) are given by

\[
B(\theta, \varphi) = \begin{bmatrix}
-\sin \theta & \cos \theta & \cos \varphi \\
\cos \theta & \sin \theta & \cos \varphi \\
\cos \theta & \sin \theta & \cos \varphi
\end{bmatrix}
\]  

(27)

for the ULA, we will use the model presented in eq. 24, that consist of up to 6 antenna elements.

The VA array will be consisting of the same number of elements. For example, the array of tripole VA will consist of tripole antennas displaced at distance \(d\) from each other and the same is true for different VA elements.

In Fig. 1 the performance of MF detector for desired user is plotted for three types of antenna configuration: 2,3-element ULA, dual and single tripole antenna ,and two dipole and a loop proposed by Konanor at a number of \(\gamma_0\) values and \(n=10\) users. The performance in terms of the BER of CLPC with step size of 1 dB is assumed. The figure shows that VA performance improve as we go from dual to tripole and gain much higher diversity compared to ULA. The tripole antenna is more effective in mitigating interference as we compare to other antenna configuration in this figure.

In Fig. 2, simulation for 3-element ULA and VA antenna for \(L = 1, 2\) multipath and \(n=10\) users is plotted. We notice that tripole antenna outperforms the ULA and maintain this performance gain over \(L = 1, 2\) multipath components. Due to the high interference from multuser and multipath channels, the ULA have a marginal improvement in BER. Increasing number of VA elements from 3 to 6 result in much lower BER than tripole antenna case. Fig. 3 is plotted for ULA with 6-element antenna and compared to an array of two tripole antenna and 6-element vector antennas. Comparing the BER attained for VA and ULA, we notice that at higher SNR values and \(L = 2\) multipath, both VA array and 6-element VA have gained diversity while ULA is approaching error floor. This shows that VA can better exploit multipath environment and can benefit from space diversity and power control more than ULA.

In Fig. 4, the performance of Pre and Post Rake are plotted for ULA and VA antennas. Since interference is higher at Pre Rake than Post Rake combining, therefore a lower BER obtained at Post Rake. In both cases of VA and ULA, the improvement in BER seems to be approximately equivalent.

Fig. 5 shows the effect of BER as a function of the number of users. In the ULA case, the benefit of increasing number of antennas have slight impact on performance and therefore BER. On the other hand, VA has shown significant improvement when we went from single tripole to an array of two tripole case, therefore, the BER for the number of users that is accommodated is much lower than in VA systems.

V. Conclusions

Performance of VA and ULA with CLPC have been analyzed in both multipath and mutliuser environment. At the receiver side, we have studied MF receiver employing vector antennas and ULA with CLPC. The assumption of equivalent SINR for the interference and threshold value is assumed in order to study the ability of proposed VA and ULA in mitigating interference. We observed various vector antennas showing a better performance of BER with power control than in the case without it. We found that VA antenna have outperform the ULA at low and high SINR over multipath fading channels. A comparison for ULA and VA at SINR with pre and post Rake have been evaluated. Also, to show the benefit of CLPC, theoretical curves for VA have been plotted and compared with simulation employing CLPC. Finally, comparing the BER for number of users at certain threshold value have shown that VA have more capacity in terms of number of users with lower BER than ULA system.

References

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Fig. 1. BER of the MF receiver for the tripole antenna (3-dipole), 2 dipole and loop, and 2,3-element ULA antennas for L=1 multipath, n=10 users, Δp=1 dB.

Fig. 2. BER of the MF receiver for the tripole antenna (3-dipole), 6-element, and 3-element ULA antenna for L=1,2 multipath, n=10 users, Δp=1 dB.

Fig. 3. BER of the MF receiver for the array of 2-tripole antenna and 6-element VA, Versus 6-element ULA antenna for L=1,2 multipath, n=10 users, Δp=1 dB.

Fig. 4. BER for Pre and Post RAKE MF receiver for the tripole antenna (3-dipole), and 3-element ULA antenna for L=1 multipath, n=10 users, Δp=1 dB.

Fig. 5. BER of the MF receiver for the tripole antenna (3-dipole), 2-tripole antenna and 3,6-element ULA antenna for L=1,2 multipath as a function of users, Δp=1 dB.