RESEARCH ARTICLE OPEN ACCESS

Regional Flood Frequency Analysis Utilizing L-Moments: A Case Study of Narmada Basin

Amit Dubey*
*(Department of Civil Engineering & Applied Mechanics, S.G.S.I.T.S, Indore)

ABSTRACT
Accurate estimation of flood frequency is needed for the designing of various hydraulic structures such as dam, spillways, barrage etc. Different approaches were presented which uses the conventional moments to extract order statistics such as mean, standard deviation, skewness, kurtosis. Due to problems arising from data quality, such as short record and outliers, conventional moments are problematic. Hosking (1990) developed L-Moments which are linear combinations of order statistics. The main advantage of L-Moments over conventional moments is that they suffer less from the effects of sampling variability. They are more robust to outliers and virtually unbiased for small samples. In this study the L-Moments based method of Regional Flood Frequency Analysis is mentioned and Narmada Basin is considered as a case. The L-Moments have been used for parameter estimation of Generalized Extreme Value (GEV) distribution. Regional flood frequency relationship for the chosen basin is developed utilizing GEV distribution. A relationship between mean annual flood and catchment area is obtained which is further employed to generate Regional Flood Formula for ungauged catchment of Narmada Basin. The developed Regional flood formula is used for T-Year return period flood estimation, knowing only one parameter i.e. catchment area of ungauged watershed.
Keywords - GEV Distribution, L-Moments, Regional Flood Frequency Analysis.

I. Introduction

The hydrologic system embodies all of the physical processes that are involved in conversion of precipitation tostream flow, as well as physical characteristics of the watershed and atmosphere that influence runoff generation. Flood and drought are two main features of hydrology which affect the human life. A significant aspect of flood hydrology is the estimation of the magnitude of stream flow at various locations in a watershed resulting from a given precipitation input. Measure might be simply the magnitude associated with a particular point in time (as in flow forecasting), magnitude associated with a non-frequency based design flood, magnitude associated with a particular exeedance or non-exeedance frequency. These are the components of design flood. Planning, designing and operation of water resources projects are based on design flood.
Floods can be estimated through the use of several methods dependent upon the structure being designed or analyzed and size of watershed. Approaches to flood hydrology include deterministic, probabilistic, conceptual or parametric analysis. The reasonable estimation of Flood has been remained one of the main problematic issues where hydrological data and information are either limited or not available. This is the typical case in Narmada Basin, where most of its tributaries are ungauged.

1.1 Regional Flood Frequency Analysis

Water resources decision making problems such as flood plain zoning, design of hydraulic structures etc., are based on design flood estimate defined as the discharge for a specified probability of exeedance is usually obtained by statistical modeling of annual peak floods at a given site. The entire analysis is based on the premise that these floods are random and the floods to follow bear similar statistical properties of floods that have occurred in the past. Most of the time the assumptions are not fully met because of techniques used to model these floods do not quantity the flood mechanism in a systematic manner.

In addition to these, when quantiles have to be estimated for sites where no observations have been recorded or observation recorded only for a very small period, and then the estimates using frequency analysis is neither possible nor reliable. Regional flood frequency analysis is one of the means to overcome such problems while reasonably quantifying the flood estimates at desired frequencies for series within a more or less hydrological homogeneous region.

There have been significant developments and studies in the area of regional flood frequency analysis in India as well as abroad. Estimation of regional flood frequency parameters is performed for a specific site for two reasons:

1). Because of the sample variations present in short hydrologic records, frequency estimates of rare events based on at site frequency analysis are subjected to large error and thus un reliable. This error can be reduced by combining data from many more sites.

2). There are many more sites in the same region where the hydrologic data are not available but design flood estimates are needed for the design of small structure. In such a situation regional flood frequency analysis helps in transferring the knowledge from gauged sites to ungauged sites.

1.2 Literature Review

As per literature review, it can be revealed that a number of techniques viz., Index Flood Method, N.E.R.C Method, Multiple Regression method for Regional Flood Frequency Analysis have been attempted by various investigators. Regional Flood Frequency Analysis using L-Moments had been proposed by many researchers in foreign catchments. In Indian catchments, not many studies on Regional Flood Frequency Analysis using L-Moments are reported with due emphasis. Some of the research work carried out by the investigators on Regional Flood frequency analysis utilizing L-Moments are Saf (2009), NIH Roorkee (1997-98), Eslamian and Feizi (2006), Lim and Voeller (2009), Chavoshi and Sulaiman (2009) etc. As reported by many researchers the Regional Flood Frequency Analysis using L-Moments performs better than other techniques, it is therefore employed in the present study to develop Regional Flood Frequency Relationships for an Indian catchment.

