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An Inventory Model for Weibull Ameliorating, Deteriorating Items under the Influence of Inflation.

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Abstract
The objective of this model is to discuss the development of an inventory model for ameliorating items. Generally fast growing animals like duck, pigs, broiler etc. in poultry farm, high-bred fishes are these types of items. This paper investigates an instantaneous replenishment model for the above type of items under cost minimization in the influence of inflation and time value of money. A time varying type of demand rate with infinite time horizon, constant deterioration and without shortage is considered. The result is illustrated with numerical example.

Keywords: Amelioration, Weibull distribution, Optimal control, Inventory system
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I. INTRODUCTION

In many inventory systems, deterioration of goods is a realistic phenomenon. Perishable items widely exist in our daily life, such as fresh products, fruits, vegetables, seafood, etc, which decrease in quantity or utility during their delivery and storage stage periods. To improve products quality and reduce deteriorating loss in the fashion goods supply chain, the emphasis is on the whole process life cycle management which includes production, storage, transportation, retailing etc. Many goods are subject to deterioration or decay during their normal storage period. Hence while determining the optimal inventory policy of such type of product, the loss due to deterioration has to be considered.

Whitin[18] considered an inventory model for fashion goods deteriorating at the end of a prescribed storage period. Ghare and Scharder [6] developed an EOQ model with an exponential decay and a deterministic demand. Thereafter, Covert and Philip[3] and Philip[12] extended EOQ (Economic Order Quantity) models for deterioration which follows Weibull distribution. Wee[17] developed EOQ models to allow deterioration and an exponential demand pattern. In last two decades the economic situation of most countries have changed to an extent due to sharp decline in the purchasing power of money, that it has not been possible to ignore the effects of time value of money. Data and Pal[5], Bose et al.[2] have developed the EOQ model incorporating the effects of time value of money, a linear time dependent demand rate. Further Sana [13] considered the money value and inflation in a new level.

In the existing literature, practitioners did not give much attention for fast growing animals like broiler, ducks, pigs etc. in the poultry farm, highbred fishes in bhery (pond) which are known as ameliorating items. When these items are in storage, the stock increases (in weight) due to growth of the items and also decrease due to death, various diseases or some other factors. Till now, only Hwang [8,9] reported this type of inventory model. Again, in 1999, Hwang [9] developed inventory models for both ameliorating and deteriorating items separately under the issuing policies, LIFO (Last input Fast output) and FIFO (First input First output).

In the present competitive market, the effect if marketing policies and conditions such as the price variations and advertisement of an item changes its selling rate amongst the public. In selecting of an item for use, the selling price of an item is one of the decisive factors to the customers. It is commonly seen that lesser selling price causes increases in the selling rate whereas higher selling price has the reverse effect. Hence, the selling rate of an item is dependent on the selling price of that item. This selling rate function must be a decreasing function with respect to the selling price. Incorporating the price variations, recently several researchers i.e. Urban [16], Ladany and Sternleib[10], Subramanyam amd kumaraswamy[14], Goyal and Gunasekaran[7], Bhunia and Maiti[1], Luo[11], and Das et.al [4] developed their models for deteriorating and non-deteriorating items. R.P.Tripathi[15] developed the model under different demand rate and holding cost.

In the paper, an economic order quantity model is developed for both the ameliorating and deteriorating items for time varying demand rate. Here the backlogging rate is assumed to be variable and dependent on the waiting time for the next replenishment. We consider the time–value of money and inflation of each cost parameter. The time horizon is classified into two intervals. In the 1st interval the given stock is decreased to zero level due to the combined effect of amelioration, deterioration and demand. In the next interval the shortages are allowed up to the time where some of the shortages are backlogged and rest are lost.

II. ASSUMPTIONS AND NOTATIONS

Following assumptions are made for the proposed model:

i. A time varying type of demand rate is used.
ii. Single inventory will be used.
iii. Lead time is zero.
iv. Shortages are allowed and partially backlogged with the backlogging rate $\frac{1}{1+\delta(T-t)}$ where the backlogging parameter δ is a non-negative constant.
v. Replenishment rate is infinite but size is finite.
vi. Time horizon is finite.
vii. There is no repair of deteriorated items occurring during the cycle.
viii. Amelioration and deterioration occur when the item is effectively in stock.
ix. The time-value of money and inflation are considered.

