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Homomorphism and Anti Homomorphism on a Bipolar Anti Fuzzy Subgroup

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ABSTRACT
In this paper, we introduce the concept of an anti image, anti pre-image of a bipolar fuzzy subgroup of a group G and discuss in detail a series of homomorphic and anti homomorphic properties of bipolar fuzzy and bipolar anti fuzzy subgroup.
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I. Introduction

The concept of fuzzy sets was initiated by Zadeh [16]. Then it has become a vigorous area of research in engineering, medical science, social science, graph theory etc. Rosenfeld [12] gave the idea of fuzzy subgroup. In fuzzy sets the membership degree of elements range over the interval [0,1]. The membership degree expresses the degree of belongingness of elements to a fuzzy set. The membership degree 1 indicates that an element completely belongs to its corresponding fuzzy set and membership degree 0 indicates that an element does not belong to fuzzy set. The membership degrees on the interval (0,1) indicate the partial membership to the fuzzy set. Sometimes, the membership degree means the satisfaction degree of elements to some property or constraint corresponding to a fuzzy set. The author W.R.Zhang [14],[15] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. In case of bipolar-valued fuzzy sets membership degree range is enlarged from the interval [0, 1] to [-1, 1]. In a bipolar-valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the membership degree (0,1] indicates that elements somewhat satisfy the property and the membership degree [-1,0) indicates that elements somewhat satisfy the implicit counter-property. M. Marudai, V.Rajendran [5] introduced the pre-image of bipolar Q fuzzy subgroup. In this paper we redefined the concept of a pre-image of a bipolar fuzzy subgroup and introduce the concept of an image , anti image and ,anti pre-image of a bipolar fuzzy subgroup and discuss some of its properties with bipolar anti fuzzy subgroup.

II. Preliminaries

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, G = (G , *) be a finite group, e is the identity element of G, xy we mean x * y.

Definition 2.1 [1]

Let X be any non-empty set. A fuzzy subset μ of X is a function $\mu : X \to [0,1]$.

Definition 2.2 [9]

Let G be a non-empty set. A bipolar-valued fuzzy set or bipolar fuzzy set μ in G is an object having the form $\mu = \{\langle x, \mu^+(x), \mu^-(x) \rangle \,/\, \text{for all } x \in G\}$,where $\mu^+ : G \to [0,1]$ and $\mu^- : G \to [-1,0]$ are mappings. The positive membership degree $\mu^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar-valued fuzzy set $\mu = \{\langle x, \mu^+(x), \mu^-(x)\rangle \,/\, \text{for all } x \in G\}$ and the negative membership degree $\mu^-(x)$ denotes the satisfaction degree of an element x to some implicit counter property corresponding to a bipolar-valued fuzzy set $\mu = \{\langle x, \mu^+(x), \mu^-(x) \rangle \,/\, \text{for all } x \in G\}$. If $\mu^+(x) \neq 0$ and $\mu^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for $\mu = \{\langle x, \mu^+(x), \mu^-(x)\rangle \,/\, \text{for all } x \in G\}$. If $\mu^+(x) = 0$ and $\mu^-(x) \neq 0$, it is the situation that x does not satisfy the property of $\mu = \{\langle x, \mu^+(x), \mu^-(x)\rangle \,/\, \text{for all } x \in G\}$, but somewhat satisfies the counter property of $\mu = \{\langle x, \mu^+(x), \mu^-(x)\rangle \,/\, \text{for all } x \in G\}$. It is possible for an element x to be such that $\mu^+(x) \neq 0$ and $\mu^-(x) \neq 0$ when the membership function of property overlaps that its counter property over some portion of G. For the sake of simplicity, we shall use the symbol $\mu = (\mu^+, \mu^-)$ for the bipolar-valued fuzzy set $\mu = \{\langle x, \mu^+(x), \mu^-(x)\rangle / \text{ for all } x\in G\}$.

Definition 2.3[9]

Let G be a group. A bipolar-valued fuzzy set or bipolar fuzzy set μ in G is a bipolar fuzzy subgroup of G if for all x, y $\in$G,

i. $\mu^+(xy) \geq \min\{\mu^+(x), \mu^+(y)\}$,
ii. $\mu^-(xy) \leq \max\{\mu^-(x), \mu^-(y)\}$,
iii. $\mu^+(x^{-1}) = \mu^+(x)$, $\mu^-(x^{-1}) = \mu^-(x)$.

