Optimization Of Hypoid Gear Using Genetic Algorithm

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Abstract

Genetic algorithm (GA) is a Non-Traditional method is useful and applicable for optimization of mechanical component design. The GA is an efficient search method which is inspired from a natural genetic selection process to explore a given search space. In this work, GA is applied to minimize the volume of hypoid gear with respect to a specified set of constraints. Module, face width and number of teeth of hypoid are used as design variables and the bending stress, contact stress and a geometric limit on the face width are set as constraints. The results showed that the optimal procedure reduced the volume of a gear designed according to ANSI/AGMA 2003-B97 to 54\% of its original volume. Further analysis was performed to study the effect of the design variables and the input parameters of the objective function.

Keywords: Genetic algorithms, hypoid gears, volume optimization.

I. INTRODUCTION

Gears are used in most types of machinery and vehicles for the transmission of power. The design of gears is highly complicated involving the satisfaction of many constraints such as strength, pitting resistance, bending stress, scoring wear, and interference in involutes gears and so on. The main concentration focuses on hypoid gear sets, which are used to transmit motion between Non-Intersecting and Non-parallel Shaft. However, geometrical design and strength evaluation of the hypoid gear depend on the machine tool of specific production companies because the geometrical design and strength evaluation of the hypoid gear are complex and difficult\cite{1}.

Hypoid gears are used in various automotive, rotorcraft and industrial applications to transmit power between two perpendicular shafts having a certain amount of offset. They also find a wide range of applications in transportation equipment such as Lorries, ships, helicopters, earth moving equipments, and construction equipments. Hypoid gears are similar to spiral bevel gears except that the shaft center lines do not intersect. The shaft offset introduces several advantages to hypoid gears such as larger pinion size with fewer numbers of teeth, higher contrast ratio, and lower contact stresses. However, higher relative sliding velocity between contacting surfaces results in high power losses and wear rates are among the most common problems found in hypoid gears\cite{2,3}.

Many numerical optimization algorithms such as GA, Simulated Annealing, Ant-Colony Optimization, and Neural Network have been developed and used for design optimization of engineering problems to find optimum design. Solving engineering problems can be complex and a time consuming process when there are large numbers of design variables and constraints. Hence, there is a need for more efficient and reliable algorithms that solve such problems. The improvement of faster computer has given chance for more robust and efficient optimization methods. Genetic algorithm is one of these methods. The genetic algorithm is a search technique based on the idea of natural selection and genetics\cite{4}.

Ki-Hun Lee,\cite{5} studied the Optimum Design Method of Hypoid Gear by Minimizing Volume and the optimum decreases is 12.5\%.

II. PRINCIPLE OF GENETIC ALGORITHM

Genetic algorithm (GA) maintains a population of encoded solutions, and guides the population towards the optimum solutions\cite{5}. Fitness function provides a measure of performance of an individual how fits. Rather than starting from a single point solution within the search space as in traditional optimization methods, the genetic algorithm starts running with an initial population which is coding of design variables. GA selects the fittest individuals and eliminates the unfit individuals in this way. The flow chart of a genetic algorithm is shown in Figure 1. An initial population is chosen randomly at the beginning, and fitness of initial population individuals is evaluated. Then an iterative process starts until the termination criteria have been run across. After the evaluation of individual fitness in the population, the genetic operators, selection, crossover and mutation are applied to breeding a new generation. Other genetic operators are applied as needed. The newly created individuals replace the existing generation and reevaluation is started in fitness of new individuals. The loop is repeated until an acceptable solution is found\cite{6}. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{flowchart.png}
\caption{Flow chart of genetic algorithm.}
\end{figure}
III. OPTIMIZATION FORMULATION

An optimization search technique based on genetic algorithms is considered in the present work to minimize the volume of hypoid gears with constraints on the bending stress, contact stress and a geometric limit on the face width. The design variables considered for optimization are the module, face width and number of teeth.

3.1 BASIC PARAMETERS

Parameters considered in the design of the hypoid gear pair include: Power, Type, Speed, Hypoid offset, Gear ratio, Pressure angle, and Shaft angle, as illustrated in Table 1.

<table>
<thead>
<tr>
<th>Table 1: Basic hypoid gear design parameter data</th>
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<tbody>
<tr>
<td>Power</td>
</tr>
<tr>
<td>Type</td>
</tr>
<tr>
<td>Speed</td>
</tr>
<tr>
<td>Hypoid offset</td>
</tr>
<tr>
<td>Gear ratio</td>
</tr>
<tr>
<td>Pressure angle</td>
</tr>
<tr>
<td>Shaft angle</td>
</tr>
</tbody>
</table>

3.2 OBJECTIVE FUNCTION

The volume considered in the calculations is the pinion volume simplified to a truncated cone at the pitch cone [7-9]. Volume (V) of a truncated cone (Frustum) shown in Figure 2 is given by:

\[ V = \frac{\pi}{3}b \cdot \cos(\gamma_f) \left( \frac{m_z}{2} + \frac{m_z R - b}{2} \right)^2 \frac{F_d}{2} \left( \frac{A_F}{A_o} \right) \]