1.3 Objectives of the Study

The main objectivesof the study are as follows:

1). Development of Regional Flood Frequency relationship using L-Moments based GEV Distribution.

2). Development of Regional relationship between mean annual peak flood and physiographic characteristics for estimating the mean annual peak flood for the ungauged catchments of Narmada Basin.

3). Coupling the relationship between mean annual peak flood and physiographic characteristicswith the L-Moments based regional flood frequency curve of GEV Distribution for developing the Regional Flood Formula.

II. Study Area

The Narmada Basin located in central India has been selected as the study area in present study. The Narmada Basin covers an area of 98796 sq. km and is located between longitudes $72^0$32'E to 81^0 45' E and latitudes $21^0$20' N to $23^0$45'N. The major part of the basin lies in Madhya Pradesh State while a small part of it lies in the states of Maharashtra and Gujrat. The Basin is bounded on north by Vindhyas, on th east by the Maikala Range, on the south by Satpura and on the west by Arabian Sea. The Narmada river is the largest river in India flowing east to west. Itsrises in the Amarkantak Plateau of Maikala range in the Shahdol district of Madhya Pradesh at an elevation of 1057 meters above mean sea level at a latitude $22^0$40' north and a longitude of $81^0$45' east. The river travels a distance of 1,312 km before it falls into Gulf of Cambay in the Arabian Sea near Bharuch in Gujarat. The first 1,079 km of its run are in Madhya Pradesh and Maharashtra. The last length of 159 km lies in Gujarat. The river has 41 tributaries in all, each having catchment area of more than 500 sq.kms out of these 41 tributaries 22 are on Southern bank and 19 on the Northern side. 39 tributaries are situated in M.P. and 2 are in Gujarat. Among all tributaries the “Hiran” is the longest having length of 188 kms. and the Tawa has largest catchment area 6067 sq.kms. The major left side tributaries are Burhner, Banjar, Hiran, Sher, Shakkar, Dudhi, Tawa, Ganjal, ChhotaTawaKundi, Goi. The major right side tributaries are Hiran, Tedoni, Barna, Kolar, Maan, Uri, Hatni, Orsang. The temperature of the basin touches a maximum of 48^0C in the month of May while minimum of 1.7^0C is recorded in the month of December. The average relative humidity value is very low in dry weather being 12% and is maximum in the monsoon season with value of 87.5%. The estimated average Rainfall of the basin is 1245mm. The monsoon rainfall from June to September is 92% of the annual rainfall.

III. Data Availability

The annual peak flood series data varying over the period 1970 to1987 for 16 gauging sites of Narmada Basin are available for the study. The range of annual peak flood data, record length for various catchments, and catchment area for various catchments are summarized in TABLE 1.

Table 1. Data Available for the study

S. No.	Gauging Station	Catchment Area (Sq. Kms.)	Length of Record Available (Years)	Range of Annual Peak flood series data (Cumecs)
1	Mannot	4300	12	960-6180
2	Jamtara	17157	17	2299.8-21355
3	Barmanghat	26458	17	1992-20658.2
4	Sandia	33953.5	12	3850-16500
5	Hoshangabad	44548	16	5853.6-33592.8
6	Handia	54027	12	3600-26240
7	Mandleshwar	72809.3	17	9979-46000
8	Mohgaon	4661	12	691.8-9000
9	Hriday Nagar	3770	12	502-2492.7
10	Belkheri	1508	12	84.5-6500
11	Gadarwara	2270	12	724-3030
12	Rajghat	77674.1	17	9600-56610
13	Garudeshwar	87892	17	7707-49500
14	Bagratawa	6018	12	299-25574
15	Chhidgaon	1729	12	197.9-4470
16	Ginnore	4815.7	17	1026-12157.8

IV. Methodology

The methodology [1,2] used in the present study for the development of Regional Flood Formula, for the estimation of flood values at different return period using L-Moments based GEV Distribution is discussed as under:

4.1 L-Moments Calculation for Data Sample

In order to calculate the L-Moments for data sample, firstly 16 gauging sites of Narmada Basin is selected and annual peak flood data at each gauging site , having record length varying from 12 to 17 years, is utilised in L-Moments calculation at each gauging site.

