Evaluation of PID Tuning Methods on Direct Gas-Fired Oven

Aborisade, D. O*, Adewuyi, P. A**
*(Department of Electronic/Electrical Engineering, Ladoke Akintola University of Technology, Nigeria)  **(Department of Mechatronics Engineering, Bells University of Technology, Nigeria)

ABSTRACT
This paper studies the temperature control of gas-fired oven using PID controller. Oven control system has the characteristics of non-linearity, time delays and setpoint response. It is difficult to overcome the effects of these factors and get the satisfactory results without appropriately tuning of the PID controller gains required for stability and good transient performance. The Ziegler-Nichols closed loop, Good Gain and Skogestad’s are the PID tuning methods implemented in this paper to control the output temperature of the gas-fired oven system. The PID tuning methods are compared, based on their rise time, maximum overshoot and settling time. The performance of Skogestad’s tuning method at different temperature set point is superior to Ziegler-Nichols closed loop and Good Gain PID tuning method.

Keywords- Gas-fired oven, PID controller, Ziegler-Nichols method, Good Gain method, Skogestad’s method.

I. Introduction
Many food service industries rely heavily on ovens to heat or preserve food. Oven exists in various configurations which can be gas-fired or electrically heated types. Gas-fired oven is cheaper to run and produce less greenhouse gas than an equivalent electric model [1-3]. Heating systems need an effective control to keep them running in the safest, most efficient and least costly manner. In this paper, we focus on temperature control of gas-fired oven only.

Accurate control of temperature in an oven without extensive operator’s involvement relies upon a controller, which accepts a temperature sensor as input and provides an output to a control element. There are various types of the controllers in the market such as On/Off, Proportional and PID. However, the PID remains the most common controller despite all the progress in advanced control [4]. Even if more sophisticated control laws are used it is common practice to have a hierarchical structure with PID control at the lowest level [5, 6]. A survey showed that 97% of regulatory controllers in the refining and industries are PID controllers because of their simple structure, robustness and high response performance [7]. PID temperature controllers can provide control action in industrial ovens if its control parameters (proportional band/gain, integral gain/reset, derivative gain/rate) are correctly tune to the optimum values in the presence of unknown nonlinearities, time delays, load disturbances and setpoint response.

There are several auto-tuning techniques to define PID controller parameters [8-20]. For a given application, each method has its advantages and disadvantages. Auto-tuning methods used in this study to obtain an optimal set of control parameters for the oven temperature model are Ziegler-Nichols closed loop method, Good Gain method and Skogestad’s method. From the dynamics of the oven temperature control system, modeled by 2nd order transfer function, the performances of the PID controller with the three auto-tuning techniques were investigated via numerical simulation implemented in MATLAB/Simulink environment.

II. OTC System Architecture
Figure 1 shows the basic architecture of the oven temperature control (OTC) system consisting of a fan-assisted burner (equipped with an igniter, a solenoid shut-off gas valve(s), an adjustable gas orifice cock and a proportional air/gas mixer), a burner by-products elimination system (vent), and control mechanisms (such as the controller, room-air sensor, and gas control valve). The feedback control of OTC system is illustrated in Figure 2. Here, the oven temperature parameter is periodically measured with resistance thermometer.
Controller

Potentiometer

Desired
Temperature

Thermometer

Vent

Burner

Control
Valve

Gas

Cable

Air

Air

Door

Figure 1: OTC System Architecture

The output signal from this sensor is digitized by an A/D converter and fed back to the PID controllers.

In order to explore the application of PID controllers in the system, a single-zone heating system is being considered. This is due to the fact that heating mode is found to perform nearly the same under most circumstances. The PID controller computes an appropriate control signal based on the changes of feedback oven temperature that continuously sensed by the sensor and then decides which action to be taken. In this study, the oven temperature is being controlled and monitored to approaching the desired temperature valued which is usually obtained from manual adjustment of a potentiometer.

III. OTC Mathematical Representation

This section presents the mathematical representation of OTC system and explains the heat equations applied in the oven temperature calculation. Considering a single zone gas-fired oven with gas burner to heat the air, temperature and heat in the oven zone are ultimately managed by controlling fuel and air flow from the external sources to the burner. The amount of air/fuel mixture (10 parts air to 1 part natural gas) delivered to the burners is based on accurate temperature sensing within the zone.

