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A Periodical Production Plan for Uncertain Orders in a Closed-Loop Supply Chain System

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ABSTRACT
Production planning is a major activity in the manufacturing or processing industries. A good plan helps the company lower its expenses, increase profit, or both. However, the worldwide economy is made up of closely related systems. Thus, a small change induces fluctuation in the supply chain. Although a production plan is based on the predicted demand, economic fluctuations make prediction difficult. Therefore, coping with production risksof uncertain demands heavily depends on the judgment and experience of the producer or customer. In addition, the reuse of recyclable products has become a major approach in reducing resource consumption because of environmental consciousness. Thus, a closed-loop supply chain has replaced the traditional supply chain to facilitate recycling, accommodate reprocess, ease environmental degradation, and save on resource costs. This study thus considers a production plan in a closed-loop supply chain, where periodic orders of retailers are adjusted and described byfuzzy quantities. The goal of the producer is to maximize profit while trying to satisfy these orders to the greatest extent. Fuzzy Set Theory is applied to construct a Fuzzy Chance-Constrained Production Mix Model (FCCPMM) to enable the risk attitude of the decision maker to be adopted to address uncertainty.Theoretical evidence is supported by numerical illustration.
Keywords*—closed loop supply chain, reprocessing, multi-order, backorder, fuzzy chance-constrained model, optimality and parametric analysis.*

I. INTRODUCTION

When a new product is due for release, information about its future demand is incomplete. Although many forecasting methods exist to estimate future demand, most of them need a large database for statistical analysis. If the historical information is limited, retailers can only estimate the demand by their own intuition or experience. A gap exists between the retailer's wants and the manner of expressing these wants to the producer because of the vagueness of natural language, which causes difficulty for the producer in determining the production plan. To resolve the problem, this study considers a multi-retailer and multi-period production planning problem in a closed-loop supply chain system, where the orders are regarded as fuzzy quantities. To cope with the fuzzy demands induced by the subjective judgment of retailers, we propose a Fuzzy Production Mix Model (FPMM) based on Fuzzy Set Theory. Using the concept of fuzzy chance-constrained programming, FPMM is transformed into a computable model, which integrates the planning preference of the decision maker to deal with uncertainty. Parametric analysis is performed in this study. The structure of this paper is as follows. Section 2 briefly reviews closed-loop supply chains and Fuzzy Set Theory and its applications. The problem is stated in Section 3, and the formulation of Fuzzy Production Mix Model and Fuzzy Chance-Constrained Production Mix Model are proposed with a discussion of their parameters. In Section 4, we use a numerical example to show how the model works. Section 5 discusses the conclusions of the study and recommendations for future research.

II. LITERATURE REVIEW

The accurate estimation of demand is an important issue because the main purpose of a production plan is to satisfy demand. However, demand forecasting is often affected by many complicated factors. The uncertain and variable nature of demand almost makes the exact estimation impossible to predict. We have proposed a deterministic model as a basisfor investigating the features of the problem (Wang and Lin, 2014), but we also want to find out the main uncertain factors that affect the result of the analysis and the complexity of the model. Demand uncertainty is the main source among these factors. Thus, this study focuses on this issue and finds a resolution in coping with this uncertainty. Fuzzy Set Theory is adopted in analysis and modeling because of the subjective preference of consumers embedded in the demand of products. In the following sections, we briefly review closed-loop supply chains and the basic idea of Fuzzy Set Theory and its applications.

2.1 Remanufacturing and the Closed-loop Supply Chain System

Environmental sustainability has become a serious issue in recent years. Therefore, researchers exerted significant efforts to ease the degradation of the environment. Remanufacturing provides many benefits, such as reducing the prices for customers and the usage of material and energy resources (Lund and Hauser, 2010). The stages from raw material processing to the delivery of products to customers are the only issues that concern traditional supply chains. However, remanufacturing entails that used products should be recycled in the factory for further reprocessing, which means that the logistics from the customer to the factory must be addressed. A closed-loop supply chain (Guide, Harrison, and Van Wassenhove, 2003) includes forward and reverse logistics; these methods facilitate the reuse of materials, reduction of energy consumption, and slowing down of environmental deterioration.

2.2 Basic Concepts of Fuzzy Set Theory

Fuzzy Set theory generalizes the notion of the crisp set to cope with vague expressions. Based on Zadeh (1965) a fuzzy set is defined as below:

Definition 1 *Fuzzy Set*

Let X be the universal set, a fuzzy set $\tilde{A}$ can be expressed as

$$\tilde{A} = \{x, \mu_{\tilde{A}}(x))|x \in X, \mu_{\tilde{A}} \in [0,1]\} \quad (1)$$

where $\mu_{\tilde{A}}$ is a membership function. It indicates that the degree of x belonging to $\tilde{A}$.

In Fuzzy Set Theory, fuzzy number plays an important role in many applications. Fuzzy number is described in terms of a number and a linguistic modifier. Before defining fuzzy number we first introduce a kind of crisp set called $\alpha-$ level set.

Definition 2 $\alpha-$*level set*(*Zimmermann, 1986*):

A crisp set of elements which belong to a fuzzy set $\tilde{A}$ at least a degree α iscalled $\alpha-$level set of $\tilde{A}$ defined by $A_\alpha = \{x|\mu_{\tilde{A}}(x) \geq \alpha, \alpha \in [0,1]\}$.

Based on Klir *et al* (1997), fuzzy number can be defined as follows:

Definition3 *Fuzzy Number:*

Let $\tilde{A}$ be a Fuzzy Set of the real line R. $\tilde{A}$ is called fuzzy number if the following conditions are satisfied

(1). A_1 is not empty.

(2). $\tilde{A}$ is a convex fuzzy set.

(3). A_α is a closed interval for $\alpha \in [0,1]$.

2.3 Fuzzy Chance-Constrained Programming

Chance-constrained programming resolves optimization problems under uncertain conditions. Instead of finding an optimal solution that satisfies the constraints, the main concept of chance-constrained programming is in finding an optimal solution that is feasible within a confidence interval. Fuzzy chance-constrained programming (Yang and Iwamura, 2008) is proposed to deal with the uncertainty induced by fuzzy quantities. We introduce Possibility Theory first because this model is developed based on this theory.

2.3.1

2.3.1*Possibility Theory*

In Possibility Theory, possibility measure is used to describe uncertainty estimation of a fuzzy event. To define possibility measure, it is necessary to define the possibility distribution. Based on Zadeh (1977), possibility distribution, possibility measure and necessity measure can be defined as follow:

Definition 4 *Possibility Distribution* (Zadeh, 1977):

Let X be a variable which takes values in a universe of discourse *U*. Let $\tilde{F}$ be a fuzzy subset of a universe of discourse with its membership function $\mu_{\tilde{F}}$.Given a proposition, *X*is $\tilde{F}$,$\tilde{F}$acts as an elastic constraint on the values that may be assigned to X. The proposition "X is$\tilde{F}$" is translated into

$$R(X) = \tilde{F} \quad (2)$$

Associates a possibility distribution, π_X, with X which is postulated to be equal to $\tilde{F}$. The possibility distribution function $\pi_X(u)$ is defined to be numerically equal to $\mu_{\tilde{F}}(u)$, i.e:

$$\pi_X(u) = \mu_{\tilde{F}}(u) \quad (3)$$

Although $\pi_X(u)$ is numerically equal to $\mu_{\tilde{F}}(u)$, they have different meanings. $\mu_{\tilde{F}}(u)$describes the degree of compatibility that u fits the fuzzy subset $\tilde{F}$,whereas $\pi_x(u)$ indicates that the degree of possibility that X is *u* given the proposition X is$\tilde{F}$.

The possibility measure and necessity measure are developed based on possibility distribution.

Definition 5 *Possibility Measure* (Zadeh, 1977):

Let $\pi_X$be the possibility distribution associates with X and A is a non-fuzzy subset of U. The possibility measure, π(A), is defined by

$$\pi(A) = POSS(X \in A) = sup_{u \in A} \pi_X(u) \quad (4)$$

Definition 6*Necessity Measure* (Zadeh, 1977):

Let $\pi_X$be the possibility distribution associates with*X*, A is a non-fuzzy subset of U. necessity measure,$NEC(X \in A)$, is defined by

$$NEC(X \in A) = 1 - sup_{u \in A^c} \pi_X(u) \quad (~~5)~~5$$

2.3.2 *Fuzzy Chance-Constrained Program*

Fuzzy chance-constrained programming uses the linear combination of possibility measure and necessity measure to describe the chance of validity of constraints. Let $\tilde{F}$ be a fuzzy number with membership function$\mu_{\tilde{F}}$, and A is a subset of real domain R. Linear combination of possibility measure and necessity measure, calledm_λ measure, is defined by

$$m_\lambda(\tilde{F} \epsilon A) = \lambda POSS(\tilde{F} \epsilon A) + (1-\lambda)\, NEC(\tilde{F} \epsilon A) \quad (2)$$

The notations used in (6) are a bit different from (4) and (5), but they imply the same thing.
λcan be regarded as optimistic indicator. Higher λ indicates that decision maker is more optimistic about the chance of a fuzzy event.
If the decision maker wants to maximized its optimistic return value, the fuzzy chance-constrained programming model can be constructed by

$$\max_{x} \max_{\bar{f}} \bar{f}$$

Subject to

$$m_\lambda\left(f(\boldsymbol{x},\boldsymbol{\xi}) \geq \bar{f}\right) \geq \alpha$$

$$m_\lambda(g_i(\boldsymbol{x},\boldsymbol{\xi}) \leq 0) \geq \alpha_j, \text{ i=1, 2,…, n.}$$

$$\mathbf{x} \in F$$

$\boldsymbol{\xi}$is a fuzzy number. (3)

$f(\boldsymbol{x},\boldsymbol{\xi})$ is the original objective function which contains a fuzzy number. $g_i(\boldsymbol{x},\boldsymbol{\xi})$ is the original constraint which contains a fuzzy number. $F$is the feasible region of **x**.
If the decision maker wants to maximized its pessimistic return value, model can be constructed by

$$\max_{x} \min_{\bar{f}} \bar{f}$$

Subject to

$$m_\lambda\left(f(\boldsymbol{x},\boldsymbol{\xi}) \leq \bar{f}\right) \geq \alpha$$

$$m_\lambda(g_i(\boldsymbol{x},\boldsymbol{\xi}) \leq 0) \geq \alpha_j, \text{ i=1, 2,…,n.} \quad (8)$$

$$\mathbf{x} \in F$$

Besides, take into consideration both optimistic and pessimistic aspect about return value, the model is constructed as following form:

$$\max_x \left(\beta \times \max_{\bar{f}} \bar{f} \;+\; (1-\beta) \times \min_{\bar{g}} \bar{g}\right)$$

Subject to

$$m_\lambda\left(f(\boldsymbol{x},\boldsymbol{\xi}) \geq \bar{f}\right) \geq \alpha$$

$$m_\lambda(f(\boldsymbol{x},\boldsymbol{\xi}) \leq \bar{g}) \geq \alpha$$

$$m_\lambda(g_i(\boldsymbol{x},\boldsymbol{\xi}) \leq 0) \geq \alpha_j, \text{ i=1, 2,…,n.} \quad (9)$$

$$\mathbf{x} \in F$$

.4 Concluding Remarks

This section reviews the concept of a closed-loop supply chain system and Fuzzy Set Theory. On the one hand, the development of a closed-loop supply chain system helps companies reduce cost and ease environmental degradation. On the other hand, Fuzzy Set Theory deals with the uncertainty that results from the vagueness of natural language.