Following notations are made for the given model:

$I(t)$ = On hand inventory at time t.
$R(t)=\lambda_0\, t^{-\beta_1}$ = Time varying demand rate where $\lambda_0>0$ .and $0<\beta_1<1.$
$\theta =$ The constant deterioration rate where $0\le\theta<1$.
$I(0)$ =Inventory at time $t=0$.
$Q=$ On-hand inventory.
$T=$ Duration of a cycle.
$A(t)$ = Instantaneous rate of amelioration of the on-hand inventory given by $\alpha\beta\cdot t^{\beta-1}$ where $0<\alpha<<1, \beta>0$.
$i=$ The inflation rate per unit time.
$r=$ The discount rate representing the time value of money.
a_c = Cost of amelioration per unit.
$p_c=$ The purchasing cost per unit item.
$d_c=$ The deterioration cost per unit item.
$o_c=$ The opportunity cost per unit item.
$h_c=$ The holding cost per unit item.
$b_c=$ The shortage cost per unit item.

III. FORMULATION

The aim of this model is to optimize the total cost incurred and to determine the optimal ordering level.In the interval $[0,t_1]$ the stock will be decreased due to the effect of amelioration, deterioration and demand. At time t_1, the inventory level reaches zero and in the next interval the shortage are allowed up to time where some of the shortage are backlogged and rest are lost. Only backlogged items are replaced in the next lot.

If $I(t)$ be the on hand inventory at time $t\ge 0$, then at time $t+\Delta t$, the on hand inventory in the interval $[0,t_1]$ will be

$$I(t+\Delta t)=$$
$$I(t)+A.I(t).\Delta t-\theta\, I(t).\Delta t-\lambda_0\, t^{-\beta_1}.\Delta t$$

Dividing by Δt and then taking as $\Delta t\to 0$ we get

(3.1)
$$\frac{dt}{dt}=\alpha\beta\, t^{\beta-1}I(t)-\theta I(t)-\lambda_0\, t^{-\beta_1} \quad \text{for } 0\le t\le t_1.$$

In the end interval , $[t_1,T]$

$$I(t+\Delta t)=I(t)-\frac{\lambda_0\, t^{-\beta_1}}{1+\delta(T-t)}.\Delta t$$

Dividing by Δt and then taking as $\Delta t\to 0$, we get,

(3.2)
$$\frac{dI}{dt}=-\frac{\lambda_0\, t^{-\beta_1}}{1+\delta(T-t)}, t_1\le t\le T .$$

Now solving equation **(3.1)**with boundary condition $I(t_1)=0$

(3.3)
$$I(t)=\lambda_0\, e^{(\alpha\, t^{\beta}-\theta\, t)}\left[\frac{\left\{t_1^{1-\beta_1}-t^{1-\beta_1}\right\}}{1-\beta_1}+\frac{\theta}{2-\beta_1}\left\{t_1^{2-\beta_1}-t^{2-\beta_1}\right\}-\frac{\alpha}{1+\beta-\beta_1}\left\{t_1^{1+\beta-\beta_1}-t^{1+\beta-\beta_1}\right\}\right]$$
for $0\le t\le t_1$.

On solving equation **(3.2)** with boundary condition $I(t_1)=0$

(3.4) $$I(t)= \lambda_0\left[\frac{(1-\delta T)}{1-\beta_1}\left\{t_1^{1-\beta_1}-t^{1-\beta_1}\right\}+\frac{\delta}{2-\beta_1}\left\{t_1^{2-\beta_1}-t^{2-\beta_1}\right\}\right] \quad \text{for } t_1 \le t \le T .$$

Form equation **(3.3)**, we obtain the initial inventory level.

(3.5) $$I(0)=\lambda_0\left[\frac{t_1^{1-\beta_1}}{1-\beta_1}+\frac{\theta}{2-\beta_1}t_1^{2-\beta_1}-\frac{\alpha}{1+\beta-\beta_1}t_1^{1+\beta-\beta_1}\right].$$