Definition 2.4[9]

Let G be a group. A bipolar-valued fuzzy set or bipolar fuzzy set μ in G is a bipolar anti fuzzy subgroup of G if for all x, y $\in$G,

i. $\mu^+(xy) \geq \min\{\mu^+(x), \mu^+(y)\}$,
ii. $\mu^-(xy) \leq \max\{\mu^-(x), \mu^-(y)\}$,
iii. $\mu^+(x^{-1}) = \mu^+(x)$, $\mu^-(x^{-1}) = \mu^-(x)$.

Definition 2.5[7]

A mapping f from a group G_1 to a group G_2 is said to be a homomorphism if f (xy) = f(x) f(y) for all x,y $\in G_1$.

Definition 2.6[7]

A mapping f from a group G_1 to a group G_2 (G_1 and G_2 are not necessarily commutative) is said to be an anti homomorphism if f (xy) = f(y) f(x) for all x,y $\in G_1$.

Theorem 2.1

Let μ be a bipolar fuzzy set of G , then μ is a bipolar anti fuzzy subgroup of G if and only if μ^c is a bipolar fuzzy subgroup of G.

Proof

Let $\mu = (\mu^+, \mu^-)$ be a bipolar anti fuzzy subgroup of G .Then for each x,y $\in$ G

Now

i. $\mu^+(xy) \leq \max\{\mu^+(x), \mu^+(y)\}$
$\Leftrightarrow 1-(\mu^+)^c(xy) \leq \max\{1-(\mu^+)^c(x), 1-(\mu^+)^c(y)\}$
$\Leftrightarrow (\mu^+)^c(xy) \geq 1-\max\{1-(\mu^+)^c(x), 1-(\mu^+)^c(y)\}$
$\Leftrightarrow (\mu^+)^c(xy) \geq \min\{(\mu^+)^c(x), (\mu^+)^c(y)\}$

ii. $\mu^-(xy) \geq \min\{\mu^-(x), \mu^-(y)\}$
$\Leftrightarrow -1-(\mu^-)^c(xy) \geq \min\{-1-(\mu^-)^c(x), -1-(\mu^-)^c(y)\}$
$\Leftrightarrow (\mu^-)^c(xy) \leq -1-\min\{-1-(\mu^-)^c(x), -1-(\mu^-)^c(y)\}$
$\Leftrightarrow (\mu^-)^c(xy) \leq \max\{(\mu^-)^c(x), (\mu^-)^c(y)\}$

iii. $\mu^+(x^{-1}) = \mu^+(x)$
$\Leftrightarrow 1-(\mu^+)^c(x^{-1}) = 1-(\mu^+)^c(x)$
$\Leftrightarrow (\mu^+)^c(x^{-1}) = (\mu^+)^c(x)$

and $\mu^-(x^{-1}) = \mu^-(x)$
$\Leftrightarrow -1-(\mu^-)^c(x^{-1}) = -1-(\mu^-)^c(x)$
$\Leftrightarrow (\mu^-)^c(x^{-1}) = (\mu^-)^c(x)$

Hence $\mu^c = ((\mu^+)^c, (\mu^-)^c)$ is a bipolar fuzzy subgroup of G.

Definition 2.7[10]

Let f be a mapping from a group G_1 to a group G_2 . Let $\mu = (\mu^+,\mu^-)$ and $\varphi = (\varphi^+, \varphi^-)$ are bipolar fuzzy subsets of G_1 and G_2 respectively, then the image of μ under f is a bipolar fuzzy subset $f(\mu) = ((f(\mu))^+, (f(\mu))^-)$ of G_2 defined by for each $u\in G$

$$(f(\mu))^+(u) = \begin{cases} \max\{\mu^+(x) : x\in f^{-1}(u)\} & , \text{ if } f^{-1}(u) \neq \phi \\ 0 & , \text{ otherwise} \end{cases}$$

and

$$(f(\mu))^-(u)) = \begin{cases} \max\{\mu^-(x) : x\in f^{-1}(u)\} & , \text{ if } f^{-1}(u) \neq \phi \\ 0 & , \text{ otherwise} \end{cases}$$

also the pre-image $f^{-1}(\varphi)$ of φ under f is a bipolar fuzzy subset of G_1 defined by for $x\in G_1$, $(f^{-1}(\varphi)^+)(x) = \varphi^+(f(x))$, $(f^{-1}(\varphi)^-)(x) = \varphi^-(f(x))$.