(1 a)

\[ V = \frac{\pi}{3} \cdot F \cdot \cos(\gamma_f) \left( \frac{n}{2P_o} \right)^2 \left( \frac{n}{2P_o} \frac{A_F}{A_o} \right) \]

(1 b)

The optimization problem can be presented as follows:

**Minimize:** Volume of pinion pitch frustum (A function in number of teeth, module and face width)

**Subject to:**

- Working contact Stress - Allowable contact stress \( \leq 0 \) \( G_1(x) \)

\[ C_p \sqrt{K_h K_v K_m C_s C_X C} \frac{2T_p}{F_d J} \leq 0 \]

- Working bending stress - Allowable bending stress \( \leq 0 \) \( G_2(x) \)

\[ \frac{2T_p}{F_d} \frac{P_d K_o K_v K_m}{K_v J} \leq 0 \]

- \( 3 \times F - A \leq 0 \) \( G_3(x) \)

3.3 CONSTRAINTS ON THE OBJECTIVE FUNCTION

Constraints are conditions that must be met in the optimum design and include restrictions on the design variables. These constraints define the boundaries of the feasible and infeasible design space domain. The constraints considered for the optimum design of minimizing pinion volume are the following:
\[ \beta = \frac{S_{uc}}{S_{ut}} \quad (2a) \]

\[ \beta = \frac{\sigma_{H\text{lim}}}{\sigma_{F\text{lim}}} \quad (2b) \]

Constraints \( G_1 \) and \( G_2 \) are solved as follows:

First for \( G_1 \) the working contact stress is equated with the allowable contact stress as in Eq. (3).

\[ S_{ac} = \frac{P_T C_P S_H K_T C_K}{n C_L C_H} \sqrt{\frac{2T_p}{F}} K_n K_A K_m C_s C_{wc} \quad (3a) \]

\[ \sigma_{H\text{lim}} = \frac{Z_s S_h K_T Z_w}{m n Z_{NT} Z_{wc}} \sqrt{\frac{2000T_i}{b Z_1}} K_i K_A K_{H\beta} Z_s Z_{wc} \quad (3b) \]

For \( G_2 \) the working bending stress is equated with the allowable bending stress as in Eq. (4).

\[ S_{bd} = \frac{2T_p P_T^2 K_K K_v K_v S_f K_i K_R}{F n K_j K_L} \quad (4a) \]

\[ \sigma_{F\text{lim}} = \frac{2000T_i}{b m^2 Z_1} K_A K_v Y_s K_{H\beta} S_f K_B Y_Z \quad (4b) \]

From equation (2), (3), and (4), the following relations are obtained

\[ \frac{2T_p P_T^2 K_K K_v K_v S_f K_i K_R}{F n K_j K_L} = \frac{2T_p}{n C_v} K_K K_v C_s C_{wc} \quad (5a) \]

\[ Z_z S_h K_T Z_w \sqrt{\frac{2000T_i}{b Z_1}} K_A K_B Z_s Z_{wc} = \beta \frac{2000T_i}{b m^2 Z_1} K_A K_B Y_s Y_Z \quad (5b) \]

Therefore, from Eq (5), we get

\[ P_d = \frac{C_P S_H C_R K_T Z_s}{\beta C_v C_s S_f K_K K_R} \sqrt{\frac{F C_s C_{wc}}{2T_p I K_A K_m}} \quad (6a) \]

\[ m = \frac{Z_s S_h Z_w Y_s Y_{NT}}{\beta Z_{NT} S_f Y_s Z_w} \sqrt{\frac{b Z^2 Z_{wc}}{2000T_i Z_1 K_A K_B}} \quad (6b) \]

Using Eq (3) and rearranging, we get

\[ n = \frac{P_d C_P S_H K_T Z_s}{S_{ac} C_s C_{wc}} \sqrt{\frac{2T_p}{F I} K_A K_K K_m C_s C_{wc}} \quad (7a) \]

\[ z_i = \frac{Z_s S_h Z_s}{m \sigma_{H\text{lim}} Z_{NT} Z_{wc}} \sqrt{\frac{2000T_i}{b Z_1}} K_A K_K K_{H\beta} Z_s Z_{wc} \quad (7b) \]

**IV. RESULTS and DISCUSSION**

The developed CAD system along with the associated optimization module is used to study the effect of different optimization inputs on optimum gear parameters over a wide range of practical values. The results compared with results of analytical method, as shown in Table 2.

<table>
<thead>
<tr>
<th>Design variables</th>
<th>Analytical method</th>
<th>Genetic algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module (m)</td>
<td>5.621 mm</td>
<td>5.08 mm</td>
</tr>
<tr>
<td>Face width (f)</td>
<td>47.041 mm</td>
<td>34.155 mm</td>
</tr>
<tr>
<td>Number of teeth</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>Minimum volume</td>
<td>92703.3 m³</td>
<td>50429.6 m³</td>
</tr>
</tbody>
</table>

The starting point of analytical method, \( m=5.621, f =47.041, N =11 \). The programmer, Developed in MATLAB 7.0 for analytical method has been run several times for different values of design variables. The results obtained are given in Table 2. As can be seen from the results, the genetic algorithm produced much better results than analytical method.