Demand in production planning is usually uncertain and has significant variations from economic fluctuations. Thus, it seeks mathematical approaches to help companies make decisions. However, statistical analysis requires large datasetto achieve such goals. Therefore, the accuracy of prediction would be affected by the number of data on hand. Data may be inaccessible in some cases. Thus, the decision maker or customer can only predict demand based on his or her own intuition. Under these circumstances, the use of Fuzzy Set Theory is helpful in determining the optimal production plan. In the next section, we utilize Fuzzy Set Theory to address a production planning problem in a closed-loop supply chain system with demand uncertainty.

III. THE PROPOSED MODEL

3.1 Problem Statement

In this study, we consider a single-product producer with multiple retailers. The retailers estimate their own market demands and place orders to the producer at the beginning of the planning horizon. Each order indicates the number of products that a retailer needs for each period. The orders are inseparable. Thus, the producer is not allowed to satisfy only a partial quantity that each order requires. The quantities of the products that they need are estimated and undetermined until the delivery period because retailers cannot precisely forecast demands. This observation implies that retailers may alter the requirements before the delivery period (4) even though the order was placed at the beginning of the planning horizon. The producer may choose to accept or reject the orders because the capacity may be limited and outsourcing is forbidden. However, to maintain retailer loyalty and to prevent losing retailers in such an uncertain environment, the producer may determine a minimum acceptance ratio of the given orders with respect to each retailer. (9)

According to environmental protection regulations, the producer is responsible for collecting used products. Thus, collection cost exists at each period when the market lifespan of a product expires. Whenever a used product is returned to the factory, the producer can dispose of the returned products or reprocess them so that they can be sold to retailers as new ones in the following periods.

The goals of a producer include figuring out the orders that should be accepted and executing production activities to maximize profit under uncertain conditions. This study develops an analytical mathematical model to support the decision of the producer in achieving such goals. Figure 1 shows the structure of this study.

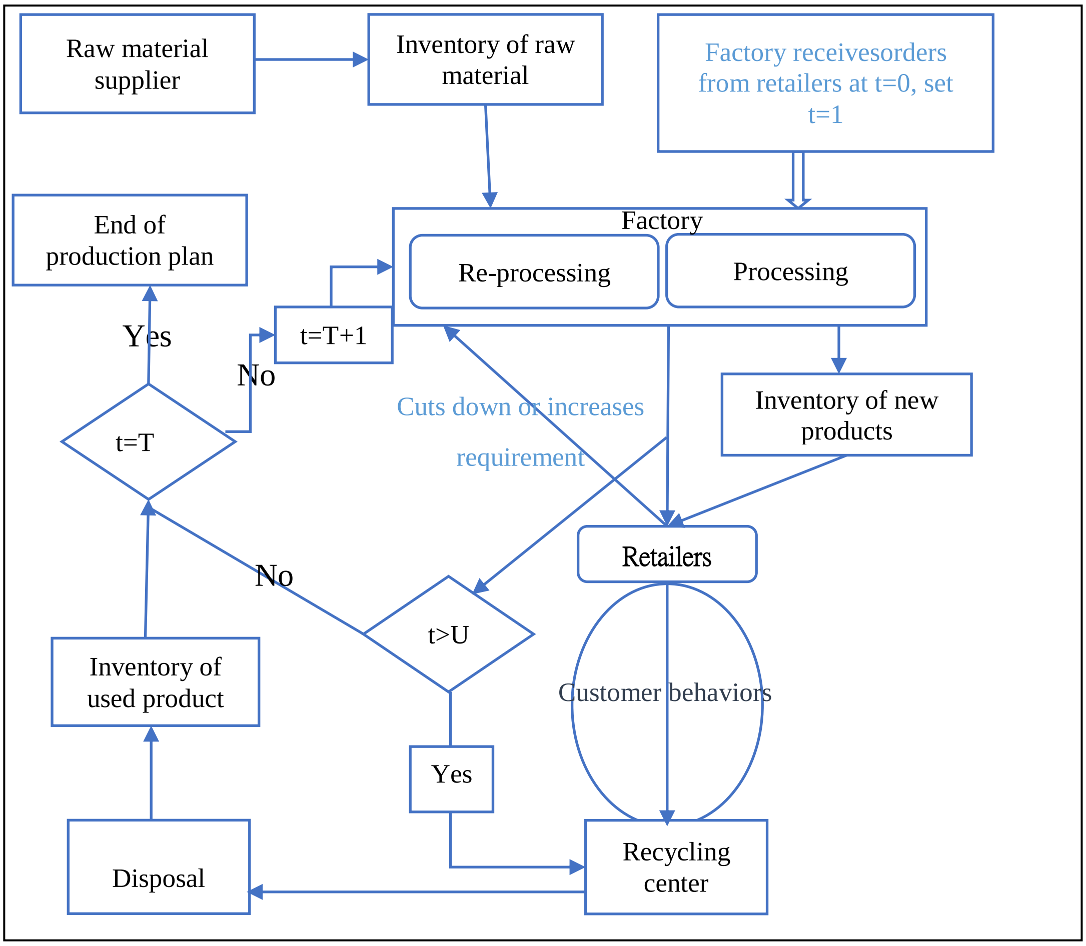

Figure 1 Activity flows of the proposed production plan

To analyze this type of production process, we first propose the Fuzzy Production Mix Model. The model is then transformed into a Fuzzy Chance Constrained Production Mix Model (FCCPMM)to facilitate the computation.

3.2Fuzzy Production Mix Model (FPMM)

The notations are first defined as below:

T	*Planning horizon.*
U	*Product lifespan period.*
P	*Net profit.*
TNO_j	*Total number of orders retailer j gives to producer, j=1,2,...J.*
$\tilde{D}_j^t$	$\tilde{D}_j^t = [\underline{d}_j^t, d_j^t, \bar{d}_j^t]$, *a triangular fuzzy number which is the quantity of products retailer j needs at period t. t=1,2,…,T and j=1,2,…,J.*
γ	*Collection rate.*
C_k	*The capacity of machine k, k=1,2,…,K.*
CRM_k	*The quantity of resource machine k used for per unit of processing, k=1,2,…,K.*
CM_k	*The quantity of machine k used for per unit of reprocessing, k=1,2,…,K.*
CL	*Inventory limit of new product.*
CRL	*Inventory limit of returned product.*
$MRSO_j$	*Minimum acceptance ratio of orders for retailer j, j=1, 2,…J.*
dl_j	*The retailer j's maximum tolerable delay periods* $(0 \le dl_j \le T-1)$.
REV	*Unit revenue of sold product.*
PC	*Unit processing cost.*

RPC *Unit reprocessing cost.*
HS *Unit holding cost of serviceable product.*
HR *Unit holding cost of returned product.*
DC *Unit disposal cost of returned products.*
SC *Setup cost for processing or reprocessing.*
OC *Ordering cost.*
BC_j *Unit backlogging cost of retailer j, j=1, 2,…, J.*
RC *Unit recycling cost.*
x^t *The quantity of products processing in period t, t=1,2,…,T.*
y^t *The quantity of products reprocessing in period t, t=1,2,…,T.*
is^t *Inventory of new product at the end of period t, t=1,2,…,T.*
ir^t *Inventory of raw material at the end of period t, t=1,2,…,T.*
im^t *Inventory of used product at the end of period t, t=1,2,…,T.*
z^t *The quantity of products disposed of in period t, t=1,2,…,T.*
m^t *Setup variable. 0-1 binary variable, t=1,2,…,T.*
n^t *Ordering variable. 0-1 binary variable, t=1,2,…,T.*
$b_j^{t,t'}$ *The quantity of products supplied in period t' for retailer j's order of period t, t=1,2,…,T, t'=t,t+1,…,T, j=1,2,…J.*
o^t *The quantity of raw materials ordered at period t, t=1,2,…,T.*
r^t *The quantity of products recycled at period t, t=U, U+1,…,T.*
s_j^t *Binary variable, t=1,2,…T and j=1,2,…J.*

Then, the Fuzzy Production Mix Model (FPMM) is proposed as follows:

FPMM

Maximize $REV \sum_{t=1}^{T}\sum_{j=1}^{J} s_j^t \widetilde{D}_j^t - \sum_{t=1}^{T}(PCx^t + RPCy^t + DCz^t + SCm^t + RCr^t + HSis^t +$
*OC×nt+HRirt+HMimt−j=1Jt′=2Tt=*max*t′−dlj, 1t′BCjbjt,t′∗(t′−t)* (10)

Subject to

$$is^t = \sum_{j}^{J}\sum_{t'=\max(1,t-dl_j)}^{t} b_j^{t',t} + is^{t-1} + x^t + y^t, t = 1,2,\ldots,T \quad (11)$$

$im^t = im^{t-1} + o^t - x^t$,*t=1,2,…., T* (12)

$ir^t = ir^{t-1} + r^t - z^t - y^t$, *t=1,2,…,T* (13)

$$r^t \le \gamma \sum_{j=1}^{J}\sum_{t'=\max(1,t-U+1-dl_j)}^{t-U+1} b_j^{t',t-U+1}, t = U, U+1, \ldots, T \quad (14)$$

$$r^t + 1 > \gamma \sum_{j=1}^{J}\sum_{t'=\max(1,t-U+1-dl_j)}^{t-U+1} b_j^{t',t-U+1}\ t = U, U+1, \ldots, T \quad (15)$$

$$\sum_{t'=t}^{\min(dl_j+t,T)} b_j^{t,t'} \ge \widetilde{D}_j^t \times s_j^t, j = 1,2,\ldots.J, \qquad t = 1,2,\ldots,T \quad (16)$$

$$CM_k x^t + CRM_k y^t \le C_k, t = 1,2,\ldots,T, k = 1,2\ldots,K \quad (17)$$

$is^t \le CL, t = 1,2,\ldots,T$ (18)

$ir^t \le CRL, t = 1,2,\ldots,T$ (19)

$im^t \le CML, t = 1,2,\ldots,T$ (20)

$y^t \le ir^{t-1}, t = 1,2,\ldots,T$ (21)

$$\sum_{t=1}^{T} s_j^t \ge MRSO_j \times TNO_j, j = 1,2,\ldots.,J \quad (22)$$

$x^t + y^t \le L \times m^t, t = 1,2,\ldots,T$ (23)

$o^t \le L \times n^t, t = 1,2,\ldots,T$ (24)

$s_j^t \le \widetilde{D}_j^t, t = 1,2,\ldots.,T, j = 1,2,\ldots.,J$ (25)

$b_j^{t,t'}$ $x^t, y^t, z^t, r^t, o^t, im^t$ are positive integers. $S_j^t, m^t, n^t, w, k_j^t, c_j^t$ are binary variables $\epsilon\{0,1\}$.