III. Properties of a Bipolar Anti Fuzzy Group of a Group under Homomorphism And Anti Homomorphism

In this section, we introduce the notion of an anti image and anti pre-image of the bipolar fuzzy subgroup of a group, and discuss the properties of a bipolar fuzzy and bipolar anti fuzzy subgroup of a group under homomorphism and anti homomorphism . Throughout this section , We mean that G_1 and G_2 are finite groups (G_1 and G_2 are not necessarily commutative) e_1 , e_2 are the identity elements of G_1 and G_2 respectively, and xy we mean x * y.

Definition 3.1

Let f be a mapping from a group G_1 to a group G_2 . Let μ and φ are fuzzy subsets of G_1 and G_2 respectively, then the anti image of μ under f is a fuzzy subset $f_a(\mu)$ of G_2 defined by for each $u\in G_2$.

$$(f_a(\mu))(u) = \begin{cases} \min\{\mu(x) : x\in f^{-1}(u)\} & , \text{ if } f^{-1}(u) \neq \phi \\ 1 & , \text{ otherwise} \end{cases}$$

also the anti pre-image $f^{-1}(\varphi)$ of φ under f is a fuzzy subset of G_1 defined by for $x\in G_1$, $(f^{-1}(\varphi))(x) = \varphi(f(x))$.

Definition 3.2

Let f be a mapping from a group G_1 to a group G_2 . Let $\mu = (\mu^+,\mu^-)$ and $\varphi = (\varphi^+, \varphi^-)$ are bipolar fuzzy subsets of G_1 and G_2 respectively, then the anti image of μ under f is a bipolar fuzzy subset $f_a(\mu) = ((f_a(\mu))^+, (f_a(\mu))^-)$ of G_2 defined by for each $u\in G_2$.

$$(f_a(\mu))^+(u) = \begin{cases} \min\{\mu^+(x) : x\in f^{-1}(u)\} & , \text{ if } f^{-1}(u) \neq \phi \\ 1 & , \text{ otherwise} \end{cases}$$

$$(f_a(\mu))^-(u) = \begin{cases} \min\{\mu^-(x) : x\in f^{-1}(u)\} & , \text{ if } f^{-1}(u) \neq \phi \\ -1 & , \text{ otherwise} \end{cases}$$

also the anti pre-image $f^{-1}(\varphi)$ of φ under f is a bipolar fuzzy subset of G_1 defined by for $x \in G_1$, $(f^{-1}(\varphi)^+)(x) = \varphi^+(f(x))$, $(f^{-1}(\varphi)^-)(x) = \varphi^-(f(x))$.

Theorem 3.1

Let f be a homomorphism from a group G_1 into a group G_2. If $\varphi = (\varphi^+, \varphi^-)$ is a bipolar fuzzy subset of G_2 then $f^{-1}(\varphi^c) = [f^{-1}(\varphi)]^c$.

Proof :

Let $\varphi = (\varphi^+, \varphi^-)$ be a bipolar fuzzy subset of G_2, then for each $x \in G_1$

i. $$\begin{aligned} [f^{-1}(\varphi^c)]^+(x) &= (\varphi^c)^+(f(x)) \\ &= 1 - \varphi^+(f(x)) \\ &= 1 - f^{-1}(\varphi^+)(x) \\ &= [f^{-1}(\varphi^+)]^c(x) \\ [f^{-1}(\varphi^c)]^+ &= [f^{-1}(\varphi^+)]^c \end{aligned}$$

ii. $$\begin{aligned} [f^{-1}(\varphi^c)]^-(x) &= (\varphi^c)^-(f(x)) \\ &= -1 - \varphi^-(f(x)) \\ &= -1 - f^{-1}(\varphi^-)(x) \\ &= [f^{-1}(\varphi^-)]^c(x) \\ [f^{-1}(\varphi^c)]^- &= [f^{-1}(\varphi^-)]^c \end{aligned}$$

Hence, $f^{-1}(\varphi^c) = (f^{-1}(\varphi))^c$.

Theorem 3.2

Let f be a homomorphism from a group G_1 into a group G_2. If $\mu = (\mu^+,\mu^-)$ is a bipolar fuzzy subset of G_1 then i. $f(\mu^c) = [f_a(\mu)]^c$.
ii. $f_a(\mu^c) = (f(\mu))^c$.

