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Fixed Point Theorem for Selfmaps On A Fuzzy 2-Metric Space Under Occasionally Subcompatibility Condition

K.P.R. Sastry [1], M.V.R. Kameswari [2], Ch. Srinivasa Rao [3], and R. Venkata Bhaskar [4]

[1] 8-28-8/1, Tamil Street, Chinna Waltair, Visakhapatnam -530 017, India
[2] Department of Engineering Mathematics, GIT, Gitam University, Visakhapatnam -530 045, India
[3] Department of Mathematics, Mrs. A.V.N. College, Visakhapatnam -530 001, India
[4] Department of Mathematics, Raghu Engineering College, Visakhapatnam -531 162, India.

Abstract

In this paper, we prove a common fixed point theorem for three selfmaps and extend it to four and six continuous selfmaps on a fuzzy 2-metric space, using the notion of occasionally sub compatible map with respect to another map. We show that the result of Surjeet Singh Chauhan and Kiran Utreja [7] follows as a corollary.

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1.Introduction

The concept of fuzzy sets was introduced by L.A.Zadeh [9] in 1965 which became active field of research for many researchers. In 1975, Karmosil and Michalek [5] introduced the concept of a fuzzy metric space based on fuzzy sets; this notion was further modified by George and Veermani [2] with the help of *t*-norms. Many authors made use of the definition of a fuzzy metric space due to George and Veermani [2] in proving fixed point theorems in fuzzy metric spaces. Gahler [1] introduced and studied 2-metric spaces in a series of his papers. Iseki, Sharma, and Sharma [3] investigated, for the first time, contraction type mappings in 2-metric spaces. In this paper we introduced the notion of occasionally sub compatible map with respect to another map. Using this notion we prove a common fixed point theorem for three selfmaps and extend it to four and six continuous selfmaps on a fuzzy 2-metric space also we show that the result of Surjeet Singh Chauhan and Kiran Utreja [8] follows as a corollary.

2. Preliminaries

We begin with some known definitions and results.

Definition 2.1 :(Zadeh. L.A [9]) A fuzzy set A in a nonempty set X is a function with domain X and values in [0,1].

Definition 2.2: (Schweizer.B and Sklar. A [6]) A function $*:[0,1]\times[0,1]\to[0,1]$ is said to be a continuous t-norm if $*$ satisfies the following conditions:

For $a,b,c,d\in[0,1]$

(i) $*$ is commutative and associative
(ii) $*$ is continuous
(iii) $a*1=a \; for \; all \; a\in[0,1]$
(iv) $a*b\le c*d \; whenever \; a\le c \; and \; b\le d$

$a*b=min\{a,b\}, a*b=a.b$ are examples of continuous *t*-norms.

Definition 2.3: (Kramosil. I and Michelek. J [5]) A triple $(X,M,*)$ is said to be a fuzzy metric space (FM space, briefly) if X is a nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2\times[0,\infty)$ which satisfies the following conditions:

For $x,y,z\in X$ and $s,t>0$.

(i) $M(x,y,t)>0, M(x,y,0)=0$
(ii) $M(x,y,t)=1 \; for \; all \; t>0 \; if \; and \; only \; if \; x=y$
(iii) $M(x,y,t)=M(y,x,t)$
(iv) $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$
(v) $M(x,y,\cdot):[0,\infty)\longrightarrow[0,1]$ is left continuous.

Then M is called a fuzzy metric space on X.

The function $M(x,y,t)$ denotes the degree of nearness between x and y with respect to t.

Example 2.4: (George and Veeramani [2]) Let (X, d) be a metric space. Define $a * b = min\{a, b\}$ and

$M(x, y, t) = \frac{t}{t+d(x,y)}$ for all $x, y \in X$ and all $t > 0$, $M(x, y, 0) = 0$

Then $(X, M,*)$ is a Fuzzy metric space. It is called the Fuzzy metric space induced by d

Definition 2.5: (Gahler [1]) Let X be a nonempty set. A real valued function d on $X \times X \times X$ is said to be a 2-metric on X if

(1) given distinct elements x, y of X, there exists an element $z \in X$

such that $d(x, y, z) \neq 0$,

(2) $d(x, y, z) = 0$ when at least two of $x, y, z \in X$ are equal,
(3) $d(x, y, z) = d(x, z, y) = d(y, z, x)\ \forall\ x, y, z \in X$,

(4) $d(x, y, z) \leq d(x, z, w) + d(x, w, z) + d(w, y, z)\ \forall\ x, y, z \in X$.