$\bar{f}$ is an integer.

L is a large number.

Fuzzy demand,which is denoted by $\widetilde{D}_j^t$, is assumed as a triangular fuzzy number. The objective function is to maximize net profit, which is equal to the difference between revenue and total cost.Constraints (11) to (13) refer to the inventory flow conservation of serviceable products, returned products, and raw materials. Constraints (14) to (15) calculate the number of products returned at the end of each period. Constraint (16) ensures that, once the order is accepted, the supply for the order must be greater than the quantity of products that the order specifies before the latest delay period or at the end of the planning horizon. Constraint (17) is the capacity limitation of a factory. Each production activity consumes resources. Constraints (18), (19), and (20) refer to the inventory limitations of serviceable products, returned products, and raw materials, respectively. Constraint (21) describes the largest number of reprocessed products for each period. Constraint (22) requires that the number of accepted orders for retailer *j* must exceed the minimum acceptance ratio of orders provided by each retailer. Constraints (23) and (24) define the setup and ordering actions:$x^t + y^t > 0$indicates that processing or reprocessing has been executed. Thus, the setup cost for this period should be counted;otherwisem^t is zero, which means thatno production activity has occurred during this period. If $o^t > 0$, then the producer orders raw materials in this period. Thus, the ordering cost of this period should be counted; otherwise, $n^t$is zero,and no ordering cost applies for this period. Constraint (25) ensures that the producer will not accept a zero-quantity order. Constraints (10) and (16) contain the fuzzy number, $\widetilde{D}_j^t$, which induces the uncertain violation of the constraints.

$\widetilde{D}_j^t$is denoted by $[\underline{d}_j^t, d_j^t, \bar{d}_j^t]$, where $\underline{d}_j^t, d_j^t$, and $\bar{d}_j^t$are the lower bound, mean, and upper bound, respectively.The membership function of $\widetilde{D}_j^t$is defined as a linear triangular form, as shown in (26). The special case,$\underline{d}_j^t = d_j^t = \bar{d}_j^t$, implies that the order is deterministic with the membership function defined by 1 if x = d_j^t; otherwise,the function is defined by 0.

$$\mu_{\widetilde{D}_j^t}(x) = \begin{cases} if\ d_j^t \neq \underline{d}_j^t \neq \overline{d}_j^t : \\ \dfrac{x - \underline{d}_j^t}{d_j^t - \underline{d}_j^t} & ,if\ \underline{d}_j^t \leq x < d_j^t \\ \dfrac{\overline{d}_j^t - x}{\overline{d}_j^t - d_j^t} & ,if\ d_j^t \leq x \leq \overline{d}_j^t \\ 0 & ,otherwise \\ if\ d_j^t = \underline{d}_j^t = \overline{d}_j^t : \\ 0 & ,if\ x \neq d_j^t \\ 1 & ,if\ x = d_j^t \end{cases} \tag{26}$$

A gap may be induced between the real profit and the estimated one because fuzzy demand is uncertain. The estimated value may depend on the risk preference of the decision maker, which could be aggressive or conservative. To incorporate the preference of the decision maker in planning, we apply fuzzy chance-constrained programming to transform FPMM into Fuzzy Chance-Constrained Production Mix Model.

3.3 Fuzzy Chance Constrained Production Mix Model (FCCPMM)

This section develops Fuzzy Chance-Constrained Production Mix Model (FCCPMM), which is based on fuzzy chance-constrained programming,to cope with the fuzzy demands induced by the ambiguity of retailer judgment.

3.3.1*Derivation of the Membership Functions & Measures*

Giventhe membership function of$\widetilde{D}_j^t \times s_j^t$in (27), the membership function,$\sum_{j=1}^{J} \sum_{t=1}^{T} \widetilde{D}_j^t \times s_j^t$,can be obtainedby applying the extension principle (Hsien, 2010 and Zadeh, 1975), as shown in (28).

$$\mu_{\widetilde{D}_j^t \times s_j^t}(x)=\begin{cases} if\ s_j^t=1\ and\ d_j^t \neq \underline{d}_j^t \neq \overline{d}_j^t: \\ \dfrac{x-\underline{d}_j^t}{d_j^t-\underline{d}_j^t} & ,if\ \underline{d}_j^t \leq x < d_j^t \\ \dfrac{\overline{d}_j^t - x}{\overline{d}_j^t - d_j^t} & ,if\ d_j^t \leq x \leq \overline{d}_j^t \\ 0 & ,otherwise \\ if\ s_j^t=1\ and\ d_j^t = \underline{d}_j^t = \overline{d}_j^t: \\ 0 & ,if\ x \neq d_j^t \\ 1 & ,if\ x = d_j^t \\ if\ s_j^t=0: \\ 0 & ,if\ x\neq 0 \\ 1 & ,if\ x=0 \end{cases} \tag{27}$$

$$\mu_{\sum_{j=1}^{J}\sum_{t=1}^{T}\widetilde{D}_j^t s_j^t}(x)=\begin{cases} if\ \sum_{j=1}^{J}\sum_{t=1}^{T}s_j^t \neq 0\ and\ \sum_{j=1}^{J}\sum_{t=1}^{T}\underline{d}_j^t s_j^t \neq \sum_{j=1}^{J}\sum_{t=1}^{T}d_j^t s_j^t \neq \sum_{j=1}^{J}\sum_{t=1}^{T}\overline{d}_j^t s_j^t: \\ \dfrac{x-\sum_{j=1}^{J}\sum_{t=1}^{T}\underline{d}_j^t s_j^t}{\sum_{j=1}^{J}\sum_{t=1}^{T}d_j^t s_j^t-\sum_{j=1}^{J}\sum_{t=1}^{T}\underline{d}_j^t s_j^t} & ,if\ \sum_{j=1}^{J}\sum_{t=1}^{T}\underline{d}_j^t s_j^t \leq x < \sum_{j=1}^{J}\sum_{t=1}^{T}d_j^t s_j^t \\ \dfrac{\sum_{j=1}^{J}\sum_{t=1}^{T}\overline{d}_j^t s_j^t - x}{\sum_{j=1}^{J}\sum_{t=1}^{T}\overline{d}_j^t s_j^t-\sum_{j=1}^{J}\sum_{t=1}^{T}d_j^t s_j^t} & ,if\ \sum_{j=1}^{J}\sum_{t=1}^{T}d_j^t s_j^t \leq x < \sum_{j=1}^{J}\sum_{t=1}^{T}\overline{d}_j^t s_j^t \\ 0 & ,otherwise \\ if\ \sum_{j=1}^{J}\sum_{t=1}^{T}s_j^t \neq 0\ and\ \sum_{j=1}^{J}\sum_{t=1}^{T}\underline{d}_j^t s_j^t = \sum_{j=1}^{J}\sum_{t=1}^{T}d_j^t s_j^t = \sum_{j=1}^{J}\sum_{t=1}^{T}\overline{d}_j^t s_j^t: \\ 0 & ,if\ x \neq \sum_{j=1}^{J}\sum_{t=1}^{T}d_j^t s_j^t \\ 1 & ,if\ x = \sum_{j=1}^{J}\sum_{t=1}^{T}d_j^t s_j^t \\ if\ \sum_{j=1}^{J}\sum_{t=1}^{T}s_j^t=0: \\ 0 & ,if\ x\neq 0 \\ 1 & ,if\ x=0 \end{cases} \tag{28}$$

We definecost function$h(\boldsymbol{u})$ andprofit function $P(\boldsymbol{v},\widetilde{\boldsymbol{D}})$ as below:

$$h(\boldsymbol{u}) = \sum_{t=1}^{T}(PCx^t + RPCy^t + DCz^t + SCm^t + RCr^t + HSis^t + OCn^t + HRir^t + HMim^t) - \sum_{j=1}^{J}\sum_{t'=2}^{T}\sum_{t=\max(t'-dl_j,1)}^{t'} BC_j b_j^{t,t'} * (t'-t) \tag{29}$$

$$P(\boldsymbol{v},\widetilde{\boldsymbol{D}}) = REV\sum_{t=1}^{T}\sum_{j=1}^{J} s_j^t \times \widetilde{D}_j^t - h(\boldsymbol{u}) \tag{30}$$

Since *P(**v**,$\widetilde{\boldsymbol{D}}$)* is also a fuzzy quantity, the membership function of *P(**v**,$\widetilde{\boldsymbol{D}}$)* is shown in (31):

$$\mu_{P(v,\tilde{D})}(z) = \begin{cases} \text{if } \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t \neq 0 \text{ and } \sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t \neq \sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t \neq \sum_{j=1}^{J}\sum_{t=1}^{T} \overline{d}_j^t \times s_j^t : \\ \quad \dfrac{\frac{z+h(u)}{REV} - \sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t}{\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \underline{d}_j^t)\times s_j^t}, \quad \text{if } REV\sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t - h(u) \leq z < REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \\ \quad \dfrac{\sum_{j=1}^{J}\sum_{t=1}^{T} \overline{d}_j^t \times s_j^t - \frac{z+h(u)}{REV}}{\sum_{j=1}^{J}\sum_{t=1}^{T}(\overline{d}_j^t - d_j^t)\times s_j^t}, \quad \text{if } REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \leq z \leq REV\sum_{j=1}^{J}\sum_{t=1}^{T} \overline{d}_j^t \times s_j^t - h(u) \\ \quad 0, \quad \text{otherwise} \\ \text{if } \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t \neq 0 \text{ and } \sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t = \sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t = \sum_{j=1}^{J}\sum_{t=1}^{T} \overline{d}_j^t \times s_j^t : \\ \quad 0, \quad \text{if } z \neq REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \\ \quad 1, \quad \text{if } z = REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \\ \text{if } \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t = 0 : \\ \quad 0, \quad \text{if } z \neq 0 \\ \quad 1, \quad \text{if } z = 0 \end{cases} \tag{31}$$

