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Torsional Response of Continuous Thin Walled Box Girder Structures

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ABSTRACT
This work considers the interactive response of continuous thin-walled box structures in pure and warping torsion. The derivation of the torsional equilibrium equation commenced on the basis of Vlasov's stress displacement relations by the variational approach. The existing equation due to St. Venant's classical torsion theory in which the torsional constants was based on the independent summation of the combined torsional stresses was compared with the derived equation by Osadebe which took into consideration the interactive effect of pure and warping torsion. Closed form expressions that explicitly described the torsional stresses along beam was developed. The results of the comparative analysis based on the numerical analysis using the method of initial value approach further revealed that the present equation based on the interactive effect of the combined torsional states generates a significant effect on torsional rigidity with greater contribution from the internal concentrated torque, giving rise to higher stresses.
Keywords –Continuous Box Girder, Initial Value Approach, Internal Concentrated Torque, St. Venant, Thin Walled Structures, Torsional response

I. INTRODUCTION

Thin-walled structures comprises an important and growing proportion of engineering construction with areas of application becoming increasingly diverse, ranging from aircraft, bridges, ships and oil rigs to storage vessels, industrial buildings and warehouses[1]. According to Schafer (2002),many factors, including cost and weight economy, new materials and processes and the growth of powerful methods of analysis have contributed to the growth in the use of thin walled elements, thin walled structures are structures made from thin plates joined along at their edges. The plate's thickness is small in comparison to other cross sectional dimensions which are in turn small to the overall length. the ratio of the thickness (t) to the other linear dimension (length l and width h) ranges within the limits h/t or l/t is ≡ 10 to 60. When thin wall structures are twisted, there is a phenomenon called warping of the cross section Warping is defined as the out-of-plane distortion of the beam's cross section in the direction of its longitudinal axis due to the twisting of its cross section [2-3]. Due to the limitations of Bernoulli's theory, as regards thin wall structures under generalized loading, the search for a more acceptable method of analysis was necessitated. The theory proposed by V.Z Vlasov, a Russian scientist amongst others was accepted and was adopted [4]. V.Z Vlasov (1906-1958) the pioneer of thin walled beam theory presented a solution for isotropic , closed section beams containing of flat walls, his solution in pure torsion, he assumed independent warping functions for each wall segment and hence, no cross sectional properties were presented.[5]

II. REVIEW OF PAST WORK

C.P Heins in his work considered thin wall open section where torsional response was considered in an open I section, his work did not consider close sections. Also Chidolue in his work considered also simply supported beams with torsional distortional effect but did not consider the effect in a continuous thin wall closed sections, Osadebe and Mbajiorgu also worked on finite difference formulation of torsion in thin walled elastic beam with arbitrary open and closed section, this work stemming from previous work highlights torsional effect in a continuous thin wall box girder putting into consideration the alternative formulation of the fourth order torsional equilibrium equation derived by Osadebe in his work on modal interaction using a closed section with initial value approach used in obtaining closed form solutions where constants obtained and forces explicitly showing the interactions at the box girder interface and torsional moments and bimoments generated over a typical numerical example producing the internal concentrated torque and its contribution to torsional stress in the box girder.[4-8].

III. EQUILIBRIUM TORSION AND COMPATIBILITY TORSION

Whatever the cause of the internal forces and deformations in a structure, three basic conditions must be observed. These are (i) the equilibrium of forces (ii) the compatibility of displacements and (iii) the laws of material behavior [7]. The first condition requires that the internal forces balance the externally applied loads/forces. The second condition requires that the deformed structure fits together, i.e. that the deformations of the members are compatible. The third condition shows the relation in linear elasticity in which Hooke's law is valid. Equilibrium torsion occurs when the torsional resistance is required to maintain static equilibrium. In this case, if sufficient torsional resistance is not provided, the structure will become unstable and collapse. External loads have no alternative load path and must be resisted by torsion. Compatibility torsion develops where redistribution of torsional moments to adjacent members can occur. The term compatibility refers to the compatibility of deformations between adjacent parts of the structure. In cases where equilibrium torsion exists or torsional behavior dominates the structural response, Engineers must design for maximum torsional moment