Proof :

Let $\mu = (\mu^+,\mu^-)$ be a bipolar fuzzy subset of G_1, then for each $x \in G_1$, let $f(x) = u \in G_2$

i. $$\begin{aligned} [f(\mu^c)]^+(u) &= \max\{(\mu^c)^+(x) : x \in f^{-1}(u)\} \\ &= \max\{1 - \mu^+(x) : x \in f^{-1}(u)\} \\ &= 1 - \min\{\mu^+(x) : x \in f^{-1}(u)\} \\ &= 1 - f_a(\mu^+)(u) \\ &= [f_a(\mu^+)]^c(u) \\ [f(\mu^c)]^+ &= [f_a(\mu^+)]^c \end{aligned}$$

$$\begin{aligned} [f(\mu^c)]^-(u) &= \max\{(\mu^c)^-(x) : x \in f^{-1}(u)\} \\ &= \max\{-1 - \mu^-(x) : x \in f^{-1}(u)\} \\ &= -1 - \min\{\mu^-(x) : x \in f^{-1}(u)\} \\ &= -1 - f_a(\mu^-)(u) \\ &= [f_a(\mu^-)]^c(u) \\ [f(\mu^c)]^- &= [f_a(\mu^-)]^c \end{aligned}$$

Hence, $f(\mu^c) = [f_a(\mu)]^c$

ii. $$\begin{aligned} [f_a(\mu^c)]^+(u) &= \min\{(\mu^c)^+(x) : x \in f^{-1}(u)\} \\ &= \min\{1 - \mu^+(x) : x \in f^{-1}(u)\} \\ &= 1 - \max\{\mu^+(x) : x \in f^{-1}(u)\} \\ &= 1 - f(\mu^+)(u) \\ &= [f(\mu^+)]^c(u) \\ [f_a(\mu^c)]^+ &= (f(\mu^+))^c \end{aligned}$$

$$\begin{aligned} [f_a(\mu^c)]^-(u) &= \min\{(\mu^-)^c(x) : x \in f^{-1}(u)\} \\ &= \min\{-1 - \mu^-(x) : x \in f^{-1}(u)\} \\ &= -1 - \max\{\mu^-(x) : x \in f^{-1}(u)\} \\ &= -1 - f(\mu^-)(u) \\ &= [f(\mu^-)]^c(u) \\ [f_a(\mu^c)]^- &= (f(\mu^-))^c \end{aligned}$$

Hence, $f_a(\mu^c) = (f(\mu))^c$.

Theorem 3.3

Let f be a homomorphism from a group G_1 into a group G_2. If $\mu = (\mu^+,\mu^-)$ is a bipolar anti fuzzy subgroup of G_1 then the anti image $f_a(\mu)$ of μ under f is a bipolar anti fuzzy subgroup of G_2.

Proof :

Let $\mu = (\mu^+,\mu^-)$ be a bipolar anti fuzzy subgroup of G_1. Let $\mu^+ : G_1 \to [0,1]$ and $\mu^- : G_1 \to [-1,0]$ are mappings. Let $u,v \in G_2$, since f is homomorphism and so there exist $x,y \in G_1$ such that $f(x) = u$ and $f(y) = v$ it follows that $xy \in f^{-1}(uv)$.

i. $$\begin{aligned} (f_a(\mu))^+(uv) &= \min\{\mu^+(z) : z = xy \in f^{-1}(uv)\} \\ &\le \min\{\mu^+(xy) : x \in f^{-1}(u), y \in f^{-1}(v)\} \\ &\le \min\{\max\{\mu^+(x), \mu^+(y)\} : x \in f^{-1}(u), y \in f^{-1}(v)\} \\ &= \max\{\min\{\mu^+(x) : x \in f^{-1}(u)\}, \min\{\mu^+(y) : y \in f^{-1}(v)\}\} \\ &= \max\{(f_a(\mu))^+(u), (f_a(\mu))^+(v)\} \\ (f_a(\mu))^+(uv) &\le \max\{(f_a(\mu))^+(u), (f_a(\mu))^+(v)\} \end{aligned}$$

ii. $$\begin{aligned} (f_a(\mu))^-(uv) &= \min\{\mu^-(z) : z = xy \in f^{-1}(uv)\} \\ &\ge \min\{\mu^-(xy) : x \in f^{-1}(u), y \in f^{-1}(v)\} \\ &\ge \min\{\min\{\mu^-(x), \mu^-(y)\} : x \in f^{-1}(u), y \in f^{-1}(v)\} \\ &= \min\{\min\{\mu^-(x) : x \in f^{-1}(u)\}, \min\{\mu^-(y) : y \in f^{-1}(v)\}\} \\ &= \min\{(f_a(\mu))^-(u), (f_a(\mu))^-(v)\} \\ (f_a(\mu))^-(uv) &\ge \min\{(f_a(\mu))^-(u), (f_a(\mu))^-(v)\} \end{aligned}$$