The pair (X, d) is a 2-metric space.

Example 2.6: Let $X = \mathbb{R}^3$ and let $d(x, y, z) =$ the area of the triangle spanned by x, y and z which may be given explicitly by the formula

$d(x,y,z) = \left| x_1(y_2z_3 - y_3z_2) - x_2(y_1z_3 - y_3z_1) + x_3(y_1z_2 - y_2z_1) \right|$

, where

$x = (x_1, x_2, x_3), y = (y_1, y_2, y_3), z = (z_1, z_2, z_3)$ Then pair (X, d) is called a 2-metric space.

Definition 2.7: (S.Sharma [7]) A triple $(X, M,*)$ is said to be a fuzzy metric 2-space, if X is a nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^3 \times [0, \infty)$ which satisfies the following conditions:

for $x, y, z, u \in X$ and $s, t_1, t_2, t_3 > 0$,

(1) $M(x, y, z, 0) = 0$,
(2) $M(x, y, z, t) = 1$ for all $t > 0$ if and only if at least two of the three points are equal,
(3) $M(x, y, z, t) = M(x, z, y, t) = M(y, z, x, t) \forall t > 0$,
(symmetry about first three variables)
(4) $M(x, y, z, t_1 + t_2 + t_3) \geq M(x, y, u, t_1) * M(x, u, z, t_2) * M(u, y, z, t_3)$,
(This corresponds to tetrahedron inequality in 2-metric space. The function value $M(x, y, z, t)$ may be interpreted as the probability that the area of triangle is less than t.)
(5) $M(x, y, z\ \cdot) : [0, \infty) \longrightarrow [0,1]$ is left continuous.

Example 2.8: Let (X, d) be a metric space. Define $a * b = min\{a, b\}$ and

$M(x, y, t) = \frac{t}{t+d(x,y,z)}$ for all $x, y, z \in X$ and all $t > 0$

Then $(X, M,*)$ is a fuzzy 2- metric space.

Definition 2.9: (Jinkyu Han[4]) Let $(X, M,*)$ be a fuzzy 2-metric space. Then,

(1) A sequence $\{x_n\}$ in a fuzzy 2- metric space X is said to be convergent to a point $a \in X$ if $\lim_{n\to\infty} M(x_n, x, a, t) = 1\ \forall\ a \in X, t > 0$.
(2) A sequence $\{x_n\}$ in a fuzzy 2- metric space X is called a Cauchy sequence, if for any $\lambda \in (0,1)$ and $t > 0$, there exists $n_0 \in N$ such that, for all $m, n \geq n_0$ and $a \in X, M(x_n, x_m, a, t) > 1 - \lambda$.
(3) A fuzzy 2- metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.10: Let A and B be maps from a fuzzy 2-metric space $(X, M,*)$ into itself. A point x in X is called a coincidence point of A and B if $Ax = Bx$.

Definition 2.11: Two selfmaps S and T of a fuzzy 2-metric space $(X, M,*)$ are said to be weakly compatible if they commute at their coincidence points, that is if $Sx = Tx$ for some $x \in X$, then $STx = TSx$.

Definition 2.12: Two selfmaps S and T of a fuzzy 2-metric space $(X, M,*)$ are said to be sub compatible if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = a \in X$ and $\lim_{n\to\infty} M(STx_n, TSx_n, z, t) = 1$

Definition 2.13: (Surjeet singh Chauhan and Kiran Utreja [8]) Two selfmaps S and T of a fuzzy 2-metric space $(X, M,*)$ are said to be sub compatible of type A, if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = a \in X$ and $\lim_{n\to\infty} M(STx_n, TTx_n, z, t) = 1$, and

$\lim_{n\to\infty} M(TSx_n, SSx_n, z, t) = 1$

The following two results are proved in [4].