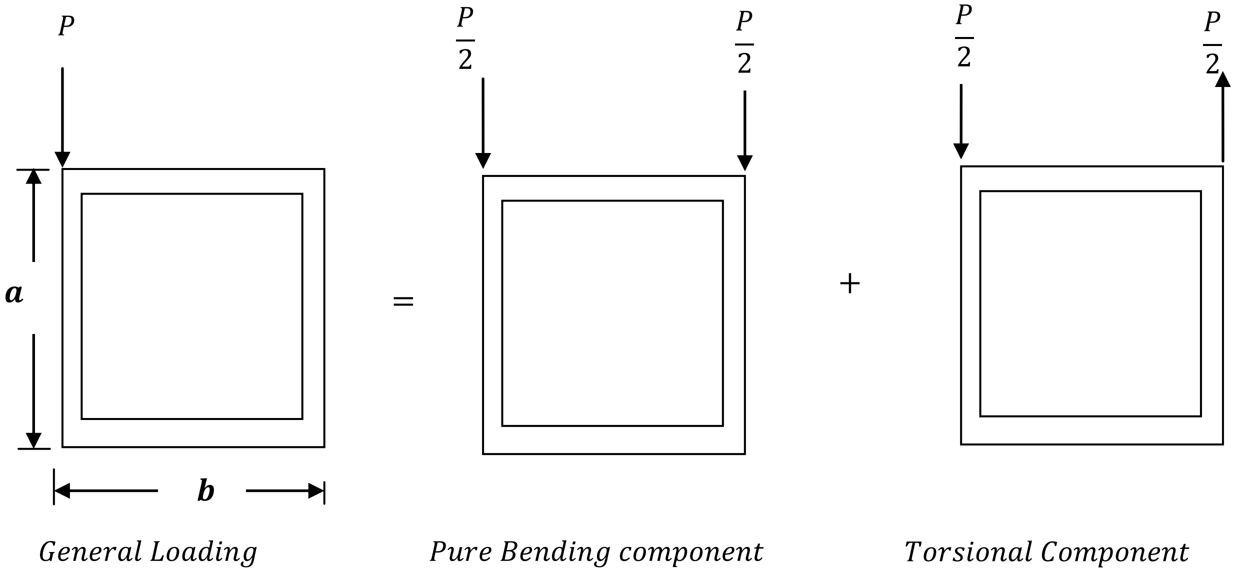

Fig 1: General loads separated into pure bending and force couple

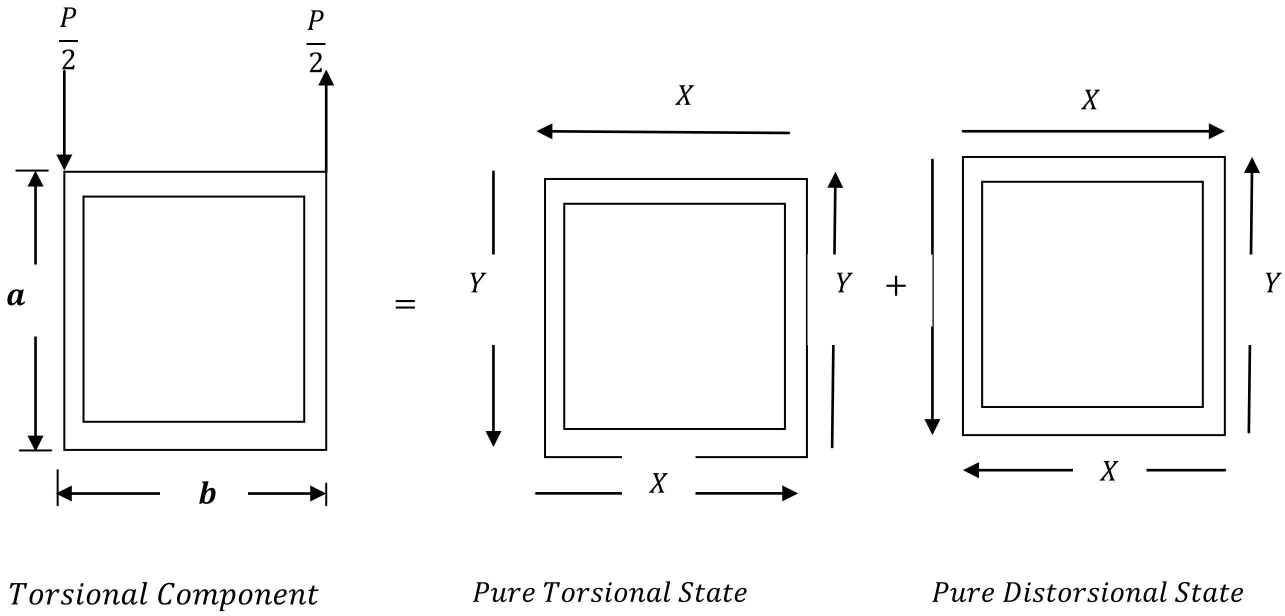

Fig 2: General loads separated into pure bending and force couple

The magnitude of the torsional shear stresses component as shown in Figure 2.3 is evaluated.