The total inventory holding during the time interval $[0,t_1]$ is given by,

(3.6) $$I_T=\int_0^{t_1} I\,dt$$

$$=\lambda_0\left\{\frac{t_1^{1-\beta_1}}{1-\beta_1}+\frac{\theta}{2-\beta_1}t_1^{2-\beta_1}-\frac{\alpha}{1+\beta-\beta_1}t_1^{1+\beta-\beta_1}\right\}.\left\{t_1+\frac{\alpha}{\beta+1}t_1^{\beta+1}-\frac{\theta}{2}t_1^{2}\right\}$$

$$-\lambda_0\left[\frac{t_1^{2-\beta_1}}{(1-\beta_1)(2-\beta_1)}-\frac{\theta}{(2-\beta_1)}t_1^{3-\beta_1}+\frac{\alpha\,\beta}{(1-\beta_1)(1+\beta-\beta_1)(2+\beta-\beta_1)}t_1^{2+\beta-\beta_1}\right.$$

$$\left.\frac{\alpha\,\theta}{(2-\beta_1)(3+\beta-\beta_1)}t_1^{3+\beta-\beta_1}-\frac{\alpha^{2}}{(1+\beta-\beta_1)(2+2\beta-\beta_1)}t_1^{2+2\beta-\beta_1}\right].$$

From of equation **(3.4)** amount of shortage during the time interval $[t_1,T]$ is

(3.7) $$B_T=\int_{t_1}^{T} -I\,dt$$

$$=-\lambda_0\left[\frac{(1-\delta T)}{(1-\beta_1)(2-\beta_1)}\left\{T^{2-\beta_1}-t_1^{2-\beta_1}\right\}+\frac{\delta}{(2-\beta_1)(3-\beta_1)}\left\{T^{3-\beta_1}-t_1^{3-\beta_1}\right\}\right.$$

$$\left.-\left\{\frac{(1-\delta T)}{(1-\beta_1)}t_1^{1-\beta_1}+\frac{\delta}{(2-\beta_1)}t_1^{2-\beta_1}\right\}(T-t_1)\right]$$

The amount of lost sell during the interval $[t_1,T]$ in given by,

(3.8) $$L_T=\int_{t_1}^{T} R[1-\frac{1}{1+\delta(T-t)}]dt$$

$$=\lambda_0\,\delta\left[\frac{T}{(1-\beta_1)}\left\{T^{1-\beta_1}-t_1^{1-\beta_1}\right\}-\frac{1}{(2-\beta_1)}\left\{T^{2-\beta_1}-t_1^{2-\beta_1}\right\}\right]$$

During the inventory cycle, generally the deteriorated units are rejected. The total number of deteriorated units during the inventory cycle is given by,

(3.9) $$D_T=\theta\int_0^{t_1} I(t)dt$$

$$=\lambda_0\,\theta\left\{\frac{t_1^{1-\beta_1}}{1-\beta_1}+\frac{\theta}{2-\beta_1}t_1^{2-\beta_1}-\frac{\alpha}{1+\beta-\beta_1}t_1^{1+\beta-\beta_1}\right\}.\left\{t_1+\frac{\alpha}{\beta+1}t_1^{\beta+1}-\frac{\theta}{2}t_1^{2}\right\}$$

$$-\lambda_0\,\theta\left[\frac{t_1^{2-\beta_1}}{(1-\beta_1)(2-\beta_1)}-\frac{\theta}{(2-\beta_1)}t_1^{3-\beta_1}+\frac{\alpha\,\beta}{(1-\beta_1)(1+\beta-\beta_1)(2+\beta-\beta_1)}t_1^{2+\beta-\beta_1}\right.$$

$$\left.\frac{\alpha\,\theta}{(2-\beta_1)(3+\beta-\beta_1)}t_1^{3+\beta-\beta_1}-\frac{\alpha^{2}}{(1+\beta-\beta_1)(2+2\beta-\beta_1)}t_1^{2+2\beta-\beta_1}\right].$$

The number of ameliorating units over the inventory cycle is given by,

(3.10) $A_T = \int_0^{t_1} \alpha\beta t^{\beta-1} I(t)dt$

$$= \lambda_0\, \alpha\, \beta \left\{ \frac{t_1^{1-\beta_1}}{1-\beta_1} + \frac{\theta}{2-\beta_1} t_1^{2-\beta_1} - \frac{\alpha}{1+\beta-\beta_1} t_1^{1+\beta-\beta_1} \right\} \cdot \left\{ \frac{t_1^{\beta}}{\beta} - \frac{\theta\, t_1^{\beta+1}}{\beta+1} \right\}$$

$$-\lambda_0 \alpha\, \beta \left\{ \frac{t_1^{\beta-\beta_1+1}}{(1-\beta)(\beta-\beta_1+1)} + \frac{\theta}{(2-\beta)(2+\beta-\beta_1)} t_1^{2+\beta-\beta_1} \right\}$$

Using the above equations into consideration the different costs under the influence of inflation and time-value of money will be as follows.