Lemma 2.14: For all $x, y, z \in X$, $M(x, y, z,\cdot)$ is a non-decreasing function.

Lemma 2.15: Let $(X, M,*)$ be a fuzzy 2-metric space. If there exists $k \in (0,1)$ such that $M(x, y, z, kt) \geq M(x, y, z, t)$ for all $x, y, z \in X$ with $z \neq x, z \neq y$ and $t > 0$, then $x = y$.

Surjeet Singh Chauhan and Kiran Utreja [6] proved the following Theorem

Theorem 2.16: (Surjeet Singh Chauhan and Kiran Utreja [8]) Let A, B, S, P, Q and T be six self mappings of a fuzzy 2-metric space$(X, M,*)$ with continuous t-norm satisfying $t * t \geq t\ \forall t \in [0,1]$. Suppose the pairs (AB, S) and (PQ, T) are sub compatible of type A having the same coincidence point and $AB = BA, BS = SB, AS = SA, PQ = QP, TQ = QT, PT = TP$,

$$M\left(Sx, Ty, z, kt\right) \geq \left\{\begin{matrix} M\left(Sx, PQy, z, t\right) * M\left(Sx, ABx, z, t\right) * M\left(PQy, Ty, z, t\right) \\ * M\left(ABx, PQy, z, t\right) * M\left(ABx, Ty, z, t\right) \end{matrix}\right\} \quad (2.16.1)$$

for all $x, y, z \in X$ and some $k \in (0,1), t > 0$.

Then $A, B, S, P, Q,$ and T have a unique common fixed point in X.

3. Main Results

In this section we first introduce the notions of

(1) occasionally sub compatible map (2) weakly sub compatible map and (3) sub compatible map with respect to another map.

We use these notions to prove some fixed point theorems. Further we show that the result of Surjeet Singh Chauhan and Kiran Utreja [7] follows as a corollary.

Definition 3.1: Let A and S be selfmaps of a fuzzy 2-metric space$(X, M,*)$. A is said to be occasionally sub compatible with respect to S, if there exists a sequence $\{x_n\}\ in\ X$ such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = a \in X\ and\ \lim_{n\to\infty} M(ASx_n, SSx_n, z, t) = 1\ \forall z \in X, t > 0$$

Definition 3.2: Let A and S be selfmaps of a fuzzy 2-metric space$(X, M,*)$. A is said to be weakly sub compatible with respect to S, if

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n \Rightarrow \lim_{n\to\infty} M(ASx_n, SSx_n, z, t) = 1\ \forall z \in X, t > 0$.

Definition 3.3: Let A and S be selfmaps of a fuzzy 2-metric space$(X, M,*)$. A is said to be sub compatible with respect to S if there exists a sequence $\{x_n\}\ in\ X$ such that

(1) $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = a \in X\ and\ \lim_{n\to\infty} M(ASx_n, SSx_n, z, t) = 1$ and further

(2) $\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Sy_n \Rightarrow \lim_{n\to\infty} M(ASy_n, SSy_n, z, t) = 1\ \forall z \in X, t > 0$

Now we state and prove our main results.

Theorem 3.4: Let A, B and S be three continuous self mappings of a fuzzy 2-metric space$(X, M,*)$with continuous t-norm $*$ satisfying $t * t \geq t\ \forall t \in [0,1]$. Suppose A, B are occasionally sub compatible with respect to S and

$$M\left(Ax, By, z, kt\right) \geq \left\{M\left(By, Sx, z, t\right) * M\left(Ax, Sy, z, t\right) * M\left(Ax, By, z, t\right)\right\} \quad (3.4.1)$$

for all $x, y, z \in X$ and some $k \in (0,1), t > 0$.

Then A, B and S have a coincidence point in X.

If further (A, S) and (B, S) are weakly compatible then A, B and S have a unique common fixed point in X .

