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Acceptance sampling for the secretion of Gastrin using crisp and fuzzy Weibull distribution

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Abstract
The acceptance sampling plan problem is an important tool in the statistical quality control. The theory of probability distribution and fuzzy probability distribution may be used to solve it. In this paper, we compare the significant elevation of plasma concentrations of stimulatory hormone gastrin in the pancreatic secretion after the meal using acceptance sampling in the Weibull distribution with crisp and fuzzy parameter.
Key words: Acceptance sampling, Fuzzy Weibull distributions, gastrin, Weibull distributions
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I. Introduction

Statistical quality control is one of the important topics in statistics which is used by many practitioners in manufacturing industry. Acceptance sampling is an inspecting procedure applied in statistical quality control. It is used in the decision making process for the purpose of quality management. Acceptance sampling plans have many applications in the field of industries and bio-medical sciences. The main concern in an acceptance sampling plan is to minimize the cost and time required for the quality control.

Single acceptance sampling plan for acceptance or rejection of a lot is characterized by the sample size and the acceptance number. An acceptance sampling plan involves quality contracting on product orders between the producers and the consumers. In a time-truncated sampling a random sample is selected from a lot and put on the test where the number of failures is recorded by the specified time. The lot will be accepted if the number of failures observed is less than or equal to the acceptance number.

Usual sampling plans have crisp parameters, however the probability of defective items is often not known precisely in decision making problems, while there also some uncertainty exists in the values of p. The theory of fuzzy set is used in solving such problems. The uncertainty existing in the problem is resolved by assuming that the parameter is a fuzzy number. In humans, gastrin is one of the best studied gut hormones. It occurs in many forms in human serum. Gastrin and vagal nerves are the main regulators of gastric acid secretion. Gastrin is a peptide hormone that stimulates secretion of gastric acid by the parietal cells of the stomach and acids in gastric motility.

Acceptance single sampling plan for truncated life test have been discussed by many authors in the literature using different probability distribution. The acceptance sampling plan was discussed by Epstein[1] for exponential distribution. Gupta et al. [3] found the acceptance sampling plan using gamma distribution. Life test sampling plan for normal and log normal distribution was developed by Gupta[4]. Kandam et al[5] used half logistic distribution in acceptance sampling. Goode et al.[2] developed the acceptance sampling plan using the Weibull probability distribution. Aslam et al. [7, 8] developed group acceptance sampling plan and two-stage group acceptance sampling plan using Weibull distribution. E. Baloui et al.[11] have been discussed acceptance single sampling plan with fuzzy environment. Zdenek et al [9] described the use of Weibull fuzzy distribution for reliability of concrete structures. Konturek SJ [10] have been discussed the elevation of plasma concentrations of Gastrin in postprandial.

II. Notation

n	–	sample size
c	–	acceptance number
d	–	observed defective item
λ	–	scale parameter
β	–	shape parameter
t	–	test termination time
$\overline{\lambda}[\alpha]$	–	alpha cut of scale parameter
$\overline{\beta}[\alpha]$	–	alpha cut of shape parameter
p	–	probability of failure
P_A	–	probability of acceptance in crisp
$\overline{P}[\alpha]$	–	Probability of acceptance in fuzzy

III. Preliminaries and Definitions

In fuzzy set theory, the concepts of membership function are most important and are used to represent various fuzzy sets. Many membership functions such as triangular, trapezoidal, normal, gamma etc. have been used to represent fuzzy numbers. However triangular and trapezoidal fuzzy numbers are mostly used, as they can easily represent the imprecise information.

A triangular fuzzy number is denoted by the triplet A = (a_1, a_2, a_3) with the membership function

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & if\ a_1 \le x \le a_2 \\ 1 & if\ x = a_2 \\ \dfrac{a_2-x}{a_3-a_2} & if\ a_2 \le x \le a_3 \end{cases}$$

3.1 Definition

The fuzzy subset $\overline{N}$ of a real ling R with the membership function $\mu_N : R \to [0,1]$ is a fuzzy number iff (i) $\overline{N}$ is normal (ii) $\overline{N}$ is fuzzy convex (iii) μ_N is upper semi continuous (iv) sup($\overline{N}$) is bounded.

3.2 Definition

The α- cut of a fuzzy number $\overline{A}$ is non –fuzzy set defined as $M[\alpha] = \{x \in R :\ \mu_A(x) \ge \alpha\}$. Hence we have $M[\alpha] = [M_{\alpha 1}, M_{\alpha 2}]$. The interval of confidence defined by alpha cuts can be written as
$M[\alpha] = [(a_2 - a_1)\alpha + a_1, (a_2 - a_3)\alpha + a_3]$

IV. Crisp and Fuzzy Weibull Distribution

The Weibull distribution is widely used in statistical model for live data. Among all statistical techniques it may be in use for engineering analysis with smaller sample sizes than any other method.