Probability Weighted moments defined by Greenwood et al. (1979) are the precursors of L-Moments. Sample probability weighted moments computed from data values $x_1, x_2, x_3 \ldots x_n$ arranged in ascending order as given :

$$b_0 = \sum_{j=1}^{n} x_j \;/\; n^{-1}$$

$$b_r = n^{-1} \sum_{j=r+1}^{n} (j-1)(j-2)..(j-r)\, x_j \,/\, (n-1)(n-2)..(n-r)$$

It is the combined form their respective forms are as given :

$$b_1 = n^{-1} \sum_{j=2}^{n} (j-1)\, x_j \;/\; (n-1)$$

$$b_2 = n^{-1} \sum_{j=3}^{n} (j-1)(j-2)\, x_j \,/\, (n-1)(n-2)$$

$$b_3 = n^{-1} \sum_{j=4}^{n} (j-1)(j-2)(j-3)\, x_j \,/\, (n-1)(n-2)(n-3)$$

Where, b_0, b_1, b_2, b_3 are the sample probability weighted moments, x_j is the annual peak flood value at j^{th} observation, n is no. of observations record length.

The first four L-Moments are given as:

$l_1 = b_0$ (1)

$l_2 = 2b_1 - b_0$ (2)

$l_3 = 6b_2 - 6b_1 + b_0$ (3)

$l_4 = 20b_3 - 30b_2 + 12b_1 - b_0$ (4)

The first L-Moment (l_1) is the measure of mean and second L-Moment (l_2) is measure of dispersion. Third and Fourth L-Moments are l_3 and l_4 respectively.

L-Moment ratios such as L-CV, L-Skewness, L-Kurtosis are defined as following :

L-Coefficient of Variation (L-CV), (t) = l_2 / l_1
L-Coefficient of Skewness (L-Skewness),(t_3) = l_3 / l_2
L-Coefficient of Kurtosis (L-Kurtosis), (t_4) = l_4 / l_2
t_3 is L-Skewness, is a measure of degree of symmetry of a sample. Symmetric distributions have $t_3 = 0$ and its value lies between -1 and +1.
$t_4$is L-Kurtosis, is a measure of peakedness or the flatness of the frequency distribution near its centre.
After the calculation of L-Moments for each gauging site, Regional L-Moments are evaluated. Regional L-Moments are evaluated by taking the weighted average of the L-Moments of the group of gauging sites. Where, n(i) is the record length of i^{th} site, N is total number of sites:

$$\text{Regional } l_1 = = \sum_{i=1}^{N} n_{(i)}\, l_1^{(i)} \,/\, \sum_{i=1}^{N} n_{(i)}$$

$$\text{Regional } l_2 = \sum_{i=1}^{N} n_{(i)}\, l_2^{(i)} \,/\, \sum_{i=1}^{N} n_{(i)}$$

$$\text{Regional } l_3 = \sum_{i=1}^{N} n_{(i)}\, l_3^{(i)} \,/\, \sum_{i=1}^{N} n_{(i)}$$

$$\text{Regional } l_4 = \sum_{i=1}^{N} n_{(i)}\, l_4^{(i)} \,/\, \sum_{i=1}^{N} n_{(i)}$$

Table 2.L-Moments and L-Moments Ratios for each Gauging Site

S.No.	Gauging Station	l_1	l_2	l_3	l_4	t	t_3	t_4
1	MANNOT	3120.2	1169.55	69.86	249.15	0.374832	0.164	0.213
2	JAMTARA	8631.711	3352.845	689.373	277.935	0.388433	0.2056	0.0829
3	BARMANGHAT	9704.1	3371.784	409.308	-396.096	0.34746	0.1214	-0.117
4	SANDIA	9451.25	2634.65	431.636	-37.08	0.278762	0.1638	-0.014
5	HOSHANGABAD	18638.75	4881.996	391.586	161.356	0.261927	0.0802	0.0331
6	HANDIA	15901.35	3740.592	-205.596	436.502	0.235237	-0.055	0.1167
7	MANDLESHWAR	23766.517	5680.711	483.697	15.931	0.239022	0.0851	0.0028
8	MOHGAON	2655.73	1273.57	612.07	335.07	0.479556	0.4806	0.2631