Proof: We may suppose, without loss of generality, that X has at least three elements. Since A is occasionally sub compatible with respect to S, there exists a sequence $\{x_n\}$ in X such that

$\mathrm{Lim}_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = a$

and $\lim_{n\to\infty} M(ASx_n, SSx_n, z, t) = 1$

$\Rightarrow M(Aa, Sa, z, t) = 1\ \forall t > 0$ (since A and S are continuous)

$\Rightarrow Aa = Sa$

Thus a is coincidence point of A and S.

Also since B is occasionally sub compatible with respect to S, there exists a sequence $\{y_n\}$ in X such that

$\mathrm{Lim}_{n\to\infty} By_n = \lim_{n\to\infty} Sy_n = b$

and $\lim_{n\to\infty} M(BSy_n, SSy_n, z, t) = 1$

$\Rightarrow M(Bb, Sb, z, t) = 1\ \forall t > 0$

$\Rightarrow Bb = Sb$

Thus b is coincidence point of B and S.

Take $x = a, y = b$ in (3.4.1). Then

$M(Aa,Bb,z,kt) \geq \{M(Bb,Sa,z,t)*M(Aa,Sb,z,t)*M(Aa,Bb,z,t)\}$

$\therefore M(Aa,Bb,z,kt) \geq \{M(Bb,Aa,z,t)*M(Aa,Bb,z,t)*M(Aa,Bb,z,t)\}$

$$\therefore M(Aa,Bb,z,kt) \geq M(Aa,Bb,z,t) \forall t > 0$$

(since, by given condition, $*$ is minimum t-norm)

$\therefore M(Aa,Bb,z,t) = 1 \; \forall t > 0$ and for all $z \in X$

$\therefore \quad Aa = Bb$

Since $Aa = Sa, Bb = Sb$ and $Aa = Bb$

$$Aa = Sa = Bb = Sb$$

Take $x = a, y = a$ in (3.4.1). Then

$M(Aa,Bb,z,kt) \geq \{M(Bb,Sa,z,t)*M(Aa,Sb,z,t)*M(Aa,Bb,z,t)\}$

$\therefore M(Aa,Ba,z,kt) \geq \{M(Ba,Aa,z,t)*M(Aa,Aa,z,t)*M(Aa,Ba,z,t)\}$

$\therefore M(Aa,Ba,z,kt) \geq \{M(Ba,Aa,z,t)*1*M(Aa,Ba,z,t)\}$

$\therefore M(Aa,Ba,z,kt) \geq M(Aa,Ba,z,t) \forall t > 0$

$\therefore M(Aa,Ba,z,t) = 1 \; \forall t > 0$

$\therefore \quad Aa = Ba$

Since $Aa = Sa$, $Aa = Ba = Sa$

Therefore a is coincidence point of A, B and S.

Take $x = b, y = b$ in (3.4.1), then, as above we can show that b is coincidence point of A, B and S.

Since (A, S) and (B, S) are weakly compatible, they commute at their coincidence points.

Thus we have,

$$ASa = SAa \text{ and } SBb = BSb$$

Take $x = a, y = Bb$ in (3.4.1). Then

$M(Aa,BBb,z,kt) \geq \{M(BBb,Sa,z,t)*M(Aa,SBb,z,t)*M(Aa,BBb,z,t)\}$

$$\therefore M(Aa, BBb, z, kt) \geq \left\{ \begin{matrix} M(BBb, Aa, z, t) * M(Aa, BSb, z, t) \\ *M(Aa, BBb, z, t) \end{matrix} \right\}$$

$$\therefore M(Aa, BBb, z, kt) \geq \left\{ \begin{matrix} M(BBb, Aa, z, t) * M(Aa, BBb, z, t) \\ *M(Aa, BBb, z, t) \end{matrix} \right\}$$

$\therefore M(Aa, BBb, z, kt) \geq M(Aa, BBb, z, t) \forall t > 0$

$\therefore M(Aa, BBb, z, t) = 1 \forall t > 0$

$\therefore \quad Aa = BBb$

$\therefore \quad Bb = BBb$

Therefore Bb is fixed point of B.