To formulate FCCPMM, we recall that one of the features in chance-constrained programming [Constraint (7)] is to maximize $\bar{f}$, provided that the chance of the original objective function greater than $\bar{f}$ is at least greaterthan a given confidence level. Hence, the first step is to construct the m_λ-measure for the fuzzy event, $P(\boldsymbol{v},\widetilde{\boldsymbol{D}}) \geq \bar{f}$. We first derive thepossibility and necessity measures for the fuzzy event, $P(v,\widetilde{D}) \geq \bar{f}$, as shown in (32).

$$POSS(P(v,\tilde{D}) \geq \bar{f}) = \begin{cases} if \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t \neq 0 \; and \sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t \neq \sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t \neq \sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t : \\ \quad 1, \qquad , if \; REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \geq \bar{f} \\ \quad \dfrac{\sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t - \dfrac{\bar{f}+h(u)}{REV}}{\sum_{j=1}^{J}\sum_{t=1}^{T}(\bar{d}_j^t - d_j^t)\times s_j^t} \qquad , \; if \; REV\sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t \geq \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \\ \quad 0 \qquad , \; if \; \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t - h(u) \\ if \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t \neq 0 \; and \sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t = \sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t = \sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t : \\ \quad 1 \qquad , if \; REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \geq \bar{f} \\ \quad 0 \qquad , if \; \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \\ if \; \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t = 0 \\ \quad 1 \qquad , if \; 0 \geq \bar{f} \\ \quad 0 \qquad , if \; \bar{f} > 0 \end{cases} \qquad (32)$$

$$NEC(P(v,\tilde{D}) \geq \bar{f}) = \begin{cases} if \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t \neq 0 \; and \sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t \neq \sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t \neq \sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t : \\ \quad 1, \qquad , if \; REV\sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t - h(u) \geq \bar{f} \\ \quad \dfrac{\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - \dfrac{\bar{f}+h(u)}{REV}}{\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \underline{d}_j^t)\times s_j^t} \qquad , \; if \; REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t \geq \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t - h(u) \\ \quad 0 \qquad , \; if \; \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \\ if \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t \neq 0 \; and \sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t = \sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t = \sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t : \\ \quad 1 \qquad , if \; REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \geq \bar{f} \\ \quad 0 \qquad , if \; \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \\ if \; \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t = 0 \\ \quad 1 \qquad , if \; 0 \geq \bar{f} \\ \quad 0 \qquad , if \; \bar{f} > 0 \end{cases} \qquad (33)$$

The m_λ-measure for fuzzy event $P(v,\tilde{D}) \geq \tilde{f}$ is defined by (34)

$$M_{\lambda}(P(v,\tilde{D}) \geq \bar{f}) = \begin{cases} case\,1: if \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t \neq 0 \ and \sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t \neq \sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t \neq \sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t : & \\ 1 & ,if \ REV\sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t - h(u) \geq \bar{f} \\ \dfrac{\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \lambda \underline{d}_j^t) \times s_j^t - (1-\lambda) \times \dfrac{f + h(u)}{REV}}{\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \underline{d}_j^t) \times s_j^t} & ,if \ REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) > \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t - h(\\ \lambda \dfrac{\sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t - \dfrac{f + h(u)}{REV}}{\sum_{j=1}^{J}\sum_{t=1}^{T}(\bar{d}_j^t - d_j^t) \times s_j^t} & ,if \ REV\sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t - h(u) > \bar{f} \geq REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(\\ 0 & ,if \ \bar{f} \geq REV\sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t - h(u) \\ case\,2: if \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t \neq 0 \ and \sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t = \sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t = \sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t : & \\ 1 & ,if \ REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \geq \bar{f} \\ 0 & ,if \ \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \\ case\,3: if \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t = 0 & \\ 1 & ,if \ 0 \geq \bar{f} \\ 0 & ,if \ \bar{f} > 0 \end{cases} \quad (34)$$

We apply a similar procedurefor the other original constraints with fuzzy numbers. Possibility and necessity measures for the fuzzy event, $\sum_{t'=t}^{\min(t+dl_j,T)} b_j^{t,t'} \geq \tilde{D}_j^t s_j^t$,are defined by (35) and (36) because Constraint (6) is the only constraint with a fuzzy demand.

$$POSS(P(v,\tilde{D}) \geq \bar{f}) = \begin{cases} if \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t \neq 0 \; and \sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t \neq \sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t \neq \sum_{j=1}^{J}\sum_{t=1}^{T} \overline{d}_j^t \times s_j^t : & \\ \quad 1, & , if \; REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \geq \bar{f} \\ \quad \dfrac{\sum_{j=1}^{J}\sum_{t=1}^{T} \overline{d}_j^t \times s_j^t - \dfrac{\bar{f}+h(u)}{REV}}{\sum_{j=1}^{J}\sum_{t=1}^{T} (\overline{d}_j^t - d_j^t) \times s_j^t} & , \; if \; REV\sum_{j=1}^{J}\sum_{t=1}^{T} \overline{d}_j^t \times s_j^t \geq \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u \\ \quad 0 & , \; if \;\; \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T} \overline{d}_j^t \times s_j^t - h(u) \\ if \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t \neq 0 \; and \sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t = \sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t = \sum_{j=1}^{J}\sum_{t=1}^{T} \overline{d}_j^t \times s_j^t : & \\ \quad 1 & , if \; REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \geq \bar{f} \\ \quad 0 & , if \;\; \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \\ if \;\; \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t = 0 & \\ \quad 1 & , if \; 0 \geq \bar{f} \\ \quad 0 & , if \;\; \bar{f} > 0 \end{cases} \quad (35)$$

$$NEC(P(v,\tilde{D}) \geq \bar{f}) = \begin{cases} if \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t \neq 0 \; and \sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t \neq \sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t \neq \sum_{j=1}^{J}\sum_{t=1}^{T} \overline{d}_j^t \times s_j^t : & \\ \quad 1, & , if \; REV\sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t - h(u) \geq \bar{f} \\ \quad \dfrac{\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - \dfrac{\bar{f}+h(u)}{REV}}{\sum_{j=1}^{J}\sum_{t=1}^{T} (d_j^t - \underline{d}_j^t) \times s_j^t} & , \; if \; REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t \geq \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t - h(\\ \quad 0 & , \; if \;\; \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \\ if \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t \neq 0 \; and \sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t = \sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t = \sum_{j=1}^{J}\sum_{t=1}^{T} \overline{d}_j^t \times s_j^t : & \\ \quad 1 & , if \; REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \geq \bar{f} \\ \quad 0 & , if \;\; \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \\ if \;\; \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t = 0 & \\ \quad 1 & , if \; 0 \geq \bar{f} \\ \quad 0 & , if \;\; \bar{f} > 0 \end{cases} \quad (36)$$

The m_λ-measure for fuzzy event $P(v,\tilde{D}) \geq \tilde{f}$ is defined by (37)

$$M_{\lambda}(P(v,\tilde{D}) \geq \bar{f}) = \begin{cases} case\,1: if \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t \neq 0 \; and \sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t \neq \sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t \neq \sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t : \\ 1 \qquad if\; REV\sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t - h(u) \geq \bar{f} \\ \dfrac{\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \lambda \underline{d}_j^t)\times s_j^t - (1-\lambda)\times \dfrac{f+h(u)}{REV}}{\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \underline{d}_j^t)\times s_j^t} \qquad if\; REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) > \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t - h(\\ \lambda \dfrac{\sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t - \dfrac{f+h(u)}{REV}}{\sum_{j=1}^{J}\sum_{t=1}^{T}(\bar{d}_j^t - d_j^t)\times s_j^t} \qquad if\; REV\sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t - h(u) > \bar{f} \geq REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(\\ 0 \qquad ,if\; \bar{f} \geq REV\sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t - h(u) \\ case\,2: if \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t \neq 0 \; and \sum_{j=1}^{J}\sum_{t=1}^{T} \underline{d}_j^t \times s_j^t = \sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t = \sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t : \\ 1 \qquad ,if\; REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \geq \bar{f} \\ 0 \qquad ,if\; \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(u) \\ case\,3: if \sum_{j=1}^{J}\sum_{t=1}^{T} s_j^t = 0 \\ 1 \qquad ,if\; 0 \geq \bar{f} \\ 0 \qquad ,if\; \bar{f} > 0 \end{cases} \tag{37}$$

For other original constraints with fuzzy numbers, we apply the same procedure as above. Since constraint (6) is the only constraint with fuzzy demand, possibility and necessity measures for fuzzy event $\sum_{t'=t}^{\min(t+dl_j,T)} b_j^{t,t'} \geq \tilde{D}_j^t s_j^t$ are defined by (38) and (39).

$$POSS(\sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \geq \tilde{D}_j^t s_j^t) = \begin{cases} if\; s_j^t = 1 \; and \; d_j^t \neq \underline{d}_j^t \neq \bar{d}_j^t : \\ 1 \qquad ,if \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \geq d_j^t \\ \dfrac{\sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} - \underline{d}_j^t}{d_j^t - \underline{d}_j^t} \qquad ,if\; d_j^t \geq \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} > \underline{d}_j^t \\ 0 \qquad ,if\; \underline{d}_j^t > \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \\ if\; s_j^t = 1 \; and \; d_j^t = \underline{d}_j^t = \bar{d}_j^t : \\ 1 \qquad ,if \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \geq d_j^t \\ 0 \qquad ,if\; d_j^t > \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \\ if\; s_j^t = 0 \\ 1 \qquad ,if \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \geq 0 \\ 0 \qquad ,if\; 0 > \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \end{cases} \tag{38}$$

$$NEC\left(\sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \geq \tilde{D}_j^t s_j^t\right) = \begin{cases} if\ s_j^t = 1\ and\ d_j^t \neq \underline{d}_j^t \neq \overline{d}_j^t : \\ 1 \quad , if \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \geq \overline{d}_j^t \ and \\ \dfrac{\sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} - d_j^t}{\overline{d}_j^t - d_j^t} \quad , if\ \overline{d}_j^t \geq \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} > d_j^t \\ 0 \quad , if\ d_j^t > \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \\ if\ s_j^t = 1\ and\ d_j^t = \underline{d}_j^t = \overline{d}_j^t : \\ 1 \quad , if \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \geq d_j^t \\ 0 \quad , if\ d_j^t > \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \\ if\ s_j^t = 0 \\ 1 \quad , if \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \geq 0 \\ 0 \quad , if\ 0 > \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \end{cases} \tag{39}$$