A continuous random variable T with probability density function

$$f(t,\lambda,\beta) = \beta\lambda^{-\beta} t^{\beta-1} e^{-\left(\frac{t}{\lambda}\right)^{\beta}},\ \ t \ge 0, \lambda \ge 0, \beta \ge 0$$

where $\beta > 0$ is the shape parameter, $\lambda > 0$ is the scale parameter is called Weibull distribution and is denoted by $W(\beta,\lambda)$.

The cumulative distribution function of the Weibull distribution is

$$F(t) = 1 - e^{-\left(\frac{t}{\lambda}\right)^{\beta}}$$

The shape parameter gives the flexibility of Weibull distribution by changing the value of the shape parameter. However sometimes we face situations when the parameter is imprecise. Therefore we consider the Weibull distribution with fuzzy parameters by replacing the scale parameter λ into the fuzzy number $\overline{\lambda}$ and shape parameter β into $\overline{\beta}$.

If a random variable T has a crisp Weibull distribution $W(\beta,\lambda)$ then the corresponding fuzzy random variable $\overline{T}$ with fuzzy weibull distribution $W(\overline{\beta},\overline{\lambda})$ has cumulative distribution function $\overline{F}(t) = 1 - e^{-\left(\frac{t}{\overline{\lambda}}\right)^{\overline{\beta}}}$. So that for $\alpha \in [0,1]$ the alpha cuts of fuzzy weibull distribution function is $\overline{P}[\alpha] = [P_1[\alpha], P_2[\alpha]]$ where

$$P_1[\alpha] = \inf\{1 - e^{-\left(\frac{t}{\lambda}\right)^{\beta}}, \lambda \in \overline{\lambda}[\alpha]\ \ \beta \in \overline{\beta}[\alpha]\}$$

$$P_2[\alpha] = \sup\{1 - e^{-\left(\frac{t}{\lambda}\right)^{\beta}}, \lambda \in \overline{\lambda}[\alpha]\ \ \beta \in \overline{\beta}[\alpha]\}$$

V. Acceptance Sampling

All sampling plans are formulated to provide a specific producer’s and consumer’s risk. However, it is a consumer’s best interest to keep the average number of items examined to a minimum, because that keeps the cost of inspection low. Sampling plans differ with respect to the average number of items examined.

The single acceptance sampling plan is a decision rule to accept or reject a lot based on the results of one random sample from the lot. The procedure is to take a random sample of size n and check each item. If the number of defective items does not exceed a specified number c, the consumer accepts the entire lot. Any defects found in the sample are either repaired or returned to the producer. If the number of defects in the sample is greater than c, the consumer rejects the entire lot and returns it to the producer. The single sampling plan is easy to use but usually result in a larger average number of items examined than other plans.

In this section, we have used the single sampling plan for classical attributes suppose that we like to check a lot of size N, first we select and check a random sample of size n and observe the number of defective items. If the number of observed defective items d is less than or equal to the acceptance number c, the lot is accepted otherwise the lot is rejected. If the size of the lot is very large the probability of acceptance of the lot is

$$P_A = P(d \le c) = \sum_{d=0}^{c} \binom{n}{d} p^d (1-p)^{n-d}$$

where p is the probability of the defective item.

The probability p for the Weibull distribution is given by $p = 1 - e^{-\left(\frac{t}{\lambda}\right)^{\beta}}$. If the probability of defective item is not known precisely, we represent this parameter with a fuzzy number $\overline{p}$ as follows $\overline{p} = (a_1, a_2, a_3)$.

VI. Application

Let us consider an example of the plasma concentration of stimulatory hormone gastrin in the pancreatic secretion after the meal. Fig. 1 describes the plasma concentrations of gastrin in the pancreatic secretion after the meal.