From torsional components
$\frac{P}{2} * a = M$ (a)
From pure torsional state:
$X * a + Y * b = M$ (b)
From pure distortional state:
$X * a = Y * b$ (c)
From equation c
$X = \frac{b}{a} Y$ *and substitute in b to have*
$\frac{b}{a} Y * a + Yb = M$
$2Yb = M$
From equation a, $M = \frac{Pa}{2} \rightarrow 2Yb = M = \frac{Pa}{2}$
Stresses on the box beam obtained by fig. 2 evaluated is given by equations d and e
$Y = \frac{Pa}{2} * \frac{1}{2b} = \frac{Pa}{4b}$ (d)
$X = \frac{b}{a} * Y = \frac{b}{a} * \frac{Pa}{4b} = \frac{P}{4}$ (e)
Torsional component obtained from Eccentric generalized loads is separated into pure torsional state and distortional states. A box girder is a Special Case of a folded plate structure in which the plates are arranged to form a closed section [8].
Osadebe (1993) used Vlasov's stress- displacement functions and related stress –strain expressions to derive the equation of torsional equilibrium of a single cell box structure subjected to pure and warping torsion. The obtained equation is given by
$$\boldsymbol{q_x = EI_W\varphi^{IV} - GD_T\varphi''} \quad 1$$
$\boldsymbol{q_x}$= Line load per unit area applied in the plane of the box girder plates E = Modulus of elasticity G = Shear modulus
The conventional equation by St. Venant's classical torsion theory is
$$\boldsymbol{q_x = EI_W\varphi^{IV} - GK_T\varphi''} \quad 2$$
Osadebe (1993) on the basis of Vlasov's theory derived expressions of Torsional equilibrium by the calculus of variations.
From Lagrange's principle, Vlasov expressed displacement in the longitudinal and transverse direction in a series form for a thin-walled closed structure subjected to an external torque [9].
$$\boldsymbol{U(x,s) = \sum_{i=1}^{m} \varphi_{i(s)} U_{i(x)}} \quad 3$$
$$\boldsymbol{V(x,s) = \sum_{k=1}^{n} \psi_{k(s)} V_{k(x)}} \quad 4$$
Where $\boldsymbol{U_{i(x)}}$ ***and*** $\boldsymbol{V_{k(x)}}$ are the unknown functions which express the laws governing the variation of the displacement along the X-axis of the box structure.

$$\boldsymbol{K_T} = \oint t(s)\,ds = B - C = \frac{2h_f^2h_w^2t_ft_w}{[h_ft_w+h_wt_f]} \quad 5$$
$$\boldsymbol{D_T} = B - \frac{C^2}{B} = \frac{2h_f^2h_w^2t_ft_w}{[h_ft_f+h_wt_w]} \quad 6$$
$$\boldsymbol{\lambda_K} = \sqrt{\frac{GK_T}{EI_W}} \quad 7$$
$$\boldsymbol{\lambda_D} = \sqrt{\frac{GD_T}{EI_W}} \quad 8$$
As obtained from the derived equation by Osadebe:
$$\boldsymbol{B = \frac{h_fh_w}{2}\left[h_ft_f + h_wt_w\right]} \quad 9$$
$$\boldsymbol{C = \frac{h_fh_w}{2}\left[h_ft_f - h_wt_w\right]} \quad 10$$
The strain energy $\boldsymbol{S_E}$ of the box structure is given by:
$$\boldsymbol{S_E = \frac{1}{2}\iint_0^l \left[\frac{{\sigma_x}^2}{E} + \frac{{\tau_{xy}}^2}{G}\right] t(s)\,ds} \quad 11$$
Work done $\boldsymbol{W_E}$ by the externally applied torque is given by
$$\boldsymbol{W_E = \frac{1}{2}\iint_0^l [q(x).V_K(x)]t(s)\,ds} \quad 12$$
Potential energy of the torsionally loaded thin wall box beam is given by
$$\boldsymbol{\Pi = S_E - W_E} \quad 13$$

For a thin-walled box beam, St. Venant pure torsion constant is given by:

$$\boldsymbol{\Pi = \frac{1}{2}\iint_0^l \left[\left[\frac{{\sigma_x}^2}{E} + \frac{{\tau_{xy}}^2}{G}\right] t(s) - q(x).V_K(x)\right] dx\,ds} \quad 14$$
Integrating (14) and simplifying further to obtain.
$$\boldsymbol{V^{IV} - \frac{GK_T}{EI_W}V'' = \frac{q_x}{EI_W}} \quad 15$$

The bimoment is a function of the second derivative of the rotation or the out of plane displacement

IV. INITIAL VALUE APPROACH

In engineering practice and applications we are not only interested in the general solution (complimentary solution and particular integral) of a given differential equations , but also in the particular solution satisfying the given initial conditions.

The initial value approach is adopted such that all the conditions necessary for obtaining the desired particular solution are all given at the origin[10].

Initial conditions.
At $x = 0$

Consider a single cell box girder subjected to a torque per unit length with arbitrary support conditions as shown in figure 3.