As the zone temperature deviates from the temperature control set-point, the quantity of the mixed air/fuel delivered to the burner is adjusted by increasing or decreasing the zone pressure of the combustion air. Therefore, a reduction in zone combustion air pressure reduces the quantity of fuel supplied to the burner, lowering the heat input to the zone, and an increase in combustion air pressure increases the quantity of fuel supplied to the burner and raises the heat input to the zone.

In control system design, it is important to simplify a single-zone space thermal system which is exposed to certain outdoor conditions [21-23]. The simplified oven temperature model in this study is obtained by applying the principle of energy balance:

\[ C_T \frac{d\theta}{dt} = Q_i(t) - Q_o(t) \]  

where

\[ Q_i(t) = \text{Heat input into oven (J/s)} \]

\[ Q_o(t) = \text{Heat removed from the oven (J/s)} \]

\[ C_T = \text{Oven thermal capacitance (J/K)} \]

The physical interpretation of (1) is that the rate of change of energy in the oven is equal to the difference between the heat supplied to and removed from the oven. The heat removed from the oven (W) is given by:
\[ Q_s(t) = \frac{(\theta_s(t) - \theta_o(t))}{R_T} \]  
where
\[ \theta_s(t) = \text{Temperature of surroundings (°C)} \]
\[ \theta_o(t) = \text{Internal oven temperature (°C)} \]
\[ R_T = \text{Thermal resistance of the walls} \]

Therefore, the equation describing the dynamic behavior of the oven is given by:
\[ R_T Q_s(t) + \theta_s(t) = \theta_o(t) + R_T C_T \frac{d\theta_o}{dt} \]  
(3)

simplify and taking the ratio of \( \theta_o(s) / \phi(s) \) gives the transfer function of the open loop system given by:
\[ G(s) = \frac{\theta_o(s)}{\phi(s)} = \frac{1}{R_T C_T + s} \]
(4)

where
\[ \phi(s) = R_T Q_s(s) + \theta_o(s) \]

The transfer function of the gas control valve and burner unit is given by:
\[ \frac{Q_v(s)}{P(s)} = \frac{K_v K_b}{T_s + 1} \]
(5)

where
\[ K_v = \text{valve constant (m}^3/\text{sV}) , \]
\[ K_b = \text{burner constant (Ws/m}^3) , \]
\[ Q_v(s) = \text{heat flow to the oven} . \]

Hence, the overall dynamics equation of the system is given by:
\[ \frac{\theta_o(s)}{P(s)} = \frac{R_T K_v K_b}{(T_s + 1)(R_T C_T + s + 1)} \]
(6)

where
\[ K_v = \text{valve constant (m}^3/\text{sV}) , \]
\[ K_b = \text{burner constant (Ws/m}^3) , \]
\[ Q_v(s) = \text{heat flow to the oven} . \]

Oven temperature system is an integration of computation, networking and physical dynamics, in which embedded devices such as sensors and actuators are networked to sense, monitor and control the oven. The ideal of feedback path in the control architecture is to exploit the measurements of the system’s output to determine the control commands that yield the desired system behavior. The sensor senses the temperature of the oven, using its resistive element, then generates a voltage which is linearized and sent to the transmitter unit, which eventually converts the thermocouple output to a standardized signal. This output of the transmitter unit is given to the controller unit. The voltage signal generated by the sensor is obtained by mounting a current source across the resistive element. The voltage magnitude, based on Ohm’s Law, is equal to the product of the magnitudes of the current source and the sensing element’s resistance. The linearized resistance response to temperature versus time is approximated by a linear first order system with a Laplace transform of the following structure:
\[ \theta_m(s) = \theta_o(s) \times \frac{K_d}{\tau S + 1} \]
(7)

where
\[ \theta_m(s) = \text{Room Temperature Sensed} \]
\[ \theta_o(s) = \text{Actual Room Temperature} \]
\[ K_d = \text{is the gain of the linearized sensor, and} \]
\[ \tau = \text{is the time constant of the sensor.} \]

In the system, 3-wire PT-100 RTD with a range of –200 to 600ºC is used as it can withstand high temperature while maintaining excellent stability. Temperature coefficient of platinum wire is 0.00385 ohm per ºC. The sensor has a calibrated range of 0ºC to 200ºC and a time-constant of 1 to 2 sec.