The m_λ-measure for fuzzy event $(\sum_{t'=t}^{\min(t+dl_j,T)} b_j^{t,t'} \geq \tilde{D}_j^t s_j^t)$ is defined by (40)

$$M_\lambda\left(\sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \geq \tilde{D}_j^t s_j^t\right) = \begin{cases} if\ s_j^t = 1\ and\ d_j^t \neq \underline{d}_j^t \neq \overline{d}_j^t : \\ \quad 1 & ,if \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \geq \overline{d}_j^t \\ \quad \dfrac{\lambda \overline{d}_j^t + (1-\lambda)\times \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} - d_j^t}{\overline{d}_j^t - d_j^t} & ,if\ \overline{d}_j^t \geq \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} > d_j^t \\ \quad \lambda \times \dfrac{\sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} - \underline{d}_j^t}{d_j^t - \underline{d}_j^t} & ,if\ d_j^t \geq \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} > \underline{d}_j^t \\ \quad 0 & ,if\ \underline{d}_j^t > \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \\ if\ s_j^t = 1\ and\ d_j^t = \underline{d}_j^t = \overline{d}_j^t : \\ \quad 1 & ,if \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \geq d_j^t \\ \quad 0 & ,if\ d_j^t > \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \\ if\ s_j^t = 0 \\ \quad 1 & ,if \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \geq 0 \\ \quad 0 & ,if\ 0 > \sum_{t'=t}^{min(t+dl_j,T)} b_j^{t,t'} \end{cases} \quad (40)$$

3.3.2 *Property of m_λ Measure for Uncertain Event*

Let $g_\lambda(P(v,\tilde{D}) \geq \tilde{f}) = \frac{dm_\lambda(P(v,\tilde{D})\geq\bar{f})}{d\bar{f}}$, where $\underline{d}_j^t s_j^t \neq d_j^t s_j^t \neq \bar{d}_j^t s_j^t$ for some t and j, then g_λ can be derived as follows:

$$g_\lambda(P(v,\tilde{D}) \geq \overline{f}) = \begin{cases} 0 & ,if\ \bar{f} < REV\sum_{j=1}^{J}\sum_{t=1}^{T}\underline{d}_j^t \times s_j^t - h(\mathbf{u}) \\ \dfrac{-(1-\lambda)}{REV\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \underline{d}_j^t)\times s_j^t} & ,if\ REV\sum_{j=1}^{J}\sum_{t=1}^{T}\underline{d}_j^t \times s_j^t - h(\mathbf{u}) < \bar{f} < REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(\mathbf{u}) \\ \dfrac{-\lambda}{REV\sum_{j=1}^{J}\sum_{t=1}^{T}(\overline{d}_j^t - d_j^t)\times s_j^t} & ,if\ REV\sum_{j=1}^{J}\sum_{t=1}^{T} d_j^t \times s_j^t - h(\mathbf{u}) < \bar{f} < REV\sum_{j=1}^{J}\sum_{t=1}^{T}\overline{d}_j^t \times s_j^t - h(\mathbf{u}) \\ 0 & ,if\ \bar{f} > REV\sum_{j=1}^{J}\sum_{t=1}^{T}\overline{d}_j^t \times s_j^t - h(\mathbf{u}) \end{cases} \quad (41)$$

Let S_1 and S_2 be two different sets, where

$$S_1 = \{x| \sum_{t=1}^{T}\sum_{j=1}^{T}\underline{d}_j^t s_j^t < x < \sum_{t=1}^{T}\sum_{j=1}^{T} d_j^t s_j^t\} \quad (42)$$

and

$$S_1 = \{y | \sum_{t=1}^{T} \sum_{j=1}^{T} d_j^t s_j^t < y < \sum_{t=1}^{T} \sum_{j=1}^{T} \overline{d}_j^t s_j^t \}$$

Theorem 1

$\lambda \geq \frac{\sum_{t=1}^{T}\sum_{j=1}^{T}(\overline{d}_j^t - d_j^t)s_j^t}{\sum_{t=1}^{T}\sum_{j=1}^{T}(\overline{d}_j^t - \underline{d}_j^t)s_j^t}$, $|g_\lambda(P(v,\tilde{D}) \geq \tilde{f})|$ *is maximized when* $\overline{f} \in S_1$, *otherwise* $g_\lambda(P(v,\tilde{D}) \geq \tilde{f})$ *is maximized when* $\overline{f} \in S_2$.

Proof: see Appendix A.

λ is the optimistic value, a larger λ makes the higher m_λ measure. Figure 2 shows m_λ measure with different λ settings under case 1 in m_λ-measure of event $P(v,\tilde{D}) \geq \tilde{f}$.

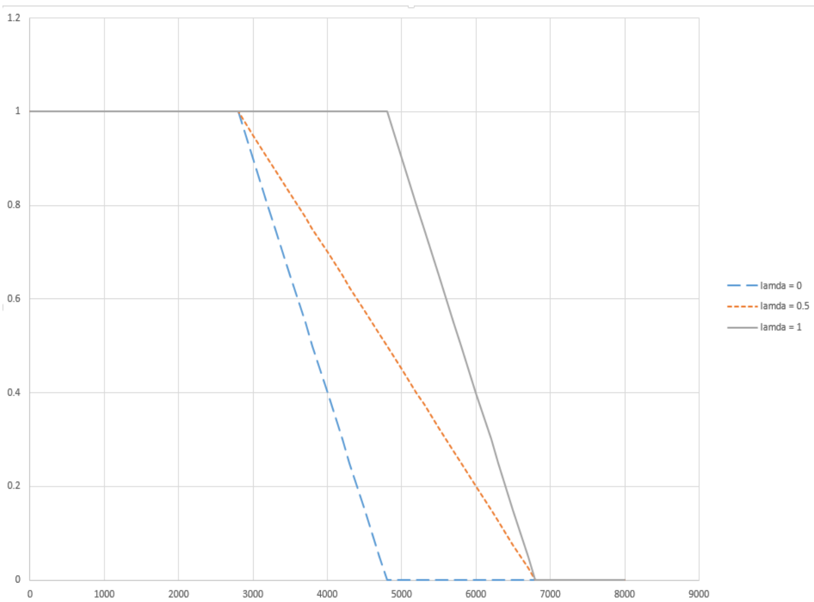

Figure 2 $m_\lambda(P(v,\tilde{D}) \geq \tilde{f})$ with Different λ

3.4 The Proposed Fuzzy Chance-Constrained Production Mix Model

First, some assumptions in modelling FCCPMM are given below

(1). All orders should be placed at the beginning of a planning horizon; otherwise, a fuzzy demand of $\tilde{0}(0,0,0)$ will be assumed.

(2). If the order of the retailer is not accepted by the producer, i.e., s_j^t is zero or the order is deterministic, which means that $\underline{d}_j^t s_j^t = d_j^t s_j^t = \overline{d}_j^t s_j^t$, then the total supply for the order should be the same as $d_j^t s_j^t$.

(3). The total supply for any order, $\tilde{D}_j^t$, is less than $\overline{d}_j^t s_j^t$.

(4). The confidence levels, such as β, α_j^U, *and* α_j^L, are greater than zero.

Then, the complete Fuzzy Chance-Constrained Production Mix Model is formulated:

FCCPMM

Maximize $\overline{f}$

Subject to

$$\max\left(0, \lambda \times \left(\sum_{t=1}^{T}\sum_{j=1}^{J} \overline{d}_j^t s_j^t - \frac{\overline{f} + h(\boldsymbol{u})}{REV}\right)\right) \geq -\mathrm{L} \times (1-\mathrm{w}) + \beta \times \sum_{t=1}^{T}\sum_{j=1}^{J} (\overline{d}_j^t - d_j^t) s_j^t \quad (44)$$

$$\max\left(0,\sum_{t=1}^{T}\sum_{j=1}^{J}(d_j^t-\lambda\underline{d}_j^t)s_j^t-(1-\lambda)\frac{\overline{f}+h(\boldsymbol{u})}{REV}\right)\geq -\mathrm{L}\times \mathrm{w}+\beta\times\sum_{t=1}^{T}\sum_{j=1}^{J}(d_j^t-\underline{d}_j^t)s_j^t \quad (45)$$

$$\overline{f}-REV\times\sum_{t=1}^{T}\sum_{j=1}^{J}d_j^t s_j^t+h(\boldsymbol{u})\leq \mathrm{REV}\sum_{t=1}^{T}\sum_{j=1}^{J}(\overline{d}_j^t-d_j^t)\times\min(\mathrm{w},s_j^t) \quad (46)$$

$$\mathrm{w}\leq\max(0,\overline{f}-REV\times\sum_{t=1}^{T}\sum_{j=1}^{J}d_j^t s_j^t+h(\boldsymbol{u})) \quad (47)$$

$$is^t=is^{t-1}+x^t+y^t-\sum_{j}^{J}\sum\nolimits_{t'=\max(t-dl_j,1)}^{t}b_j^{t',t},t=1,2,\dots,T. \quad (48)$$

$$im^t=im^{t-1}+o^t-x^t,t=1,2,\dots,T. \quad (49)$$

$$ir^t=ir^{t-1}+r^t-z^t-y^t,t=1,2,\dots,T. \quad (50)$$

$$\mathrm{r^t}\leq\gamma\times\sum_{j=1}^{J}\sum_{\max(t-U+1-dl_j,1)}^{t-U+1}b_j^{t',t-U+1},\mathrm{t}=U,U+1,\dots,T. \quad (51)$$

$$\mathrm{r^t}+1>\gamma\times\sum_{j=1}^{J}\sum_{\max(t-dl_j,1)}^{t-U+1}b_j^{t',t-U+1},t=U,U+1,\dots T. \quad (52)$$

$$\min\left(1,\max\left(1-s_j^t,\lambda\left(\frac{\sum_{t'=t}^{\min(dl_j+t,T)}b_j^{t,t'}-\underline{d}_j^t}{max(d_j^t-\underline{d}_j^t,1)}\right)\right)\right)\geq\ \alpha_j^L-L\times k_j^t-L\times\left(1-c_j^t\right),t=1,2,\dots,T,j=1,2,\dots,J. \quad (53)$$

$$\min\left(1,\max\left(1-s_j^t,\frac{\lambda\overline{d}_j^t s_j^t+(1-\lambda)\sum_{t'=t}^{\min(dl_j+t,T)}b_j^{t,t'}-d_j^t s_j^t}{max(\overline{d}_j^t-d_j^t,1)}\right)\right)\geq\alpha_j^L-L\times\left(1-k_j^t\right),\ t=1,2,\dots,T,j=1,2,\dots,J. \quad (54)$$

$$\min\left(1,\max\left(1-s_j^t,\lambda\left(\frac{\sum_{t'=t}^{\min(dl_j+t,T)}b_j^{t,t'}-\underline{d}_j^t}{max(d_j^t-\underline{d}_j^t,1)}\right)\right)\right)\leq\alpha_j^U-s_j^t+1+L\times k_j^t+L\times(1-c_j^t),\ \mathrm{t}=1,2,\dots,\mathrm{T},\ j=1,2,\dots,J. \quad (55)$$