We have.

$ABb = ASa = SAa = SBb = BSb = BBb = Bb$

Therefore Bb is fixed point of A.

We have

$SBb = BSb = BBb = Bb$

Therefore Bb is fixed point of S.

Thus Bb is common fixed point of A, B and S.

For uniqueness, suppose w is another common fixed point of A , B and S.

Then

$M(Bb, w, z, kt) = M(ABb, Bw, z, kt)$

$\geq \{M(Bw,SBb,z,t)*M(ABb,Sw,z,t)*M(ABb,Bw,z,t)\}$

$= \{M(w,Bb,z,t)*M(Bb,w,z,t)*M(Bb,w,z,t)\}$

$$\therefore M(Bb,w,z,kt) \geq M(Bb,w,z,kt) \forall t > 0$$

$$\therefore M(Bb,w,z,kt) = 1 \; \forall t > 0$$

$$\therefore \quad Bb = w$$

This completes the proof of Theorem.

Theorem 3.5: Let A, B, S, P, Q and T be six self mappings of a fuzzy 2-metric space

$(X, M, *)$ with continuous t-norm satisfying $t * t \geq t \; \forall t \in [0,1]$. Suppose

$(i) AB, PQ, S$ and T are continuous and $(ii) AB$ and PQ are occasionally sub compatible with respect to S and T respectively and

$$M(Sx, Ty, z, kt) \geq \left\{ \begin{matrix} M(Sx, PQy, z, t) * M(Sx, ABx, z, t) * \\ M(PQy, Ty, z, t) * M(ABx, PQy, z, t) \\ *M(ABx, Ty, z, t) \end{matrix} \right\}$$

(3.5.1)

for all $x, y, z \in X$ and some $k \in (0,1), t > 0$.

Then (AB, S) and (PQ, T) have a coincidence point in X.

If further (AB, S) and (PQ, T) are weakly compatible then AB, PQ, S and T have a unique common fixed point in X.

Proof: We may suppose, without loss of generality, that X has at least three elements. Since AB is occasionally sub compatible with respect to S, there exists a sequence $\{x_n\}$ in X such that

$\text{Lim}_{n\to\infty} ABx_n = \lim_{n\to\infty} Sx_n = a$

and $\lim_{n\to\infty} M(ABSx_n, SSx_n, z, t) = 1$

$$\Rightarrow M(ABa, Sa, z, t) = 1 \; \forall t > 0$$

$$\Rightarrow ABa = Sa$$

Thus a is coincidence point of AB and S.

Also since PQ is occasionally sub compatible with respect to T, there exists a sequence $\{y_n\}$ in X such that

$\mathrm{Lim}_{n\to\infty} PQy_n = \lim_{n\to\infty} Ty_n = b$

and $\lim_{n\to\infty} M(PQTy_n, TTy_n, z, t) = 1$

$$\Rightarrow M(PQb, Tb, z, t) = 1 \;\forall t > 0$$

$$\Rightarrow PQb = Tb$$

Thus b is coincidence point of PQ and T.

Take $x = a \; and \; y = b$ in (3.5.1). Then

$$M(Sa, Tb, z, kt) \ge \begin{Bmatrix} M(Sa, PQb, z, t) * M(Sa, ABa, z, t) \\ *M(PQb, Tb, z, t) * M(ABa, PQb, z, t) \\ *M(ABa, Tb, z, t) \end{Bmatrix}$$

$$\therefore M(Sa,Tb,z,kt) \ge \begin{Bmatrix} M(Sa,Tb,z,t) * M(Sa,Sa,z,t) * M(Tb,Tb,z,t) \\ *M(Sa,Tb,z,t) * M(Sa,Tb,z,t) \end{Bmatrix}$$

$$\therefore M(Sa,Tb,z,kt) \ge \{M(Sa,Tb,z,t) * 1 * 1 * M(Sa,Tb,z,t) * M(Sa,Tb,z,t)\}$$

$$\therefore M(Sa,Tb,z,kt) \ge M(Sa,Tb,z,t) \;\forall t > 0$$

$$\therefore M(Sa,Tb,z,kt) = 1 \;\forall t > 0$$

$$\therefore \quad Sa = Tb$$

Since (AB, S) and (PQ, T) are weakly compatible, they commute at their coincidence points.