1. **Purchasing cost per cycle**

(3.11) $p_c I(0) \int_0^{T} e^{-(r-i)t} dt$

$$= \lambda_0 \frac{p_c\left[1-e^{-(r-i)T}\right]}{r-i} \left[\frac{t_1^{1-\beta_1}}{1-\beta_1} + \frac{\theta}{2-\beta_1} t_1^{2-\beta_1} - \frac{\alpha}{1+\beta-\beta_1} t_1^{1+\beta-\beta_1} \right]$$

2. **Holding cost per cycle**

(3.12) $h_c \int_0^{t_1} I(t) e^{-(r-i)t} dt$

$$= \lambda_0\, h_c \left\{ \frac{t_1^{1-\beta_1}}{1-\beta_1} + \frac{\theta}{2-\beta_1} t_1^{2-\beta_1} - \frac{\alpha}{1+\beta-\beta_1} t_1^{1+\beta-\beta_1} \right\} \cdot \left\{ t_1 + \alpha \frac{t_1^{\beta+1}}{\beta+1} - \frac{\theta\, t_1^{2}}{2} \right\}$$

$$-\lambda_0\, h_c \left\{ \frac{t_1^{2-\beta_1}}{(1-\beta_1)(2-\beta_1)} + \frac{\theta\, t_1^{3-\beta_1}}{(2-\beta_1)(3-\beta_1)} + \frac{\alpha\, \beta\, t_1^{2+\beta-\beta_1}}{(1-\beta_1)(1+\beta-\beta_1)(2+\beta-\beta_1)} + \frac{\alpha\, \theta\, t_1^{3+\beta-\beta_1}}{(2-\beta_1)(3+\beta-\beta_1)} \right\}$$

$$-(r-i)\, \lambda_0\, h_c \left\{ \frac{t_1^{1-\beta_1}}{1-\beta_1} + \frac{\theta}{2-\beta_1} t_1^{2-\beta_1} - \frac{\alpha}{1+\beta-\beta_1} t_1^{1+\beta-\beta_1} \right\} \cdot \left\{ \frac{t_1^{2}}{2} + \alpha \frac{t_1^{\beta+2}}{\beta+2} - \frac{\theta\, t_1^{3}}{3} \right\}$$

$$+(r-i)\lambda_0\, h_c \left\{ \frac{t_1^{3-\beta_1}}{(1-\beta_1)(3-\beta_1)} + \frac{\theta\, t_1^{4-\beta_1}}{(2-\beta_1)(4-\beta_1)} + \frac{\alpha\, \beta\, t_1^{3+\beta-\beta_1}}{(1-\beta_1)(1+\beta-\beta_1)(3+\beta-\beta_1)} + \frac{\alpha\, \theta\, t_1^{4+\beta-\beta_1}}{(2-\beta_1)(4+\beta-\beta_1)} \right\}$$

3. **Deterioration cost per cycle**

(3.13) $d_c \int_0^{t_1} \theta I(t) e^{-(r-i)t} dt$

$$= \lambda_0\, d_c\, \theta \left\{ \frac{t_1^{1-\beta_1}}{1-\beta_1} + \frac{\theta}{2-\beta_1} t_1^{2-\beta_1} - \frac{\alpha}{1+\beta-\beta_1} t_1^{1+\beta-\beta_1} \right\} \cdot \left\{ t_1 + \alpha \frac{t_1^{\beta+1}}{\beta+1} - \frac{\theta\, t_1^{2}}{2} \right\}$$

$$-\lambda_0\, d_c\, \theta \left\{ \frac{t_1^{2-\beta_1}}{(1-\beta_1)(2-\beta_1)} + \frac{\theta\, t_1^{3-\beta_1}}{(2-\beta_1)(3-\beta_1)} + \frac{\alpha\, \beta\, t_1^{2+\beta-\beta_1}}{(1-\beta_1)(1+\beta-\beta_1)(2+\beta-\beta_1)} + \frac{\alpha\, \theta\, t_1^{3+\beta-\beta_1}}{(2-\beta_1)(3+\beta-\beta_1)} \right\}$$