$$\min\left(1,\max\left(1-s_j^t,\frac{\lambda\overline{d}_j^t s_j^t+(1-\lambda)\sum_{t'=t}^{\min(dl_j+t,T)}b_j^{t,t'}-d_j^t s_j^t}{max(\overline{d}_j^t-d_j^t,1)}\right)\right)\leq\alpha_j^U-s_j^t+1+L\times(1-k_j^t),t=1,2,\dots,T,j=1,2,\dots,J. \quad (56)$$

$$\sum_{t'=t}^{\min(dl_j+t,T)}b_j^{t,t'}-d_j^t s_j^t\leq\left(\overline{d}_j^t-d_j^t\right)\times\min\left(k_j^t,s_j^t\right),t=1,2,\dots,T,j=1,2,\dots,J. \quad (57)$$

$$k_j^t\leq\max\left(0,\sum_{t'=t}^{dl_j+t}b_j^{t,t'}-d_j^t s_j^t\right),t=1,2,\dots,T,j=1,2,\dots,J. \quad (58)$$

$$c_j^t=\min(d_j^t-\underline{d}_j^t,1),t=1,2,\dots,T,j=1,2,\dots,J. \quad (59)$$

$$\sum_{t'=t}^{dl_j+t}b_j^{t,t'}\geq\underline{d}_j^t s_j^t,\mathrm{t}=1,2,\dots\mathrm{T},j=1,2,\dots,J. \quad (60)$$

$$CM_k x^t+CRM_k y^t\leq C_k,t=1,2,\dots,T,k=1,2..,K. \quad (61)$$

$is^t \leq CL, t = 1,2, \dots, T.$ (62)

$ir^t \leq CRL, t = 1,2, \dots, T.$ (63)

$im^t \leq CML, t = 1,2, \dots, T.$ (64)

$y^t \leq ir^{t-1}, t = 1,2, \dots, T.$ (65)

$\sum_{j=1}^{J} \sum_{t=1}^{T} s_j^t \geq MRSO_j \times TNO, j = 1,2, \dots, J.$ (66)

$x^t + y^t \leq L \times m^t, t = 1,2, \dots, T.$ (67)

$o^t \leq L \times n^t, t = 1,2, \dots, T.$ (68)

$s_j^t \leq d_j^t, t = 1,2, \dots, T, j = 1,2, \dots, J.$ (69)

$x^t, y^t, o^t, im^t, ir^t, is^t, z^t, r^t$ *are positive integers,* $S_j^t, m^t, n^t, w, k_j^t, c_j^t$ *are binary variables* $\epsilon\{0,1\}$.

$b_j^{t,t'}$ *is positiveand integral.*

λ, β *and* $\alpha_j^U, \alpha_j^L \in (0,1]$

L is a large number.

$\bar{f}$*is a large number.*

Constraints (45) to (47) represent $M_\lambda(P(\boldsymbol{v}, \tilde{D}) \geq \bar{f}) \geq \beta$. If $\bar{f} + h(\boldsymbol{u})$ is greater than $REV \times \sum_{t=1}^{T} \sum_{j=1}^{J} d_j^t s_j^t$, Constraint (45) has to be satisfied; otherwise, Constraint (46) must be satisfied. If $\sum_{t=1}^{T} \sum_{j=1}^{J} s_j^t$ is equal to zero because β is assumed to be greater than zero, then $\bar{f}$ must be less than zero. Constraints (53) to (60) represent Constraints $\alpha_j^U \geq M_\lambda \left(\sum_{t'=t}^{\min(dl_j+t,T)} b_j^{t,t'} \geq \tilde{D}_j^t \times s_j^t\right) \geq \alpha_j^L$. If $\sum_{t'=t}^{\min(dl_j+t,T)} b_j^{t,t'} > d_j^t$, which indicates that Constraints (54) and (56) have to be satisfied; otherwise, (53) and (55) shall be satisfied. In addition, if s_j^t is zero, then constraint (57) also ensures that $\sum_{t'=t}^{\min(dl_j+t,T)} b_j^{t,t'}$ must be zero. Furthermore, if s_j^t is zero, then $M_\lambda \left(\sum_{t'=t}^{\min(dl_j+t,T)} b_j^{t,t'} \geq \tilde{D}_j^t \times s_j^t\right)$ is always 1, which extends the upper bound to least 1 to satisfy the constraint. Constraint (59) ensures that if the demand is deterministic, then the total supply for the order should be $d_j^t s_j^t$.

Theorem 2:

If $\sum_{t=1}^{T} \sum_{j=1}^{J} s_j^t \neq 0, \underline{d}_j^t s_j^t \neq d_j^t s_j^t \neq \bar{d}_j^t s_j^t$ *for some* $\beta > 0$, *then* the *maxumum* $\bar{f}$ *is*

$$\bar{f} = \begin{cases} \dfrac{REV(\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \lambda \underline{d}_j^t) \times s_j^t - \beta \sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \underline{d}_j^t) \times s_j^t)}{1-\lambda} - h(u), \text{if } \lambda < \beta \\ REV(\sum_{j=1}^{J}\sum_{t=1}^{T} \bar{d}_j^t \times s_j^t - \dfrac{\beta}{\lambda}\sum_{j=1}^{J}\sum_{t=1}^{T}(\bar{d}_j^t - d_j^t) \times s_j^t) - h(u) \quad , \text{if } \lambda \geq \beta \end{cases} \quad (70)$$

Proof: See Appendix B

3.5 Parametric Analysis

λ and β are based on the risk attitude of the decision maker. The goal of this section is to analyze the effects of these two factors on cost structures and objective values to assist a decision maker in setting the parameters. Suppose that the decision maker changes his attitude towards the risk parameters after the production plan is made and wants to adjust λ and β, the feasibility of the original optimal solution after the adjustments are made is in the interest of the current researchers because adjusting the production plan may be costly.

3.5.1 *Feasibility of* β

Theorem 2indicates that once β is increased to β', Constraint $m_\lambda(P(v, \tilde{D}) \geq \tilde{f}) \geq \beta'$ is always infeasible. Therefore, the adjustment region of β for model feasibility is $(0, \beta]$.

3.5.2*Feasibility of λ*

To find such a region, we first notice that λ affects the feasibility of two types of constraints, namely, $\alpha_j^L \le M_\lambda\left(B_j^t \ge \tilde{D}_j^t \times s_j^t\right) \le \alpha_j^U$ and $m_\lambda\left(P\left(v, \tilde{D}\right) \ge \tilde{f}\right) \ge \beta$.Therefore, the adjustment of λ must satisfy these constraints. To satisfy Constraint $m_\lambda\left(P\left(v, \tilde{D}\right) \ge \tilde{f}\right) \ge \beta$, the region of λ must be $[\lambda, 1]$. This region indicates that the adjustment of λ is at least positive so that the constraint will not be violated. Hence, we only focus on $M_\lambda\left(B_j^t \ge \tilde{D}_j^t \times s_j^t\right) \le \alpha_j^U$ because a higher λproduces a higher value of $M_\lambda\left(B_j^t \ge \tilde{D}_j^t \times s_j^t\right)$. We then suppose that $M_{\lambda^*}\left(B_j^t \ge \tilde{D}_j^t \times s_j^t\right)^*$ is the optimal value. The value of λ', $\lambda' \ge \lambda^*$then becomes of interest, such that $m_{\lambda'}\left(B_j^t \ge \tilde{D}_j^t \times s_j^t\right) \le \alpha_j^U$. To achieve this goal, we let $\lambda' = \lambda^* + \delta$, where $\delta \ge 0$. We know that $m_{\lambda'}(\cdot) - m_{\lambda^*}(\cdot) = \delta(POS(\cdot) - NEC(\cdot))$. For any λ', $\delta(POS(\cdot) - NEC(\cdot))$ must be less than $\alpha_j^U - m_{\lambda^*}(\cdot)$ so that the original optimal basis is still feasible. Therefore, δ must be less than min $\left(\frac{\alpha_j^U - m_\lambda(\cdot)}{POS(\cdot) - NEC(\cdot)}\right)$. In addition, δ must be less than $1 - \lambda^*$because $\lambda \in [0, 1]$. The overall minimum δ to satisfy the constraints $M_\lambda\left(B_j^t \ge \tilde{D}_j^t \times s_j^t\right) \le \alpha_j^U$ is

$$\delta_{\min} \le \min_{t,j}\left(\frac{\alpha_j^U - m_{\lambda^*}(\cdot)}{POS(\cdot) - NEC(\cdot)}\right), 1 - \lambda^*) \qquad (71)$$

3.6 Concluding Remarks

This section develops the Fuzzy Production Mix Model, where the orders provided by retailers are assumed to be fuzzy quantities.To deal with the uncertainty issue, Fuzzy Chance-Constrained Production Mix Model is introduced using the concept of fuzzy chance-constrained programming. The properties and parameter analysis are investigated. The advantage of such a model is the incorporation of the preferences of the decision maker into the planning stage.

IV. NUMERIACAL EXAMPLE

In this section, we use a numerical example to illustrate the operation of FCCPMM by giving the input parameters. Table 1 is the periodic-order list placed by retailers. Each order is a triangular fuzzy number that is represented in the form of $(\underline{d}_j^t, d_j^t, \bar{d}_j^t)$.Tables 2 and 3 define the input parameters and capacities, respectively. Without loss of generality, the confidence levels are assumed to be $\alpha^U = \alpha_j^U$ and $\alpha^L = \alpha_j^L$ for *j*=1,2,…,J.

Using the optimization software package, ILOG Cplex v12.5, Tables 4~8 list the output of the results of which the optimal value for $\bar{f}$ is 4,756, which means that the realprofit of at least 4,756 has a probablility of 50%. While the shadow blocks in Table 4show the ordersthat should be accepted by the producer to maximize net profit; Table 5 summarizes the optimal production activities. Then, each entity (t, t') in Tables 6, 7, and 8 shows the number of products supplied to the retailers' t^{th}-period orders from *t*' period.