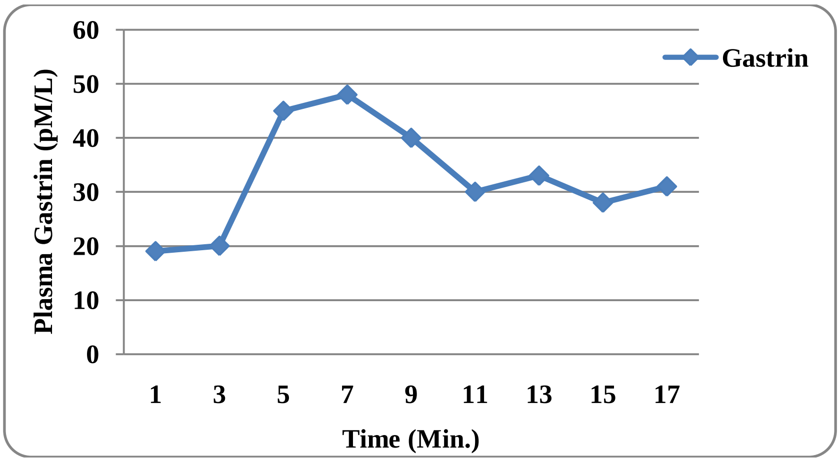

Fig. 1 Stimulating hormone Gastrin in the control of pancreatic secretion

6.1 Solution by crisp Weibull distribution

From the Fig. 1 the scale and shape parameter of Weibull distributions are $\lambda = 36.54$ and $\beta = 3.43$. By assuming the value of t as 14, the probability of failure using Weibull distribution is p = 0.0365. The probability of acceptance of various sample sizes for the acceptance numbers 1, 2 and 3 are shown in the Table – 1.

Table – 1 Probability of acceptance in crisp parameter

sample size (n)	Probability of acceptance		
	c = 1	c = 2	c = 3
5	0.9876	0.9995	0.999991
10	0.9507	0.9952	0.999688
15	0.8978	0.9841	0.998248
20	0.8355	0.9652	0.994623
25	0.7686	0.9385	0.98785
30	0.7002	0.9049	0.977198
35	0.633	0.8654	0.962218
40	0.5684	0.8214	0.942745
45	0.5075	0.7741	0.918862

50	0.4509	0.7248	0.890855
55	0.3989	0.6746	0.859162
60	0.3516	0.6245	0.824319
65	0.3088	0.5751	0.786923
70	0.2705	0.5272	0.74759
75	0.2362	0.4812	0.706926
80	0.2058	0.4374	0.665511
85	0.1789	0.3962	0.623876
90	0.1553	0.3576	0.582495
95	0.1345	0.3218	0.541784
100	0.1162	0.2887	0.502093
105	0.1003	0.2583	0.46371
110	0.0865	0.2305	0.426867
115	0.0744	0.2052	0.391735
120	0.064	0.1822	0.358439
125	0.055	0.1615	0.327059

6.2 Solution by Fuzzy Weibull distribution

In some situations the value of the scale and shape parameters of the Weibull distribution are not known precisely. Therefore we consider triangular numbers for the scale and shape parameter. The triangular fuzzy number of the scale and the shape parameters respectively are $\overline{\lambda}$ = [36, 36.54, 37] and $\overline{\beta}$ = [3.3, 3.43, 3.6]

The alpha cut of scale and shape parameters respectively are

$\overline{\lambda}[\alpha] = [36 + 0.54\,\alpha, 37 - 0.46\alpha]$ and

$\overline{\beta}[\alpha] = [3.3 + 0.13\alpha, 3.6 - 0.17\alpha]$

Under the alpha cut zero, the fuzzy probability of acceptance of various sample sizes for the acceptance number 1, 2 and 3 are shown in Table – 2

Table – 2 Probability of acceptance in fuzzy parameter

sample size (n)	c = 1		c = 2		c = 3	
	$\boldsymbol{P_1[\alpha]}$	$\boldsymbol{P_2[\alpha]}$	$\boldsymbol{P_1[\alpha]}$	$\boldsymbol{P_2[\alpha]}$	$\boldsymbol{P_1[\alpha]}$	$\boldsymbol{P_2[\alpha]}$
5	0.9828	0.9917	0.9992	0.9997	0.999983	0.999996
10	0.933	0.9661	0.9923	0.9973	0.999402	0.999859
15	0.8643	0.9283	0.975	0.9909	0.996734	0.999184
20	0.7861	0.8821	0.9466	0.9795	0.990246	0.997426
25	0.7048	0.8307	0.908	0.963	0.978539	0.994028
30	0.6249	0.7764	0.861	0.9414	0.960759	0.988493
35	0.5489	0.721	0.8077	0.9151	0.936619	0.980433
40	0.4784	0.666	0.7504	0.8848	0.906323	0.969583
45	0.4143	0.6122	0.691	0.851	0.870455	0.955801
50	0.3568	0.5604	0.6312	0.8146	0.829853	0.939064
55	0.3058	0.5111	0.5724	0.7761	0.785501	0.919446
60	0.261	0.4647	0.5156	0.7363	0.738434	0.89711
65	0.2219	0.4212	0.4617	0.6958	0.689673	0.872279
70	0.188	0.3808	0.4112	0.655	0.640168	0.845229