Thus we have,

$$ABSa = SABa \text{ and } PQTb = TPQb$$

Take $x = Sa, y = b$ in (3.5.1). Then

$$M(SSa, Tb, z, kt) \ge \begin{Bmatrix} M(SSa, PQb, z, t) * M(SSa, ABSa, z, t) \\ *M(PQb, Tb, z, t) * M(ABSa, PQb, z, t) \\ *M(ABSa, Tb, z, t) \end{Bmatrix}$$

$$\therefore M(SSa, Tb, z, kt) \ge \begin{Bmatrix} M(SSa, Tb, z, t) * M(SSa, SABa, z, t) \\ *M(Tb, Tb, z, t) * M(SABa, Tb, z, t) \\ *M(SABa, Tb, z, t) \end{Bmatrix}$$

$$\therefore M(SSa, Tb, z, kt) \ge \begin{Bmatrix} M(SSa, Tb, z, t) * M(SSa, SSa, z, t) \\ *M(Tb, Tb, z, t) * M(SSa, Tb, z, t) \\ *M(SSa, Tb, z, t) \end{Bmatrix}$$

$$\therefore M(SSa,Tb,z,kt) \ge \{M(SSa,Tb,z,t) * 1 * 1 * M(SSa,Tb,z,t) * M(SSa,Tb,z,t)\}$$

$$\therefore M(SSa,Tb,z,kt) \ge M(SSa,Tb,z,t) \;\forall t > 0$$

$$\therefore M(SSa,Tb,z,kt) = 1 \;\forall t > 0$$

$$\therefore \quad SSa = Tb$$

$$\therefore \quad SSa = Sa$$

Therefore Sa is fixed point of S.

We have.

$ABSa = SBAa = SSa = Sa$

Therefore Sa is fixed point of AB.

Take $x = a, y = Tb$ in (3.5.1). Then

$$M(Sa, TTb, z, kt) \ge \begin{Bmatrix} M(Sa, PQTb, z, t) * M(Sa, ABa, z, t) \\ *M(PQTb, TTb, z, t) \\ *M(ABa, PQTb, z, t) \\ *M(ABa, TTb, z, t) \end{Bmatrix}$$

$$\therefore M(Sa, TTb, z, kt) \ge \begin{Bmatrix} M(Sa, TPQb, z, t) * M(Sa, Sa, z, t) \\ *M(TPQb, TTb, z, t) * M(Sa, TPQb, z, t) \\ *M(Sa, TPQb, z, t) \end{Bmatrix}$$

$$\therefore M(Sa, TTb, z, kt) \ge \begin{Bmatrix} M(Sa, TTb, z, t) * M(Sa, Sa, z, t) \\ *M(TTb, TTb, z, t) * M(Sa, TTb, z, t) \\ *M(Sa, TTb, z, t) \end{Bmatrix}$$

$$\therefore M(Sa, TTb, z, kt) \ge \begin{Bmatrix} M(Sa, TTb, z, t) * 1 * 1 * \\ M(Sa, TTb, z, t) * M(Sa, TTb, z, t) \end{Bmatrix}$$

$$\therefore M(Sa, TTb, z, kt) \ge M(Sa, TTb, z, t) \;\forall t > 0$$

$$\therefore M(Sa, TTb, z, kt) = 1 \;\forall t > 0$$

$$\therefore \quad Sa = TTb$$

$$\therefore \quad Sa = TSa$$

Therefore Sa is fixed point of T.

We have

$PQSa = PQTb = TPQb = TTb = TSa = Sa$

Therefore Sa is fixed point of PQ.

Thus Sa is common fixed point of AB, PQ, S and T .