Table 1 Periodic Orders Given by Retailers

Retailer / Order	R1	R2	R3	R4
1st	(80,85,90)	(60,95,100)	(70,78,85)	(77,88,99)
2nd	(80,96,100)	(100,110,120)	(80,90,110)	(110,115,120)
3rd	(120,150,170)	(130,160,190)	(160,170,180)	(140,152,200)
4th	(140,160,190)	(150,180,190)	(150,170,180)	(100,200,300)
5th	(100,110,120)	(50,60,70)	(20,30,50)	(100,250,400)
6th	(50,60,70)	(50,70,80)	(10,20,30)	(200,300,530)
7th	(1,2,3)	(0,1,2)	(1,2,3)	(3,4,36)
8th	(80,85,90)	(60,95,100)	(70,78,85)	(77,88,99)
9th	(80,96,100)	(100,110,120)	(80,90,110)	(110,115,120)
10th	(120,150,170)	(130,160,190)	(160,170,180)	(140,152,200)
11th	(140,160,190)	(150,180,190)	(150,170,180)	(100,200,300)
12th	(100,110,120)	(50,60,70)	(20,30,50)	(100,250,400)
13th	(50,60,70)	(50,70,80)	(10,20,30)	(200,300,530)

Table 2 Input Parameters

Parameters	value	Parameters	value
T	13 periods	$BC_j, j=1,\ldots,4$	[4, 5, 6, 5] dollars/per product
J	4 retailers	RC	2 dollars/per product
K	4 machines	OC	20 dollars /per product
U	3 periods	REV	15 dollars /per product
$TNO_j, j=1,\ldots,4$	[13,13,13,13] order	PC	10 dollars /per product
Γ	0.7	RPC	7 dollars /per product
CL	500 new product	HS	7 dollars /per product
CRL	150 returned products	HR	5 dollars /per product
$MRSO_j, j=1,\ldots,4$	[0.8 0.7 0.5 0.4]	HM	3 dollars /per product
CML	150 raw materials	DC	2 dollars /per product
$dl_j, j=1,\ldots,4$	[1 2 1 0] periods	SC	30 dollars/per period
α^U	0.5	β	0.5
α^L	0.8	λ	0.5

Table 3 Resource and Consumptions of Processing and Reprocessing

machine k	1	2	3	4
Total Capacity of machine k	800	1000	1400	500
Unit consumption of processing 1 unit product for machine k	3	5	7	2
Unit consumption of reprocessing unit product for machine k	1	2	4	1

Table 4Optimal Orders Combinationwith Confidence Level $\beta = 0.5$ and$\alpha^U = \alpha^L = 0.5$

Retailer / Order	R1	R2	R3	R4
1st	(80,85,90)	(60,95,100)	(70,78,85)	(77,88,99)
2nd	(80,96,100)	(100,110,120)	(80,90,110)	(110,115,120)
3rd	(120,150,170)	(130,160,190)	(160,170,180)	(140,152,200)
4th	(140,160,190)	(150,180,190)	(150,170,180)	(100,200,300)
5th	(100,110,120)	(50,60,70)	(20,30,50)	(100,250,400)
6th	(50,60,70)	(50,70,80)	(10,20,30)	(200,300,530)
7th	(1,2,3)	(0,1,2)	(1,2,3)	(3,4,36)
8th	(80,85,90)	(60,95,100)	(70,78,85)	(77,88,99)
9th	(80,96,100)	(100,110,120)	(80,90,110)	(110,115,120)
10th	(120,150,170)	(130,160,190)	(160,170,180)	(140,152,200)
11th	(140,160,190)	(150,180,190)	(150,170,180)	(100,200,300)
12th	(100,110,120)	(50,60,70)	(20,30,50)	(100,250,400)
13th	(50,60,70)	(50,70,80)	(10,20,30)	(200,300,530)
Total accepted orders	11	10	7	7
Accepted/Total orders	0.85	0.77	0.54	0.54

Table 5 Optimal Production Activities

Period t / number of	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th
Process, x^t	200	200	200	120	121	121	38	114	104	125	113	116	114
Reprocess, y^t	0	0	0	104	138	138	150	150	150	131	150	147	150
Disposed, z^t	0	0	0	0	0	34	2	59	0	34	0	59	132
Recycled, r^t	0	0	140	139	137	184	152	209	131	184	147	209	132
Inventory, is^t	0	0	3	0	41	0	0	0	44	0	73	0	0
Inventory, ir^t	0	0	140	139	138	150	150	150	131	150	147	150	0
Inventory, im^t	0	0	0	0	0	0	0	0	0	0	0	0	0
Materials, o^t	200	200	200	120	121	121	38	114	104	125	113	116	114

Setup, m^t	1	1	1	1	1	1	1	1	1	1	1	1	1
Ordering, n^t	1	1	1	1	1	1	1	1	1	1	1	1	1
w	0												

Table 6Optimal Periodic Allocation for Retailers (R1, R2, R3, R4)−Supply From Periods 1,2,3,4 with Confidence Level $\beta = 0.5$ and$\alpha^U = \alpha^L = 0.5$

Supply / Order	1st	2nd	3rd	4th
1st	(34,0,78,88)	(51,0,0,0)	**0**	**0**
2nd		(59,0,90,0)	(37,0,0,0)	**0**
3rd			(0,160,0,0)	**0**
4th				(113,0,0,150)

Table 7Optimal Periodic Allocation for Retailers (R1, R2, R3, R4)−Supply From Periods 5,6,7,8 with Confidence Level $\beta = 0.5$ and$\alpha^U = \alpha^L = 0.5$

Supply / Order	5th	6th	7th	8th
1st	**0**	**0**	**0**	**0**
2nd	**0**	**0**	**0**	**0**
3rd	**0**	**0**	**0**	**0**
4th	(47,0,0,0)	**0**	**0**	**0**
5th	(110,31,30,0)	(7,0,0,0)	(0,29,0,0)	**0**
6th		(60,70,20,300)	(56,0,0,0)	**0**
7th			(2,1,2,4)	**0**
8th				(81,95,0,88)

Table 8 Optimal Periodic Allocation for Retailers (R1, R2, R3, R4)−Supply From Periods 9,10,11,12,13 with Confidence Level $\beta = 0.5$ and$\alpha^U = \alpha^L = 0.5$

Supply / Order	9th	10th	11th	12th	13th
1st	**0**	**0**	**0**	**0**	**0**
2nd	**0**	**0**	**0**	**0**	**0**
3rd	**0**	**0**	**0**	**0**	**0**
4th	**0**	**0**	**0**	**0**	**0**
5th	**0**	**0**	**0**	**0**	**0**
6th	**0**	**0**	**0**	**0**	**0**
7th	**0**	**0**	**0**	**0**	**0**
8th	(4,0,0,0)	**0**	**0**	**0**	**0**
9th	(96,110,0,0)	**0**	**0**	**0**	**0**
10th		(140,160,0,0)	(10,0,0,0)	**0**	**0**
11th			(0,180,0,0)	(0,13,0,0)	**0**
12th				(110,56,30,250)	(0,4,0,0)
13th					(60,70,20,0)

V. CONCLUSION AND FUTURE RESEARCH

In this study, we consider a production planning problem with multiple retailers and multiple planning periods in a closed-loop system. We apply Fuzzy Set Theory to deal with uncertainty, which is induced from the ambiguity of retailer judgment, because demands are usually not deterministic at the beginning of a planning horizon. We develop the Fuzzy Production Mix Model according to Fuzzy Set Theory and employ the concept of fuzzy chance-constrained programming to construct Fuzzy Chance-Constrained Production Mix Model. Properties and parametric analysis of the FCCPMM are investigated. Therefore, by allocating right amount of products to right retails at right time, the satisfactory level of the retailers could reach to the highest.

Future research should consider that many companies produce multiple types of products, and the interrelationship between different products will become more complex. For example, the parts of one product may be used in another type of product. In such cases, modeling a multi-product production planning will be more useful in terms of reducing resource cost and easing environmental degradation.

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APPENDIX A

Let $\widetilde{D}_j^t = (\underline{d}_j^t, d_j^t, \overline{d}_j^t)$, where the membership function is

$$\mu_{\widetilde{D}_j^t}(x_j^t) = \begin{cases} if \ d_j^t \neq \underline{d}_j^t \neq \overline{d}_j^t : & \\ \dfrac{x_j^t - \underline{d}_j^t}{d_j^t - \underline{d}_j^t} & , if \ \underline{d}_j^t \le x_j^t < d_j^t \\ \dfrac{\overline{d}_j^t - x_j^t}{\overline{d}_j^t - d_j^t} & , if \ d_j^t \le x_j^t \le \overline{d}_j^t \\ 0 & , otherwise \\ else : & \\ 0 & , if \ x_j^t \neq d_j^t \\ 1 & , if \ x_j^t = d_j^t \end{cases}$$

the membership function of $\widetilde{D}_j^t \times s_j^t$ is

$$\mu_{\widetilde{D}_j^t \times s_j^t}(x_j^t) = \begin{cases} if\ s_j^t = 1\ and\ d_j^t \neq \underline{d}_j^t \neq \overline{d}_j^t : \\ \dfrac{x_j^t - \underline{d}_j^t}{d_j^t - \underline{d}_j^t} & , if\ \underline{d}_j^t \leq x_j^t < d_j^t \\ \dfrac{\overline{d}_j^t - x_j^t}{\overline{d}_j^t - d_j^t} & , if\ d_j^t \leq x_j^t \leq \overline{d}_j^t \\ 0 & , otherwise \\ if\ s_j^t = 1\ and\ d_j^t = \underline{d}_j^t = \overline{d}_j^t : \\ 0 & , if\ x_j^t \neq d_j^t \\ 1 & , if\ x_j^t = d_j^t \\ if\ s_j^t = 0 : \\ 0 & , if\ x_j^t \neq 0 \\ 1 & , if\ x_j^t = 0 \end{cases}$$

Now we are going to show the membership function of fuzzy addition of two fuzzy sets. Take two fuzzy sets, says $\widetilde{D}_j^t \times s_j^t$ and $\widetilde{D}_i^t \times s_i^t$, we deal with five different cases

Case 1: $s_k^t \neq 0$ and $\underline{d}_k^t \neq d_k^t \neq \bar{d}_k^t$ for k=j,t

In this situation, two fuzzy sets becomes fuzzy numbers, using $\alpha - level\ cut,$ we can calculate the fuzzy addition of these two fuzzy numbers:

(1) $\frac{x_k^t - \underline{d}_k^t}{d_k^t - \underline{d}_k^t} \geq \alpha \rightarrow x_k^t \geq \alpha(d_k^t - \underline{d}_k^t) + \underline{d}_k^t$, for k=j,i.

(2) $\frac{\bar{d}_k^t - x_k^t}{\bar{d}_k^t - d_k^t} \geq \alpha \rightarrow x_k^t \leq \bar{d}_k^t - \alpha(\bar{d}_k^t - d_k^t)$, for k=j,i.