$$-(r-i)\, \lambda_0\, d_c\, \theta \left\{ \frac{t_1^{1-\beta_1}}{1-\beta_1} + \frac{\theta}{2-\beta_1} t_1^{2-\beta_1} - \frac{\alpha}{1+\beta-\beta_1} t_1^{1+\beta-\beta_1} \right\} \cdot \left\{ \frac{t_1^{2}}{2} + \alpha \frac{t_1^{\beta+2}}{\beta+2} - \frac{\theta\, t_1^{3}}{3} \right\}$$

$$+(r-i)\lambda_0\, d_c\, \theta\left\{\frac{t_1^{3-\beta_1}}{(1-\beta_1)(3-\beta_1)}+\frac{\theta\, t_1^{4-\beta_1}}{(2-\beta_1)(4-\beta_1)}+\frac{\alpha\,\beta\, t_1^{3+\beta-\beta_1}}{(1-\beta_1)(1+\beta-\beta_1)(3+\beta-\beta_1)}+\frac{\alpha\,\theta\, t_1^{4+\beta-\beta_1}}{(2-\beta_1)(4+\beta-\beta_1)}\right\}$$

4. **Amelioration cost per cycle**

(3.14) $$a_c\int_0^{t_1}\alpha\beta t^{\beta-1}I(t)e^{-(r-i)t}dt$$

$$=\lambda_0\, a_c\,\alpha\,\beta\left\{\frac{t_1^{1-\beta_1}}{1-\beta_1}+\frac{\theta}{2-\beta_1}t_1^{2-\beta_1}-\frac{\alpha}{1+\beta-\beta_1}t_1^{1+\beta-\beta_1}\right\}\cdot\left\{\frac{t_1^{\beta}}{\beta}-\frac{\theta\, t_1^{\beta+1}}{\beta+1}\right\}$$

$$-\lambda_0\, a_c\,\alpha\,\beta\left\{\frac{t_1^{1+\beta-\beta_1}}{(1-\beta_1)(1+\beta-\beta_1)}+\frac{\theta\, t_1^{2+\beta-\beta_1}}{(2-\beta_1)(2+\beta-\beta_1)}\right\}$$

$$-\lambda_0\,(r-i)a_c\,\alpha\,\beta\left\{\frac{t_1^{1-\beta_1}}{1-\beta_1}+\frac{\theta}{2-\beta_1}t_1^{2-\beta_1}-\frac{\alpha}{1+\beta-\beta_1}t_1^{1+\beta-\beta_1}\right\}\cdot\left\{\frac{t_1^{\beta+1}}{\beta+1}-\frac{\theta\, t_1^{\beta+2}}{\beta+2}\right\}$$

$$-\lambda_0(r-i)\, a_c\,\alpha\,\beta\left\{\frac{t_1^{2+\beta-\beta_1}}{(1-\beta_1)(2+\beta-\beta_1)}+\frac{\theta\, t_1^{3+\beta-\beta_1}}{(2-\beta_1)(3+\beta-\beta_1)}\right\}$$

5. **Shortage cost per cycle**

(3.15) $$-b_c\int_{t_1}^{T}I(t)e^{-(r-i)t}dt$$

$$=\lambda_0\, b_c\,\frac{(1-\delta T)}{(1-\beta_1)(2-\beta_1)}\left\{T^{2-\beta_1}-t_1^{2-\beta_1}\right\}+\frac{\lambda_0\, b_c\delta}{(2-\beta_1)(3-\beta_1)}\left\{T^{3-\beta_1}-t_1^{3-\beta_1}\right\}+\left\{\frac{\lambda_0\, b_c(1-\delta T)\, t_1^{2-\beta_1}}{(1-\beta_1)}+\frac{\lambda_0\, b_c\delta\, t_1^{2-\beta_1}}{(2-\beta_1)}\right\}(T-t_1)$$

$$-(r-i)\left\{\frac{\lambda_0\, b_c(1-\delta T)}{(1-\beta_1)(3-\beta_1)}\left\{T^{3-\beta_1}-t_1^{3-\beta_1}\right\}\backslash-\frac{\lambda_0\, b_c\delta}{(2-\beta_1)(4-\beta_1)}\left\{T^{4-\beta_1}-t_1^{4-\beta_1}\right\}+\frac{1}{2}\left\{\frac{\lambda_0\, b_c(1-\delta T)\, t_1^{2-\beta_1}}{(1-\beta_1)}+\frac{\lambda_0\, b_c\delta\, t_1^{2-\beta_1}}{(2-\beta_1)}\right\}(T^2-t_1^2)\right\}$$