Sensor gain = \[
\frac{(20 - 4) \text{mA}}{200 - 0} \text{C} = 0.08 \text{mA/ºC}
\]
(8)

Transfer function of the sensor \( H(s) \) is given by:
\[ H(s) = \frac{\theta_m(s)}{\theta_o(s)} = \frac{0.08}{2s + 1} \]
(9)

The PID control algorithm remains the most popular approach for industrial process control despite continual advances in control theory. This is not only due to the simple structure which is conceptually easy to understand and, which makes manual tuning possible, but also to the fact that the algorithm provides adequate performance in the vast majority of applications. PID controller in continuous time is given as:
\[ u(t) = K_p \left[ e(t) + \frac{1}{T_n} \int_0^t e(\tau) d\tau + T_v \frac{de(t)}{dt} \right] \]
(10)

where \( u(t) \) is the control signal, \( K_p, T_n \) and \( T_v \) are positive parameters, which are respectively referred to as proportional gain, integral time and derivative time, and the error \( e(t) \) is the control error. The Laplace transformation representation of the approximate PID controller can be written as:
\[ U(s) = K_p \left[ 1 + \frac{1}{T_n s} + \frac{s T_v}{T_s + 1} \right] e(s) \]
(11)
where \( U(s) \) and \( e(s) \) is respectively the output and input signals in frequency domain. The transfer function of PID controller is:

\[
W_{\text{PID}}(s) = \frac{U(s)}{e(s)} = K_p \left( 1 + \frac{1}{T_n s} + \frac{sT_v}{T_s s + 1} \right)
\]

(12)

The PID parameters (the proportional gain \( K_p \), the integral time \( T_n \), and the derivative time gain \( T_v \)) are determined by the well-known tuning methods based on step response.

Having obtained the mathematical models for each of the components shown in Fig.1, they are combined to yield the closed loop transfer function for the system:

\[
\frac{\theta_e(s)}{\theta_i(s)} = \frac{3.75W_{\text{PID}}}{(6s + 1)(5s + 1)}
\]

(13)

\[
1 + \frac{3.75W_{\text{PID}}}{(6s + 1)(5s + 1)} = 0.08
\]

(13)

The parameters of the oven temperature control system are shown in Table 1.

IV. Tuning of PID Controller

Tuning a control loop is the adjustment of its control parameters to the optimum values in order to achieve the desired control response. Designing and tuning a proportional-integral-derivative (PID) controller appears to be conceptually intuitive, but do, however, present some challenges to control in the aspect of tuning of the gains required for stability and good transient performance. There are many methods proposed for tuning of PID controller based on experiments executed on a simulated system. In this section we have used the following three methods for tuning [15, 20, 24, 14, and 25].

- **Ziegler-Nichols’ Closed Loop Method**
- **Good Gain method**
- **Skogestad’s method**

4.1 Ziegler Nichol’s Method

Ziegler and Nichols suggested that we set the values of the parameter \( K_p \), \( T_n \), and \( T_v \) according to the formula shown in Table 2. In the Ziegler Nichol’s Closed-Loop method we first set \( T_n = \infty \) and \( T_v = 0 \). Since the system have a mathematical model, then we use the root-locus method to find the critical gain \( K_{p_c} \) and the frequency of the sustained oscillations \( \omega_c \), where \( 2\pi / \omega_c = T_c \). This value is evaluated from the crossing points of the root-locus branches with the \( j\omega \) axis.

4.2 Good Gain Method

“Good Gain” PID tuning method was developed by Finn Haugen in 2010 [20]. It is a simple method that based on experiments on a real or simulated control system. The theoretical background of the method is described in detail by Haugen [24].

In this method, the process is first brought close to the specified operation point with the controller in manual mode. Then, ensure that the controller is a P-controller with \( K_p = 0 \) (set \( T_n = \infty \) and \( T_v = 0 \)). The value of \( K_p \) was increased until some overshoot and a barely observable undershoot shown in Figure 3 is observed due to a small step change of the set-point. By trial-and-error, we find the gain value \( K_{p_{GG}} \). This gives the control loop good stability as seen in the response in the measurement signal due to a step in the setpoint.