iii. $$\begin{aligned} (f_a(\mu))^+(u^{-1}) &= \min\{\mu^+(x) : x \in f^{-1}(u^{-1})\} \\ &= \min\{\mu^+(x^{-1}) : x^{-1} \in f^{-1}(u)\} \\ &= (f_a(\mu))^+(u) \quad \text{and} \\ (f_a(\mu))^-(u^{-1}) &= \min\{\mu^-(x) : x \in f^{-1}(u^{-1})\} \\ &= \min\{\mu^-(x^{-1}) : x^{-1} \in f^{-1}(u)\} \\ &= (f_a(\mu))^-(u) \end{aligned}$$

Hence, $f_a(\mu)$ is a bipolar anti fuzzy subgroup of G_2.

Theorem 3.4

Let f be an anti homomorphism from a group G_1 into a group G_2. If $\mu = (\mu^+,\mu^-)$ is a bipolar anti fuzzy subgroup of G_1 then the anti image $f_a(\mu)$ of μ under f is a bipolar anti fuzzy subgroup of G_2.

Proof :

Let $\mu = (\mu^+,\mu^-)$ be a bipolar anti fuzzy subgroup of G_1. Let $\mu^+ : G_1 \to [0,1]$ and $\mu^- : G_1 \to [-1,0]$ are mappings, Let $u,v \in G_2$, since f is an anti homomorphism and so there exist $x,y \in G_1$ such that $f(x) = u$ and $f(y) = v$ it follows that $xy \in f^{-1}(vu)$.

i. $(f_a(\mu))^+(uv) = \min\{\mu^+(z) : z = xy \in f^{-1}(vu)\}$

$\leq \min \{ \mu^+(xy) : x\in f^{-1}(u), y\in f^{-1}(v)\}$
$\leq \min \{\max \{ \mu^+(x), \mu^+(y)\} : x\in f^{-1}(u), y\in f^{-1}(v)\}$
$= \max\{\min\{\mu^+(x):x\in f^{-1}(u)\}, \min\{\mu^+(y):y\in f^{-1}(v)\}\}$
$= \max \{(f_a(\mu))^+(u) , (f_a(\mu))^+(v)\}$
$(f_a(\mu))^+(uv) \leq \max \{(f_a(\mu))^+(u) , (f_a(\mu))^+(v)\}$

ii. $(f_a(\mu))^-(uv) = \min \{ \mu^-(z) : z = xy \in f^{-1}(vu)\}$
$\geq \min \{ \mu^-(xy) : x\in f^{-1}(u), y\in f^{-1}(v)\}$
$\geq \min \{\min \{ \mu^-(x), \mu^-(y)\}: x\in f^{-1}(u), y\in f^{-1}(v)\}$
$= \min\{\min\{\mu^-(x):x\in f^{-1}(u)\},\min\{\mu^-(y) : y\in f^{-1}(v)\}\}$
$= \min \{(f_a(\mu))^-(u) , (f_a(\mu))^-(v)\}$
$(f_a(\mu))^-(uv) \geq \min \{(f_a(\mu))^-(u) , (f_a(\mu))^-(v)\}$

iii. $(f_a(\mu))^+(u^{-1}) = \min \{ \mu^+(x) : x\in f^{-1}(u^{-1})\}$
$= \min \{ \mu^+(x^{-1}) : x^{-1}\in f^{-1}(u)\}$
$= (f_a(\mu))^+(u)$ and
$(f_a(\mu))^-(u^{-1}) = \min \{ \mu^-(x) : x\in f^{-1}(u^{-1})\}$
$= \min \{ \mu^-(x^{-1}) : x^{-1}\in f^{-1}(u)\}$
$= (f_a(\mu))^-(u)$

Hence, $f_a(\mu)$ is a bipolar anti fuzzy subgroup of G_2.

Theorem 3.5

Let f be a homomorphism from a group G_1 into a group G_2. If $\varphi = (\varphi^+, \varphi^-)$ is a bipolar anti fuzzy subgroup of G_2 then the anti pre-image $f^{-1}(\varphi)$ of φ under f is a bipolar anti fuzzy subgroup of G_1.