6. **Opportunity cost due to lost sales per cycle**

(3.16) $$o_c\int_{t_1}^{T}R\left[1-\frac{1}{1+\delta(T-t)}\right]e^{-(r-i)t}dt$$

$$=\lambda_0\, o_c\,\delta\left[\frac{T}{1-\beta_1}\left\{T^{1-\beta_1}-t_1^{1-\beta_1}\right\}-\frac{1}{2-\beta_1}\left\{T^{2-\beta_1}-t_1^{2-\beta_1}\right\}\right.$$
$$\left.-(r-i)\left\{\frac{T}{2-\beta_1}\left\{T^{2-\beta_1}-t_1^{2-\beta_1}\right\}-\frac{1}{3-\beta_1}\left\{T^{3-\beta_1}-t_1^{3-\beta_1}\right\}\right\}\right]$$

The average total cost per unit time of the model will be

(3.17) $$C(t_1)=\frac{1}{T}\left[\lambda_0\,\frac{p_c\left[1-e^{-(r-i)T}\right]}{r-i}\left[\frac{t_1^{1-\beta_1}}{1-\beta_1}+\frac{\theta}{2-\beta_1}t_1^{2-\beta_1}-\frac{\alpha}{1+\beta-\beta_1}t_1^{1+\beta-\beta_1}\right]\right.$$

$$+\lambda_0\,(h_c+d_c\,\theta)\left\{\frac{t_1^{1-\beta_1}}{1-\beta_1}+\frac{\theta}{2-\beta_1}t_1^{2-\beta_1}-\frac{\alpha}{1+\beta-\beta_1}t_1^{1+\beta-\beta_1}\right\}\cdot\left\{t_1+\alpha\frac{t_1^{\beta+1}}{\beta+1}-\frac{\theta\, t_1^{2}}{2}\right\}$$

$$-\lambda_0\,(h_c+d_c\,\theta)\left\{\frac{t_1^{2-\beta_1}}{(1-\beta_1)(2-\beta_1)}+\frac{\theta\, t_1^{3-\beta_1}}{(2-\beta_1)(3-\beta_1)}+\frac{\alpha\,\beta\, t_1^{2+\beta-\beta_1}}{(1-\beta_1)(1+\beta-\beta_1)(2+\beta-\beta_1)}+\frac{\alpha\,\theta\, t_1^{3+\beta-\beta_1}}{(2-\beta_1)(3+\beta-\beta_1)}\right\}$$

$$-(r-i)\,\lambda_0\,(h_c+d_c\,\theta)\left\{\frac{t_1^{1-\beta_1}}{1-\beta_1}+\frac{\theta}{2-\beta_1}t_1^{2-\beta_1}-\frac{\alpha}{1+\beta-\beta_1}t_1^{1+\beta-\beta_1}\right\}.\left\{\frac{t_1^2}{2}+\alpha\frac{t_1^{\beta+2}}{\beta+2}-\frac{\theta\,t_1^3}{3}\right\}$$

$$+(r-i)\lambda_0\,(h_c+d_c\,\theta)\left\{\frac{t_1^{3-\beta_1}}{(1-\beta_1)(3-\beta_1)}+\frac{\theta\,t_1^{4-\beta_1}}{(2-\beta_1)(4-\beta_1)}+\frac{\alpha\,\beta\,t_1^{3+\beta-\beta_1}}{(1-\beta_1)(1+\beta-\beta_1)(3+\beta-\beta_1)}+\frac{\alpha\,\theta\,t_1^{4+\beta-\beta_1}}{(2-\beta_1)(4+\beta-\beta_1)}\right\}$$

$$+\lambda_0\,a_c\,\alpha\,\beta\left\{\frac{t_1^{1-\beta_1}}{1-\beta_1}+\frac{\theta}{2-\beta_1}t_1^{2-\beta_1}-\frac{\alpha}{1+\beta-\beta_1}t_1^{1+\beta-\beta_1}\right\}.\left\{\frac{t_1^{\beta}}{\beta}-\frac{\theta\,t_1^{\beta+1}}{\beta+1}\right\}$$

$$-\lambda_0\,a_c\,\alpha\,\beta\left\{\frac{t_1^{1+\beta-\beta_1}}{(1-\beta_1)(1+\beta-\beta_1)}+\frac{\theta\,t_1^{2+\beta-\beta_1}}{(2-\beta_1)(2+\beta-\beta_1)}\right\}$$