- **Effect of Torque on the Optimum Design Parameters**

  Figs. (3) To (6) shows that the optimum design parameters (module, face width, number of teeth, and minimum pinion volume) increases with increasing the torque and decreases with decreasing the rpm.

- **Effect of Material Property Ratio at Different Torque Value (\( \beta \))**

  Figs. (7) To (10) shows that the optimum design parameters (module, face width, number of teeth, and minimum volume) increases with increasing input torque, and decreases with decreasing the material property factor (\( \beta \)).
Fig. (3) Effect of torque on optimum module for max. & min. rpm, $\beta = 5$, $S_{ult}(\sigma_{F_{lim}}) = 205$ MPa, $m_G(r) = 4$, face width to cone distance $\leq 1/3$

Fig. (4) Effect of torque on optimum face width for max. & min. rpm, $\beta = 5$, $S_{ult}(\sigma_{F_{lim}}) = 205$ MPa, $m_G(r) = 4$, Face Width to cone distance $\leq 1/3$

Fig. (5) Effect of torque on the optimum number of teeth for max. & min. rpm, $\beta = 5$, $S_{ult}(\sigma_{F_{lim}}) = 205$ MPa, $m_G(r) = 4$, face width to cone distance $\leq 1/3$
Fig. (6) Effect of torque on optimum volume for max. & min. rpm, $\beta = 5, S_w (\sigma_{F_{lim}}) = 205$ MPa, $m_G (r) = 4$, face width to cone distance $\leq 1/3$

Fig. (7) Effect of torque on optimum module for different material property factor, $S_w (\sigma_{F_{lim}}) = 205$ MPa, $n_p (n_1) = 1700$ rpm, $m_G (r) = 4$

Fig. (8) Effect of torque on optimum face width for different material property factor, $S_w (\sigma_{F_{lim}}) = 205$ MPa, $n_p (n_1) = 1700$ rpm, $m_G (r) = 4$
V. CONCLUSIONS
The aim of this study was to minimize the volume of the hypoid gear system using the genetic algorithm and numerical optimization method. The results obtained showed that the genetic algorithm to provide better solution than those obtained from numerical optimization method. It can be concluded that the genetic algorithm that can be successfully and efficiently used for the hypoid gear system design.

The results showed that the optimal procedure reduced the volume of a gear designed according to ANSI/AGMA 2003-B97 to 54% of its original volume.

References


Abbreviations

\(A_0 (R_0)\) Outer cone distance, in (mm).

\(P_d (m)\) Outer diametral pitch, in \(^{-1}\).

\(F (b)\) Face width, in (mm).

\(z_1\) Pinion number of teeth.

\(\gamma_p\) Pinion pitch angle.

\(d (d_{el})\) Pinion outer pitch diameter, in (mm);

\(T_p (T_1)\) Operating pinion torque, lb in (Nm).

\(\beta\) Material property factor.

\(S_c (\sigma_{H})\) Calculated working contact stress, lb/in\(^2\) (N/mm\(^2\)).

\(S_l (\sigma_{F})\) Calculated working bending stress at the root of the tooth, lb/in\(^2\) (N/mm\(^2\)).

\(S_{ac} (\sigma_{HP})\) Allowable contact stress, lb/in\(^2\) (N/mm\(^2\)).

\(S_{at} (\sigma_{FP})\) Allowable bending stress, lb/in\(^2\) (N/mm\(^2\)).

\(S_{ac} (\sigma_{HP})_{lim}\)

\(S_{at} (\sigma_{FP})_{lim}\)

\(K_0 (K_A)\) Overload factor.

\(K_1 (Q_v)\) Dynamic factor.

\(K_s (Y_\beta)\) Tooth lengthwise curvature factor.

\(K_m (K_{HP})\) Load distribution factor.

\(K_i (Y_j)\) Size factor.

\(K_L (Y_{NT})\) Stress cycle factor.

\(K_T (K_0)\) Temperature factor.

\(K_R (Y_j)\) Reliability factor.

\(C_{sc} (Z_{xc})\) Crowning factor.

\(C_p (Z_\xi)\) Elastic coefficient, [lb/in\(^2\)]\(^{0.5}\) ([N/mm\(^2\)]\(^{0.5}\)).

\(C_s (Z_X)\) Size factor.

\(C_t\) Stress cycle factor.

\(C_N (ZNT)\)

\(C_H (Z_m)\) Hardness ratio factor.

\(C_R (Z_j)\) Reliability factor.

\(S_{HI}\) Contact safety factor.

\(S_{FH}\) Bending safety factor.

\(J (Y_j)\) Bending strength geometry factor.

\(I (Z_j)\) Pitting resistance geometry factor.