For uniqueness, suppose w is another common fixed point of AB, PQ, S and T .

Then

$$M\left(Sa,w,z,kt\right)=M\left(SSa,Tw,z,kt\right)$$
$$\geq\begin{Bmatrix}M\left(SSa,PQw,z,t\right)*M\left(SSa,ABSa,z,t\right)\\ *M\left(PQw,Tw,z,t\right)*M\left(ABSa,PQw,z,t\right)\\ *M\left(ABSa,Tw,z,t\right)\end{Bmatrix}$$
$$=\begin{Bmatrix}M\left(Sa,w,z,t\right)*M\left(Sa,Sa,z,t\right)*M\left(w,w,z,t\right)\\ *M\left(Sa,w,z,t\right)*M\left(Sa,w,z,t\right)\end{Bmatrix}$$
$$=\begin{Bmatrix}M\left(Sa,w,z,t\right)*1\\ *1*M\left(Sa,w,z,t\right)\\ *M\left(Sa,w,z,t\right)\end{Bmatrix}$$
$$M\left(Sa,w,z,kt\right)\geq M\left(Sa,w,z,t\right)\forall t>0$$
$$M\left(Sa,w,z,kt\right)=1\forall t>0$$
$$Sa=w$$

This completes the proof of Theorem.

Now we extend Theorem 3.5 to six continuous selfmaps.

Theorem 3.6: Let A,B,S,P,Q and T be six continuous self mappings of a fuzzy 2-metric space$(X,M,*)$ with continuous t-norm satisfying $t*t\geq t\ \forall t\in[0,1]$. Suppose AB and PQ are occasionally sub compatible with respect to S and T respectively

$$M\left(Sx,Ty,z,kt\right)\geq\begin{Bmatrix}M\left(Sx,PQy,z,t\right)*M\left(Sx,ABx,z,t\right)\\ *M\left(PQy,Ty,z,t\right)*M\left(ABx,PQy,z,t\right)\\ *M\left(ABx,Ty,z,t\right)\end{Bmatrix}$$
(3.6.1)

for all $x,y,z\in X$ and some $k\in(0,1),t>0$.

Then (AB,S) and (PQ,T) have a coincidence point in X.

If further $AB=BA, BS=SB, AS=SA, PQ=QP, TQ=QT, PT=TP$, then A,B,S,P,Q, and T have a unique common fixed point in X.

Proof: From Theorem 3.5, we can prove the following

a is coincidence point of AB and S, b is coincidence point of PQ and T, and $Sa=Tb$.

Since A,B and S commute, the pair (AB,S) is weakly compatible.

Therefore, from Theorem 3.5, Sa is fixed point of S.

Also since P,Q and T commute, the pair (PQ,T) is weakly compatible.

Therefore , from Theorem 3.5, Sa is fixed point of T.

Now we prove that Sa is fixed point of $A,B.P,$ and Q.

Take $x=Aa, y=b$ in (3.6.1). Then

$$M\left(SAa,Tb,z,kt\right)\geq\begin{Bmatrix}M\left(SAa,PQb,z,t\right)*M\left(SAa,ABAa,z,t\right)\\ *M\left(PQb,Tb,z,t\right)*M\left(ABAa,PQb,z,t\right)\\ *M\left(ABAa,Tb,z,t\right)\end{Bmatrix}$$

Since A,B and S commute, $ABAa=AABa=ASa=SAa$

$$\therefore M\left(SAa,Tb,z,kt\right)\geq\begin{Bmatrix}M\left(SAa,Tb,z,t\right)*M\left(SAa,SAa,z,t\right)\\ *M\left(Tb,Tb,z,t\right)*M\left(SAa,Tb,z,t\right)\\ *M\left(SAa,Tb,z,t\right)\end{Bmatrix}$$
$$\therefore M\left(SAa,Tb,z,kt\right)\geq\begin{Bmatrix}M\left(SAa,Tb,z,t\right)*1*1\\ *M\left(SAa,Tb,z,t\right)*M\left(SAa,Tb,z,t\right)\end{Bmatrix}$$
$$\therefore M\left(SAa,Tb,z,kt\right)\geq M\left(SAa,Tb,z,t\right)\forall t>0$$
$$\therefore M\left(SAa,Tb,z,kt\right)=1\forall t>0$$
$$\therefore\quad SAa=Tb$$
$$\therefore\quad ASa=Sa$$

Therefore Sa is fixed point of A.