$$-\lambda_0\,(r-i)a_c\,\alpha\,\beta\left\{\frac{t_1^{1-\beta_1}}{1-\beta_1}+\frac{\theta}{2-\beta_1}t_1^{2-\beta_1}-\frac{\alpha}{1+\beta-\beta_1}t_1^{1+\beta-\beta_1}\right\}.\left\{\frac{t_1^{\beta+1}}{\beta+1}-\frac{\theta\,t_1^{\beta+2}}{\beta+2}\right\}$$

$$-\lambda_0(r-i)\,a_c\,\alpha\,\beta\left\{\frac{t_1^{2+\beta-\beta_1}}{(1-\beta_1)(2+\beta-\beta_1)}+\frac{\theta\,t_1^{3+\beta-\beta_1}}{(2-\beta_1)(3+\beta-\beta_1)}\right\}$$

$$+\lambda_0\,b_c\,\frac{(1-\delta T)}{(1-\beta_1)(2-\beta_1)}\left\{T^{2-\beta_1}-t_1^{2-\beta_1}\right\}+\frac{\lambda_0\,b_c\delta}{(2-\beta_1)(3-\beta_1)}\left\{T^{3-\beta_1}-t_1^{3-\beta_1}\right\}+\left\{\frac{\lambda_0\,b_c(1-\delta T)\,t_1^{2-\beta_1}}{(1-\beta_1)}+\frac{\lambda_0\,b_c\delta\,t_1^{2-\beta_1}}{(2-\beta_1)}\right\}(T-t_1)$$

$$-(r-i)\left\{\frac{\lambda_0\,b_c(1-\delta T)}{(1-\beta_1)(3-\beta_1)}\left\{T^{3-\beta_1}-t_1^{3-\beta_1}\right\}-\frac{\lambda_0\,b_c\delta}{(2-\beta_1)(4-\beta_1)}\left\{T^{4-\beta_1}-t_1^{4-\beta_1}\right\}+\frac{1}{2}\left\{\frac{\lambda_0\,b_c(1-\delta T)\,t_1^{2-\beta_1}}{(1-\beta_1)}+\frac{\lambda_0\,b_c\delta\,t_1^{2-\beta_1}}{(2-\beta_1)}\right\}(T^2-t_1^2)\right\}$$

$$+\lambda_0\,o_c\,\delta\left[\frac{T}{1-\beta_1}\left\{T^{1-\beta_1}-t_1^{1-\beta_1}\right\}-\frac{1}{2-\beta_1}\left\{T^{2-\beta_1}-t_1^{2-\beta_1}\right\}-(r-i)\left\{\frac{T}{2-\beta_1}\left\{T^{2-\beta_1}-t_1^{2-\beta_1}\right\}-\frac{1}{3-\beta_1}\left\{T^{3-\beta_1}-t_1^{3-\beta_1}\right\}\right\}\right]\Bigg]$$

As it is difficult to solve the problem by deriving a closed equation of the solution of equation (3.17), Matlab Software has been used to determine optimal t_1^* and hence the optimal $I(0)$, the minimum average total cost per unit time can be determined.

IV. NUMERICAL EXAMPLE

Following example is considered to illustrate the preceding theory.

Example

The values of the parameters are considered as follows:

$r=0.02, i=0.38, \theta=0.2, \delta=0.1, T=1\,Year,$

$\lambda_0=200,\ \beta_1=0.7, \alpha=0.7, \beta=0.6,$

$a_c=\$6/unit, h_c=\$4/unit/year, p_c=\$15/unit,.$

$d_c=\$9/unit, o_c=\$12/unit, b_c=\$10/unit$

According to equation (3.17), we obtain the optimal $t_1^*=0.045$ Year. In addition, the optimal $I^*(0)=262.679$ units. Moreover, from equation (3.17), we have the minimum average total cost per unit time as $C^*=138.09$ \$.

V. CONCLUSION

Here we have derived an inventory model for some special type of items like mushroom & fishes with both amelioration and deterioration. In particular amelioration is considered to be of Weibull type and constant deterioration is used under time varying demand condition. In this model, shortages are allowed and in the shortage period the backlogging rate is variable which depends on the length of the waiting time for the next replenishment. An optimal replenishment policy is derived with minimization of average total cost under the influence of inflation and time-value of money. The result is illustrated through numerical example. The model can further be studied for different demand situations and for multiple items under identical conditions.

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