S.No.	Gauging Station	l_1	l_2	l_3	l_4	t	t_3	t_4
9	HRIDAY NAGAR	1484.16	366.92	-10.26	23.62	0.247224	-0.028	0.0644
10	BELKHERI	1683.525	808.329	284.583	331.099	0.480141	0.3521	0.4096
11	GADARWARA	1869.86	734.446	-338.968	144.926	0.392781	-0.4615	0.1973
12	RAJGHAT	27207.294	6361.206	356.058	1330.086	0.233805	0.056	0.2091
13	GARUDESHWAR	24967.835	6685.493	939.095	678.893	0.267764	0.1405	0.1015
14	BAGRATAWA	5752.83	2970.47	1701.45	1576.37	0.516349	0.5728	0.5307
15	CHHIDGAON	1864.6	808.92	131.71	32.77	0.43383	0.1628	0.0405
16	GINNORE	4264.658	1267.442	619.598	-152.378	0.297197	0.4889	-0.12

4.2 Estimation of Parameters of GEV Distribution

The GEV distribution is a generalized three parameter distribution proposed by Jenkinson (1955). Its theory and application are reviewed by Flood Studies Report prepared by N.E.R.C (1975). The cumulative density function F(x) for GEV Distribution is expressed as :

$F(x) = EXP. – [1-k \{(x-u)/\alpha\}]^{1/k}$ (5)

Where, u is the location parameter, α is the scale parameter, and k is the shape parameter.

The probability distribution function of GEV Distribution is given as :

$f (x) = \alpha^{-1} [1-k \{(k-u)/\alpha \}]^{(1/k)-1} . EXP. [- \{1-k(x-u)/\alpha \}^{1/k}]$ (6)

For the estimation of parameters of GEV Distribution i.e. u, α, k , following relationships are utilized as proposed by NIH, Roorkee (1997-98) given as :

$z = (2 / 3 + t_3) – (\log 2/ \log 3)$ (7)

$k = 7.8590 z + 2.9554 z^2$ (8)

$l_2 = \alpha(1- 2^{-k}) \Gamma 1+k / k$ (9)

$l_1 = u + \alpha (1-2^{-k}) \Gamma 1+k / k$ (10)

4.3 Development of Regional Flood Frequency Relationship

After the estimation of parameters of GEV Distribution, flood frequency relationship for a region is developed. The form of regional frequency relationship is expressed as :

$Z_T = Q_T / \square = u + \alpha Y_T$ (11)

Where, Q_T is the T year return period flood estimate, $\square$ is the mean annual peak flood , Y_T is the reduced variate corresponding to T year return period.

The GEV reduced variate Y_T can be expressed as a function of return period T as:

$Y_T = [1- \{-\ln (1-1/T)\}^k] / k$ (12)

Where, T is return period, k is shape parameter of GEV Distribution.

$Q_T / \square$ values are evaluated for different return periods, then $Q_T / \square$ values are plotted against reduced variate values. The curve which is plotted is called Regional Growth Curve.

4.4 Development of Relation between Mean Annual Flood and Catchment Area

For the estimation of T year return flood at a site, the estimate for mean annual peak flood required. For the gauged catchments such estimates can be obtained based on the at site mean of annual maximum peak flood data. However, for ungauged catchment at site mean cannot be computed in absence of flow data. In such a situation, a relationship between mean annual peak flood and catchment area of the gauged catchments is developed of the following form:

$\square = a . A^b$ (13)

Where, $\square$ is the mean annual peak flood for a catchment, A is the catchment area, and a and b are the constants estimated using least square approach.

4.5 Development of the Regional Flood Formula

The various steps involved in the derivation of Regional Flood Formula are given below.