Proof :

Let $\varphi = (\varphi^+, \varphi^-)$ be a bipolar anti fuzzy subgroup of G_2, $\varphi^+ : G_2 \to [0,1]$ and $\varphi^- : G_2 \to [-1,0]$ are mappings. Let $x,y \in G_1$

i. $(f^{-1}(\varphi))^+(xy) = \varphi^+(f(xy))$
$= \varphi^+(f(x) f(y))$
$\leq \max \{ \varphi^+(f(x)) , \varphi^+(f(y))\}$
$(f^{-1}(\varphi))^+(xy) \leq \max\{(f^{-1}(\varphi))^+(x) , (f^{-1}(\varphi))^+(y))\}$

ii. $(f^{-1}(\varphi))^-(xy) = \varphi^-(f(xy))$
$= \varphi^-(f(x)f(y))$
$\geq \min \{ \varphi^-(f(x)), \varphi^-(f(y))\}$
$= \min \{(f^{-1}(\varphi))^-(x), (f^{-1}(\varphi))^-(y))\}$
$(f^{-1}(\varphi))^-(xy) \geq \min \{(f^{-1}(\varphi))^-(x), (f^{-1}(\varphi))^-(y))\}$

iii. $(f^{-1}(\varphi))^+(x^{-1}) = \varphi^+(f(x^{-1}))$
$= \varphi^+(f(x)^{-1})$
$= \varphi^+(f(x))$
$= (f^{-1}(\varphi))^+(x)$ and
$(f^{-1}(\varphi))^-(x^{-1}) = \varphi^-(f(x^{-1}))$
$= \varphi^-(f(x)^{-1})$
$= \varphi^-(f(x))$
$= (f^{-1}(\varphi))^-(x)$

Hence, $f^{-1}(\varphi)$ is a bipolar anti fuzzy subgroup of G_1.

Theorem 3.6

Let f be an anti homomorphism from a group G_1 into a group G_2. If $\varphi = (\varphi^+, \varphi^-)$ is a bipolar anti fuzzy subgroup of G_2 then the anti pre-image $f^{-1}(\varphi)$ of φ under f is a bipolar anti fuzzy subgroup of G_1.

Proof :

Let $\varphi = (\varphi^+, \varphi^-)$ be a bipolar anti fuzzy subgroup of G_2, $\varphi^+ : G_2 \to [0,1]$ and $\varphi^- : G_2 \to [-1,0]$ are mappings. Let $x,y \in G_1$

i. $(f^{-1}(\varphi))^+(xy) = \varphi^+(f(xy))$
$= \varphi^+(f(y) f(x))$
$\leq \max \{ \varphi^+(f(y)) , \varphi^+(f(x))\}$
$= \max \{(f^{-1}(\varphi))^+(y) , (f^{-1}(\varphi))^+(x))\}$
$= \max \{(f^{-1}(\varphi))^+(x) , (f^{-1}(\varphi))^+(y))\}$
$(f^{-1}(\varphi))^+(xy) \leq \max \{(f^{-1}(\varphi))^+(x) , (f^{-1}(\varphi))^+(y))\}$

ii. $(f^{-1}(\varphi))^-(xy) = \varphi^-(f(xy))$
$= \varphi^-(f(y) f(x))$
$\geq \min \{ \varphi^-(f(y)), \varphi^-(f(x))\}$
$= \min \{(f^{-1}(\varphi))^-(y), (f^{-1}(\varphi))^-(x))\}$
$= \min \{(f^{-1}(\varphi))^-(x), (f^{-1}(\varphi))^-(y))\}$
$(f^{-1}(\varphi))^-(xy) \geq \min \{(f^{-1}(\varphi))^-(x), (f^{-1}(\varphi))^-(y))\}$

iii. $(f^{-1}(\varphi))^+(x^{-1}) = \varphi^+(f(x^{-1}))$
$= \varphi^+(f(x)^{-1})$
$= \varphi^+(f(x))$
$= (f^{-1}(\varphi))^+(x)$ and
$(f^{-1}(\varphi))^-(x^{-1}) = \varphi^-(f(x^{-1}))$
$= \varphi^-(f(x)^{-1})$
$= \varphi^-(f(x))$
$= (f^{-1}(\varphi))^-(x)$

Hence, $f^{-1}(\varphi)$ is a bipolar anti fuzzy subgroup of G_1.

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