With the inclusion of I-term, the value of the integral time, \( T_n \) was set to:

\[
T_n = 1.5 T_{ov-un}
\]

(14)

where \( T_{ov-un} \) is the time between the first overshoot and the first undershoot of the step response (a step in the setpoint) with the P controller. Due to the inclusion of the I-term, the loop with the PI controller in action probably have somewhat reduced stability than with the P controller only. To compensate for this, the value of \( K_p \) is hence computed as:

\[
K_p = n_p K_{p_{GG}}
\]

(15)

where the value of \( n_p \) is about 0.8.

With the introduction of the D (Derivative)-term, the controller becomes a PID controller, and thus \( T_v \) is set to:

\[
T_v = n_v T_n
\]

(16)

where the value of \( n_v \) is 0.25.

4.3 Skogestad’s Method

Skogestad’s PID tuning method is a model-based tuning method where the controller parameters are expressed as functions of the process model parameters. The process model is assumed to be a continuous-time transfer function. The control system tracking transfer function \( T(s) \), which is the transfer function from the setpoint to the process with sensor, is specified as a first order transfer function with time delay:

\[
T(s) = \frac{Y_m(s)}{Y_m(s)} = \frac{1}{T_c s + 1} e^{-\theta s} = \frac{C(s)G_{pf}(s)}{1 + C(s)G_{pf}(s)}
\]

(17)
where $T_c$ is the time-constant of the control system which the user must specify, and $\theta$ is the effective time delay which is given by the process model. $G_{pf}(s)$ represent all the dynamics that the controller “feels”. It is a combined transfer function of the process, gas solenoid valve and burner unit given as:

$$G_{pf}(s) = K \cdot \frac{H(s)}{(T_1s + 1)}$$  \hspace{1cm} (18)

**TABLE 1**

PARAMETERS OF THE SYSTEM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal capacitance of the air in the oven, $C_T$</td>
<td>$50 \text{ J/K}$</td>
</tr>
<tr>
<td>Thermal resistance of the oven’s air, $R_T$</td>
<td>$1.0 \text{ K/J}$</td>
</tr>
<tr>
<td>$T_i$</td>
<td>$6.0 \text{ Seconds}$</td>
</tr>
<tr>
<td>Valve constant, $K_v$</td>
<td>$1.5 \text{ m}^3/\text{s V}$</td>
</tr>
<tr>
<td>Burner constant, $K_b$</td>
<td>$2.5\text{Ws/m}^3$</td>
</tr>
</tbody>
</table>

**TABLE 2**

ZEIGLER-NICHOL’S TUNING RULES

<table>
<thead>
<tr>
<th>Type of Controller</th>
<th>$K_p$</th>
<th>$T_n$</th>
<th>$T_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>$0.5K_{pcr}$</td>
<td>$\infty$</td>
<td>$0$</td>
</tr>
<tr>
<td>PI</td>
<td>$0.45K_{pcr}$</td>
<td>$0.83T_{cr}$</td>
<td>$0$</td>
</tr>
<tr>
<td>PID</td>
<td>$0.6K_{pcr}$</td>
<td>$0.5T_{cr}$</td>
<td>$0.125T_{cr}$</td>
</tr>
</tbody>
</table>

Figure 3: Reading off the time between the first overshoot and the first undershoot of the step response with P controller

Considering second-order plus delay, the resulting model transfer function is given as:

$$G_{pf}(s) = k \frac{e^{-\theta s}}{(T_{c1}s + 1)(T_{c2}s + 1)}$$  \hspace{1cm} (19)

where $T_{c1}$ and $T_{c2}$ are the desired closed-loop time constant, and they are the tuning parameters for the controller.

From (16), the feedback part of the controller is given as:

$$C(s) = \frac{1}{G_{pf}(s)} \cdot \frac{1}{1 - \frac{Y_{mp}(s)}{Y_{mf}(s)}_{\text{desired}}}$$  \hspace{1cm} (20)