75	0.1589	0.3435	0.3644	0.6144	0.590767	0.816266
80	0.1339	0.3091	0.3215	0.5745	0.542202	0.785712
85	0.1126	0.2777	0.2824	0.5355	0.495075	0.753897
90	0.0944	0.249	0.2471	0.4978	0.449865	0.721146
95	0.0791	0.2228	0.2155	0.4614	0.406933	0.687773
100	0.0661	0.1992	0.1873	0.4266	0.366532	0.654072
105	0.0551	0.1778	0.1623	0.3935	0.328822	0.620317
110	0.0459	0.1584	0.1402	0.3622	0.293882	0.586756
115	0.0382	0.141	0.1208	0.3327	0.261724	0.55361
120	0.0317	0.1254	0.1039	0.3049	0.232307	0.521072
125	0.0263	0.1114	0.0891	0.279	0.205546	0.48931

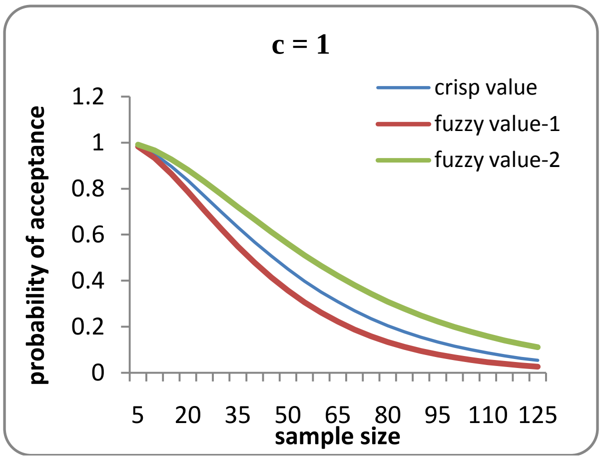

Fig. 2 Crisp and fuzzy Weibull acceptance probability for c = 1

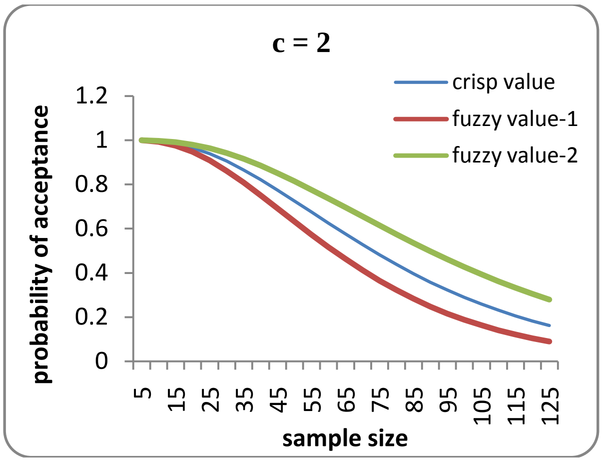

Fig. 3 Crisp and fuzzy Weibull acceptance probability for c = 2

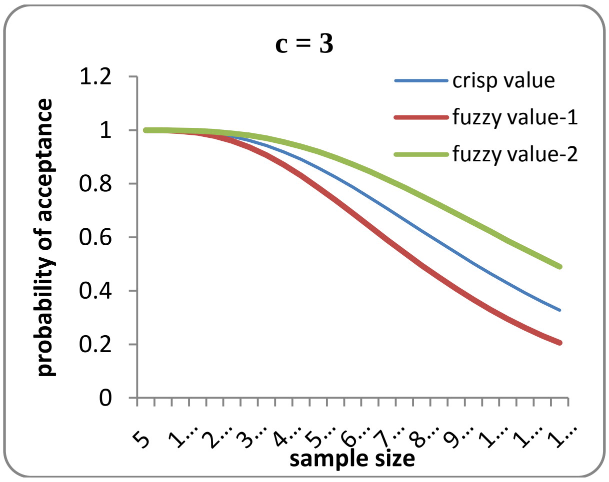

Fig. 4 Crisp and fuzzy Weibull acceptance probability for c = 3

VII. Conclusion

In this paper we have reported the results of the comparative study to predict the sample size of determining the plasma concentration of gastrin by acceptance single sampling using a Weibull distribution in crisp and fuzzy environment. The curve obtained by a Weibull distribution in crisp parameter lies between fuzzy parameters. We hope this work may be used to predict the sample size to be selected for testing the plasma concentrations of gastrin in real life.

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