The form of regional flood frequency relationship is :

$Q_T / \square = u + \alpha Y_T$ (14)

Where,

$Y_T = [1- \{ -\ln (1-1/T) \}^k] / k$ (15)

The conventional Dickens formula is given as :

$Q = A^{0.75}$
(16)

The form of developed formula may be mentioned as :

$Q_T = C_T A^b$ (17)

The form of relationship between mean annual flood and catchment area is :

$\square = a A^b$ (18)

Dividing equation 18 by equation 17 it is obtained that :

$Q_T / \square = C_T / a$ (19)

It can also be expressed as given :

$C_T = (Q_T / \square) . a$ (20)

Substituting the values of $Q_T / \square$ from equation 14 in equation 20 as :

$C_T = (u + \alpha Y_T) . a$ (21)

Substituting the value of C_T from equation 21 in equation 17 as:

$Q_T = (u + \alpha Y_T) . aA^b$ (22)

$Q_T = [ua + \alpha a Y_T] . A^b$ (23)

$Q_T = [ua + \alpha a \{1- \{-\ln (1-1/T)\}^k\}/ k] . A^b$ (24)

$Q_T = [ua+(a\alpha)/k –(a\alpha) /k\{-\ln(1-1/T)\}^k].A^b$ (25)

$Q_T = [a(\alpha/k +u)–(a\alpha/k)\{-\ln(1-1/T)\}^k] . A^b$ (26)

$Q_T = [\beta + \gamma \{ -\ln (1-1/T)\}^k] . A^b$ (27)

Where, $\beta = a (\alpha/k +u)$, and $\gamma = -(\alpha/k). a$

C_T is the coefficient for T-year return period flood estimated from regional frequency curve, and a and b are the constants for the Regional relationship between mean annual flood and catchment area.

V. Results

5.1 Regional L-Moments and L-moments Ratios

Table 3. Regional L-Moments

L-Moments	Value
l_1	1
l_2	0.282
l_3	0.04
l_4	0.028

Table 4. Regional L-Moments Ratios

L-Moments Ratio	Value
t	0.334
t_3	0.1505
t_4	0.1109

5.2 Estimated Parameters of GEV Distribution

Parameters	Estimate
u	0.718
α	0.411
k	0.03

5.3 Regional Growth Curve

For estimating the value of flood for a return period in any ungauged catchment of Narmada Basin a regional flood frequency relationship has been developed based on Gumbel'sexterme value (GEV) distribution. The developed relationship is as follows:

$$Q_T / Q_{(mean)} = 0.716 + 0.410\ Y_T \qquad (28)$$

The r^2 value for the developed curve has been obtained as 1.0 which shows the strongest correlation and its validity.

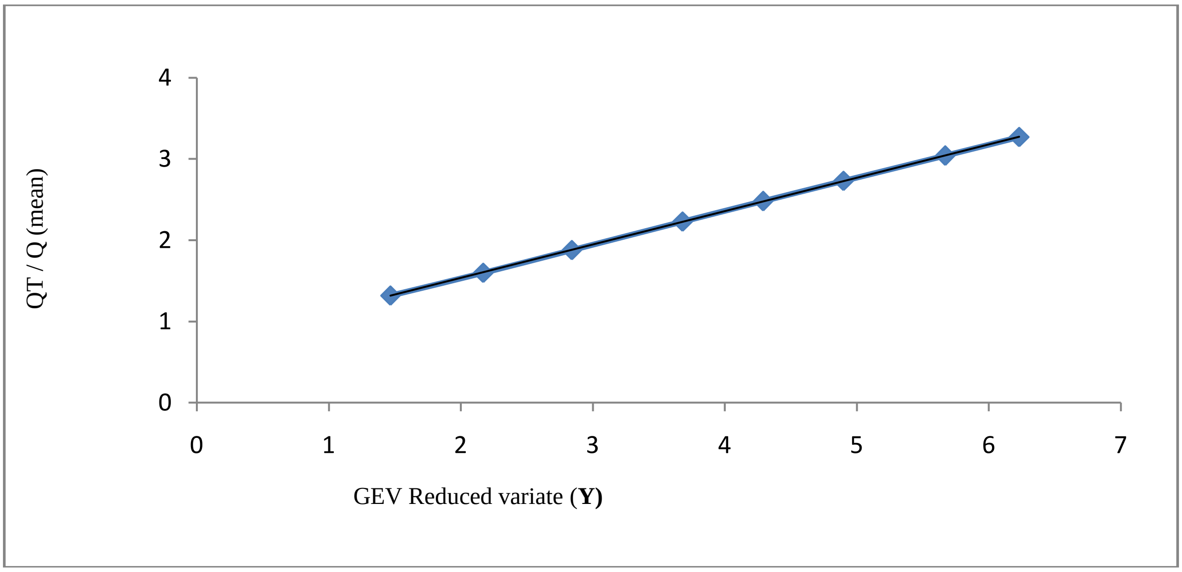

Figure 1 Regional growth curve

5.4 Mean Annual Flood – Catchment Area Relationship

The relationship between mean annual flood and catchment area has been developed using the regression technique, and utilizing the data of 16 gauging sites in Narmada Basin. Following relationship has been derived:

$$Q_{(mean)} = 8.87\ A^{0.698} \qquad (29)$$

The r^2 value of the above relationship is 0.945 which shows a strong correlation between the parameters $Q_{(mean)}$ and A and also the validity of equation.

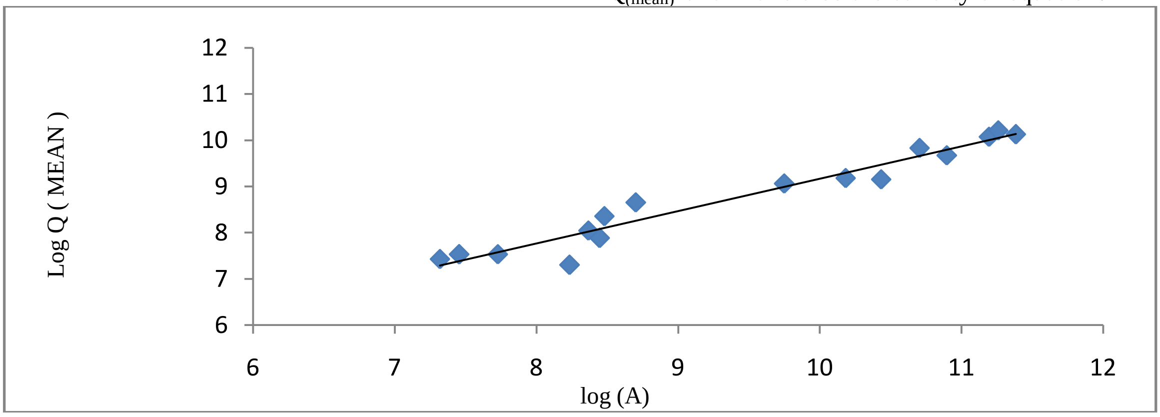

Figure2 Catchment area – mean annual flood relation

5.5 Developed Regional Flood Formula

At the end of the L-Moments analysis, a Regional Flood Formula has been developed for the ungauged catchments of Narmada Basin, by which the flood estimates for different return periods can be obtained knowing only one parameter i.e. Catchment area. The developed Regional Flood Formula is given as:

$$Q_T = [127.88 – 121.59 \{- \ln (1-1/T)\}^{0.03}] . A^{0.698}$$

VI. Conclusions

Based on analysis performed as per the methodology described, following major conclusions can be derived from the present study:

1). The L-Moments method can be considered simpler as the L-Moments statistics can be obtained by simple linear expressions.

2). The L-Moments Method uses only Generalized Extreme Value (GEV) Distribution to develop the Regional Frequency Relationship for T-Year flood and does not require the determination of frequency ratios etc., thus the procedure is simplified considerably.

3). The conventional empirical formula cannot provide floods of desirable return periods however, the flood formula developed in this study are capable of providing flood estimates for different return periods.

4). The form of developed Regional flood formula is very simple, as for the estimation of desired return period flood for an ungauged catchment it requires only catchment area.

VII. Acknowledgement

The author would like to pay deep sense of gratitude to Dr. R.K.Shrivastava, Professor, Department of Civil Engineering and Applied Mechanics, S.G.S.I.T.S, Indore for enlightening the path and for the words of encouragement throughout the study.

References

[1] J.R.Hosking, and J.R.Wallis, Some statistics useful in regional frequency analysis, *Water Resour. Res., 29(2),* 1993, 271-281.

[2] National Institute of Hydrology, *Regional Flood Frequency Analysis using L-Moments*, 1997-98, Roorkee.

[3] J.R.Hosking, and J.R.Wallis, Regional frequency analysis-An approach based on L-moment (Cambridge University Press, Cambridge,(1997).