From Theorem 3.5, we have Sa is fixed point of AB.

Therefore $Sa=ABSa=BASa=BSa$

Therefore Sa is fixed point of B.

Take $x=a, y=Pb$ in (3.6.1). Then

$$M\left(Sa,TPb,z,kt\right)\geq\begin{Bmatrix}M\left(Sa,PQPb,z,t\right)*M\left(Sa,ABa,z,t\right)\\ *M\left(PQPb,TPb,z,t\right)*M\left(ABa,PQPb,z,t\right)\\ *M\left(ABa,TPb,z,t\right)\end{Bmatrix}$$

Since P,Q and T commute, so $PQPb=PPQb=PTb=TPb$

$$\therefore M\left(Sa, TPb, z, kt\right) \geq \begin{Bmatrix} M\left(Sa, TPb, z, t\right) * M\left(Sa, Sa, z, t\right) * \\ M\left(TPb, TPb, z, t\right) * M\left(Sa, TPb, z, t\right) \\ * M\left(Sa, TPb, z, t\right) \end{Bmatrix}$$

$$\therefore M\left(Sa, TPb, z, kt\right) \geq \begin{Bmatrix} M\left(Sa, TPb, z, t\right) * 1 * 1 * \\ M\left(Sa, TPb, z, t\right) * M\left(Sa, TPb, z, t\right) \end{Bmatrix}$$

$$\therefore M\left(Sa, TPb, z, kt\right) \geq M\left(Sa, TPb, z, t\right) \forall t > 0$$

$$\therefore M\left(Sa, TPb, z, kt\right) = 1 \forall t > 0$$

$$\therefore \quad Sa = TPb$$

$$\therefore \quad Sa = PTb$$

$$\therefore \quad Sa = PSa$$

Therefore Sa is fixed point of P.

From Theorem 3.5, we have Sa is fixed point of PQ.

Therefore $Sa = PQSa = QPSa = QSa$

Therefore Sa is fixed point of Q.

Thus Sa is common fixed point of $A, B, S, P, Q,$ and T in X.

For uniqueness, suppose w is another common fixed point of $A, B, S, P, Q,$ and T in X..

Then

$$M\left(Sa, w, z, kt\right) = M\left(SSa, Tw, z, kt\right)$$

$$\geq \begin{Bmatrix} M\left(SSa, PQw, z, t\right) * M\left(SSa, ABSa, z, t\right) \\ * M\left(PQw, Tw, z, t\right) * M\left(ABSa, PQw, z, t\right) \\ * M\left(ABSa, Tw, z, t\right) \end{Bmatrix}$$

$$= \begin{Bmatrix} M\left(Sa, w, z, t\right) * M\left(Sa, Sa, z, t\right) * \\ M\left(w, w, z, t\right) * M\left(Sa, w, z, t\right) * \\ M\left(Sa, w, z, t\right) \end{Bmatrix}$$

$$= \begin{Bmatrix} M\left(Sa, w, z, t\right) * 1 * 1 * M\left(Sa, w, z, t\right) * \\ M\left(Sa, w, z, t\right) \end{Bmatrix}$$

$$\therefore M\left(Sa, w, z, kt\right) \geq M\left(Sa, w, z, t\right) \forall t > 0$$

$$\therefore M\left(Sa, w, z, kt\right) = 1 \forall t > 0$$

$$\therefore \quad Sa = w$$

This completes the proof of Theorem.

Note 3.7: It is evident that Theorem 2.16 becomes a corollary of Theorem 3.6, because the pairs (AB, S) and (PQ, T) are sub compatible of type A implies that AB and PQ are occasionally sub compatible with respect to S and T respectively.

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