Hence the range of x_k^t is $[\alpha(d_k^t - \underline{d}_k^t) + \underline{d}_k^t, \bar{d}_k^t - \alpha(\bar{d}_k^t - d_k^t)]$, and the range of interval addition is

$[\alpha \times \sum_{k\epsilon\{j,i\}}(d_k^t - \underline{d}_k^t) + \sum_{k\epsilon\{j,i\}} \underline{d}_k^t, \sum_{k\epsilon\{j,i\}} \bar{d}_k^t - \alpha \times \sum_{k\epsilon\{j,i\}}(\bar{d}_k^t - d_k^t)]$

Since $\alpha \in (0,1]$, this means that

$\sum_{k\epsilon\{j,i\}} \bar{d}_k^t - \alpha \times \sum_{k\epsilon\{j,i\}}(\bar{d}_k^t - d_k^t) = z$ for z $\in (\sum_{k\epsilon\{j,i\}} \underline{d}_k^t, \sum_{k\epsilon\{j,i\}}(d_k^t - \underline{d}_k^t) + \sum_{k\epsilon\{j,i\}} \underline{d}_k^t]$

and

$\sum_{k\epsilon\{j,i\}} \bar{d}_k^t - \alpha \times \sum_{k\epsilon\{j,i\}}(\bar{d}_k^t - d_k^t) = z$ for z $\in (\sum_{k\epsilon\{j,i\}} \bar{d}_k^t - \sum_{k\epsilon\{j,i\}}(\bar{d}_k^t - d_k^t), \sum_{k\epsilon\{j,i\}} \bar{d}_k^t]$

Therefore the membership function $\widetilde{D}_j^t \times s_j^t + \widetilde{D}_i^t \times s_i^t$ is

$$\mu_{\sum_{k\in\{j,i\}}^{J} \widetilde{D}_j^t s_j^t}(x) = \begin{cases} \dfrac{z - \sum_{k\in\{j,i\}} \underline{d}_j^t}{\sum_{k\in\{j,i\}}^{J} d_j^t - \sum_{k\in\{j,i\}}^{J} \underline{d}_j^t} & , if \sum_{k\in\{j,i\}} \underline{d}_j^t \leq z < \sum_{k\in\{j,i\}} d_j^t \\ \dfrac{\sum_{k\in\{j,i\}} \overline{d}_j^t - z}{\sum_{k\in\{j,i\}} \overline{d}_j^t - \sum_{k\in\{j,i\}}^{J} d_j^t} & , if \sum_{k\in\{j,i\}} d_j^t \leq z < \sum_{k\in\{j,i\}} \overline{d}_j^t \\ 0 & , otherwise \end{cases}$$

Case 2: $s_k^t \neq 0$ for k = j, t and $\underline{d}_j^t = d_j^t = \bar{d}_j^t$, but $\underline{d}_i^t \neq d_i^t \neq \bar{d}_i^t$

Since $\mu_{\widetilde{D}_j^t \times s_j^t}(x_j^t) = 1$ if $x_j^t = d_j^t$ and zero if $x_j^t \neq d_j^t$, by using extension principle it is obvious that the membership function of $\widetilde{D}_j^t \times s_j^t + \widetilde{D}_i^t \times s_i^t$ is

$$\mu_{\sum_{k\in\{j,i\}}^{J}\tilde{D}_j^t s_j^t}(z)=\begin{cases}\dfrac{z-\sum_{k\in\{j,i\}}\underline{d}_j^t}{\sum_{k\in\{j,i\}}^{J} d_j^t-\sum_{k\in\{j,i\}}^{J}\underline{d}_j^t} & ,if \sum_{k\in\{j,i\}}\underline{d}_j^t\le z<\sum_{k\in\{j,i\}} d_j^t\\ \dfrac{\sum_{k\in\{j,i\}}\overline{d}_j^t-z}{\sum_{k\in\{j,i\}}\overline{d}_j^t-\sum_{k\in\{j,i\}}^{J} d_j^t} & ,if \sum_{k\in\{j,i\}} d_j^t\le z<\sum_{k\in\{j,i\}}\overline{d}_j^t\\ 0 & ,otherwise\end{cases}$$

Case 3: $s_k^t \neq 0$ and $\underline{d}_k^t = d_k^t = \bar{d}_k^t$ for k=j,t

In this case, since $\mu_{\tilde{D}_k^t \times s_k^t}(x_k^t) = 1$ if $x_k^t = d_k^t$ and zero if $x_k^t \neq d_k^t$, by using extension principle the membership function of $\tilde{D}_j^t \times s_j^t + \tilde{D}_i^t \times s_i^t$ therefore is $\mu_{\sum_{k\in\{j,i\}}^{J}\tilde{D}_j^t s_j^t}(x)=\begin{cases}1 & ,if\ \ z=\sum_{k\in\{j,i\}}^{J} d_j^t\\ 0 & ,otherwise\end{cases}$

Case 4: $s_j^t = 0,\ s_i^t = 1$

Since $s_j^t = 0$, the membership function of $\tilde{D}_j^t \times s_j^t + \tilde{D}_i^t \times s_i^t$ is equals to the membership function of $\tilde{D}_i^t \times s_i^t$

Case 5: $s_k^t = 0$ for k=t,j

In this case, it is very trivial that zero is the only one number whose membership function is 1, and the remaining numbers are zeros.

Finally, by integrating these 5 cases, we get

$$\mu_{\sum_{k\in\{j,i\}}^{J}\tilde{D}_j^t s_j^t}(x)=\begin{cases} if \sum_{k\in\{j,i\}} s_k^t\neq 0, and\ \underline{d}_k^t\neq d_k^t\neq\overline{d}_k^t\ \ for\,k=t,j & \\ \dfrac{z-\sum_{k\in\{j,i\}}\underline{d}_j^t s_k^t}{\sum_{k\in\{j,i\}}^{J} d_j^t s_k^t-\sum_{k\in\{j,i\}}^{J}\underline{d}_j^t s_k^t} & ,if \sum_{k\in\{j,i\}}\underline{d}_j^t s_k^t\le z<\sum_{k\in\{j,i\}} d_j^t s_k^t\\ \dfrac{\sum_{k\in\{j,i\}}\overline{d}_j^t s_k^t-z}{\sum_{k\in\{j,i\}}\overline{d}_j^t s_k^t-\sum_{k\in\{j,i\}}^{J} d_j^t s_k^t} & ,if \sum_{k\in\{j,i\}} d_j^t s_k^t\le z<\sum_{k\in\{j,i\}}\overline{d}_j^t s_k^t\\ 0 & ,otherwise\\ if\ \sum_{k\in\{j,i\}} s_k^t\ and\ and\ \underline{d}_k^t= d_k^t=\overline{d}_k^t\ \ for\,k=t,j & \\ 1 & ,if\ \ z=\sum_{k\in\{j,i\}}\underline{d}_j^t s_k^t\\ 0 & ,if\ \ z\neq\sum_{k\in\{j,i\}}\underline{d}_j^t s_k^t\\ if\ \sum_{k\in\{j,i\}} s_k^t=0 & \\ 1 & ,if\ \ z=0\\ 0 & ,if\ \ z\neq 0\end{cases}$$

Q.E.D.

APPENDIX B

We know that $M_\lambda\left(P(\boldsymbol{v},\tilde{D}) \geq \overline{f}\right)$ is 1 within the first part of the formula, [1, λ] within the second part, [λ, 0] within the third part, and 0 for the fourth part.

Case 1:$\lambda < \beta$

In this case, the possible value of $M_\lambda\left(P(\boldsymbol{v},\tilde{D}) \geq \overline{f}\right)$ is

$$M_\lambda(P(v,\tilde{D}) \geq \overline{f}) = \begin{cases} 1 & ,if\ \overline{f} \leq REV\sum_{j=1}^{J}\sum_{t=1}^{T}\underline{d}_j^t \times s_j^t - h(u) \\ \dfrac{\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \lambda\underline{d}_j^t)\times s_j^t - (1-\lambda)\times\dfrac{f+h(u)}{REV}}{\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \underline{d}_j^t)\times s_j^t} & ,if\ REV\sum_{j=1}^{J}\sum_{t=1}^{T}\underline{d}_j^t \times s_j^t - h(u) < \overline{f} < REV\sum_{j=1}^{J}\sum_{t=1}^{T}d_j^t \times s_j^t - h(u) \end{cases}$$

ifβ is 1,due to maximization of $\overline{f}$, $\overline{f}$ is

$$\overline{f} = REV\sum_{j=1}^{J}\sum_{t=1}^{T}\underline{d}_j^t \times s_j^t - h(u)$$

ifβ is smaller than 1 (not equal to zero), then

$$M_\lambda(P(v,\tilde{D}) \geq \overline{f}) \geq \beta$$

$$=> \frac{\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \lambda\underline{d}_j^t)\times s_j^t - (1-\lambda)\times\dfrac{\overline{f}+h(u)}{REV}}{\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \underline{d}_j^t)\times s_j^t} \geq \beta$$

$$=> \overline{f} \leq \frac{REV(\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \lambda\underline{d}_j^t)\times s_j^t - \beta\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \underline{d}_j^t)\times s_j^t)}{1-\lambda} - h(u)$$

Due to maximization of $\overline{f}$

$$=> \overline{f} = \frac{REV(\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \lambda\underline{d}_j^t)\times s_j^t - \beta\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \underline{d}_j^t)\times s_j^t)}{1-\lambda} - h(u)$$

Case 2:$\lambda \geq \beta$

In this case, the possible value of $M_\lambda\left(P(\boldsymbol{v},\tilde{D}) \geq \overline{f}\right)$ could be any of four parts. However, due to maximization, the value would be

$$M_\lambda(P(v,\tilde{D}) \geq \overline{f}) = \lambda\frac{\sum_{j=1}^{J}\sum_{t=1}^{T}\overline{d}_j^t \times s_j^t - \dfrac{f+h(u)}{REV}}{\sum_{j=1}^{J}\sum_{t=1}^{T}(\overline{d}_j^t - d_j^t)\times s_j^t} = \beta$$

Hence the value of $\overline{f}$ is

$$\overline{f} = REV(\sum_{j=1}^{J}\sum_{t=1}^{T}\overline{d}_j^t \times s_j^t - \frac{\beta}{\lambda}\sum_{j=1}^{J}\sum_{t=1}^{T}(\overline{d}_j^t - d_j^t)\times s_j^t) - h(u)$$

Therefore, maximum $\overline{f}$ can be represented by:

$$\overline{f} = \begin{cases} \dfrac{REV(\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \lambda\underline{d}_j^t)\times s_j^t - \beta\sum_{j=1}^{J}\sum_{t=1}^{T}(d_j^t - \underline{d}_j^t)\times s_j^t)}{1-\lambda} - h(u), & if\ \lambda < \beta \\ REV(\sum_{j=1}^{J}\sum_{t=1}^{T}\overline{d}_j^t \times s_j^t - \dfrac{\beta}{\lambda}\sum_{j=1}^{J}\sum_{t=1}^{T}(\overline{d}_j^t - d_j^t)\times s_j^t) - h(u) & if\ \lambda \geq \beta \end{cases}$$ **Q.E.D.**