Combining (15, 17 and 18) and solving with respect to the controller gives a “Smith Predictor” controller [26]:

$$C(s) = \frac{1}{k} \frac{1}{(T_{c1}s + 1)(T_{c2}s + 1)} \cdot \frac{1}{1 - e^{-\theta s}}$$  \hspace{1cm} (21)
To get a PID-controller we introduce in (21) the following first-order Taylor approximation for the delay as [27]

\[ e^{-\theta s} \approx 1 - \theta s \]  \hspace{1cm} (22)

and compute

\[ C(s) = \frac{\left( T_{c1} s + 1 \right) \left( T_{c2} s + 1 \right)}{k} \cdot \frac{1}{(\Gamma_c + \theta) s} \] \hspace{1cm} (23)

which is a cascade form PID-controller with

\[ K_p = \frac{1}{k} \frac{T_{c1}}{\Gamma_c + \theta} \] \hspace{1cm} (24)

\[ T_n = \min\{T_{c1}, c \left( \Gamma_c + \theta \right) \} \] \hspace{1cm} (25)

\[ T_v = T_{c2} \] \hspace{1cm} (26)

where \(-\theta < \Gamma_c < \infty\) is the tuning parameter. For robust tunings we select \( \Gamma_c \geq \theta \).

V. Experimental Results

The overall model of gas-fired oven temperature control system with PID controllers is implemented in MATLAB/Simulink.

The simulink model of the PID controller is shown in Figure 4. Calculated PID parameters for all types of tuning methods are given in Table 3. Figure 5, 6, and 7 shows the response of the system with Skogestad’s PID-, Ziegler-Nichols’ Closed Loop PID-, and Good Gain PID- tunings at three different set points 60ºC, 90ºC, and 120ºC. The performance comparison of the tuning methods is given in Fig.8 and Table 4. As seen in the figure and table it is clear that Skogestad’s PID-tunings gives a much improved performance at each set point than Ziegler-Nichols’ Closed Loop PID- and Good Gain PID-tuning.

VI. Conclusion

This paper compared three kind of PID-tuning techniques to control the temperature of a gas-fired oven. Our aim is to improve the dynamic performance of the system output like settling time, rise time and maximum overshoot at three different set points 60ºC, 90ºC, and 120ºC. Skogestad’s method took only a couple of seconds to solve the problem. Compared to Ziegler-Nichols’ Closed Loop and Good Gain methods, Skogestad’s tuning method has good steady state response and performance indices.

![Simulink Experimental Setup](image-url)

**Figure 4: Simulink Experimental Setup**

<table>
<thead>
<tr>
<th>PID</th>
<th>Ziegler-Nichols’ Closed Loop Method</th>
<th>Good Gain Method</th>
<th>Skogestad Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>20.5332</td>
<td>27.3776</td>
<td>3.2</td>
</tr>
<tr>
<td>$T_n$</td>
<td>6.7498</td>
<td>11.25</td>
<td>0.75</td>
</tr>
<tr>
<td>$T_v$</td>
<td>1.6874</td>
<td>2.8125</td>
<td>5.5</td>
</tr>
</tbody>
</table>

**TABLE 3**

PID Controller parameters of all auto-tuning methods used in the experiments
TABLE 4
Analysis of system with auto-tuning method at set point 60ºC, 90ºC and 120ºC

<table>
<thead>
<tr>
<th>Set point</th>
<th>Rise time (sec.)</th>
<th>Maximum overshot (%)</th>
<th>Settling time (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ziegler-Nichols’ Closed Loop Method</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Good Gain Method</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skogestad Method</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60ºC</td>
<td>112</td>
<td>82.7</td>
<td>225</td>
</tr>
<tr>
<td>90ºC</td>
<td>65</td>
<td>81.5</td>
<td>178</td>
</tr>
<tr>
<td>120ºC</td>
<td>40</td>
<td>28.9</td>
<td>114</td>
</tr>
<tr>
<td>60ºC</td>
<td>327</td>
<td>89.4</td>
<td>552</td>
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<tr>
<td>90ºC</td>
<td>308</td>
<td>79.5</td>
<td>204</td>
</tr>
<tr>
<td>120ºC</td>
<td>179</td>
<td>41.6</td>
<td>497</td>
</tr>
<tr>
<td>60ºC</td>
<td>49</td>
<td>47.4</td>
<td>52</td>
</tr>
<tr>
<td>90ºC</td>
<td>1.0</td>
<td>58.1</td>
<td>37</td>
</tr>
<tr>
<td>120ºC</td>
<td>5.0</td>
<td>56.5</td>
<td>59</td>
</tr>
</tbody>
</table>

Figure 5: Simulation Result of PID control for Ziegler-Nichols’ Closed Loop Method

Figure 6: Simulation Result of PID control for Good Gain Method
Figure 7: Simulation Result of PID control for Skogestad Method

Figure 8: Comparison of the simulation Results over entire experimental period