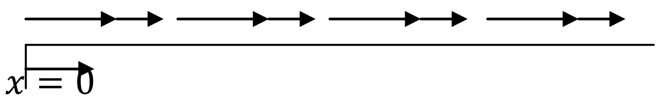

Fig 3. Single cell box girder subjected to a torque per unit length with arbitrary support conditions

$$V_\varphi = A + Bx + C\cosh(\lambda x) + D\sinh(\lambda x) - \frac{qx^2}{2\lambda^2 EI_W} \quad 16$$

$$V_\varphi = y_0 : V_\phi' = \theta_0 : V_\varphi'' = M_0 : V_\varphi''' = Q_0 \quad 17$$

$$V_\varphi = y_0 + \square_0 x + \frac{M_0}{\lambda^2}\cosh(\lambda x - 1) + \frac{Q_0}{\lambda^3}(\sinh(\lambda x) - \lambda x + q2\lambda 4EIW2\cosh\lambda x - \lambda x2 - 2 \quad 18$$

$$V_\varphi' = \square_0 + \frac{M_0}{\lambda}\sinh(\lambda x) + \frac{Q_0}{\lambda^2}(\cosh(\lambda x) - 1) + \frac{q}{\lambda^3 EI_W}(\sinh(\lambda x) - \lambda x) \quad 19$$

$$V_\varphi'' = M_0\cosh(\lambda x) + \frac{Q_0}{\lambda}\sinh(\lambda x) + \frac{q}{\lambda^2 EI_W}(\cosh(\lambda x) - 1) \quad 20$$

$$V_\varphi''' = M_0\lambda\sinh(\lambda x) + Q_0\cosh(\lambda x) + \frac{q}{\lambda EI_W}\sinh(\lambda x) \quad 21$$

V. TORSIONAL STRESS ANALYSIS

Continuous Span.

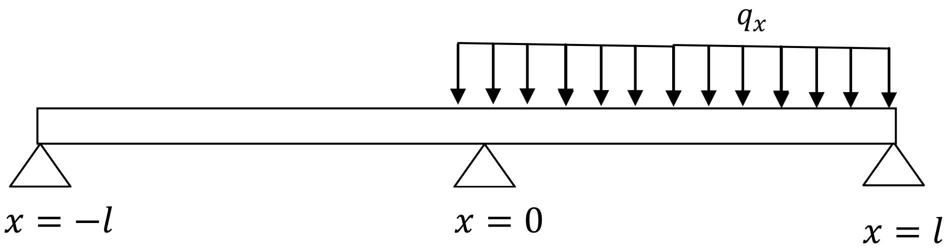

Fig 4. Continuous thin walled box beam with loaded distributed torque.

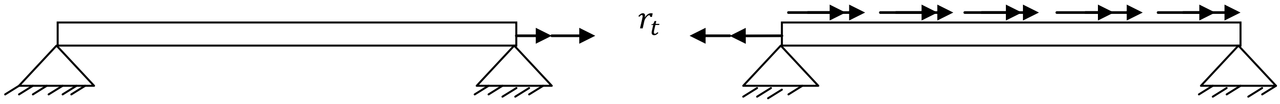

Fig 5. Discretization of the continuous span showing the internal indeterminate concentrated torque r_t that exists.

5.1 NUMERICAL ANALYSIS

$h_f = 0.3m$; $h_w = 0.15m$;
E =1.6*10$^6 KN/m^2$; $L = 5m$
t_f =0.03m ; t_w = 0.015m ;
G=6.6*10$^5 KN/m^2$;

The boundary conditions used in this numerical analysis are:

$$\left\| V_\varphi'(x)_l = \overline{V_\varphi'}(x)_r \right\| \ : \quad V_\varphi''(x) = 0 \quad 22$$

$$\left\| V_\varphi''(x)_l = \overline{V_\varphi''}(x)_r \right\| \ : \quad V_\varphi(x) = 0 \quad 23$$

Equations (22) and (23) is applied at continuous beam interface; $x = 0$ as shown in Fig 5 and applied to maintain equilibrium of forces between both girders.

Using equation 22and 23, the constants in equation 24 are uniquely determined using the initial value approach

$$\begin{bmatrix} C_1 & C_2 & C_3 \\ \overline{C_1} & \overline{C_2} & \overline{C_3} \\ \overline{C_4} & & r_t \end{bmatrix} \quad 24$$

The closed form expressions for evaluating the torsional response (girder 1) is given by

$$V_\varphi(x) = \frac{r_t L}{\lambda^2 EI_W}\left[1 - \cosh(\lambda x) - \coth\lambda L\sinh(\lambda x) + \frac{x}{L}\right] \quad 25$$

$$V_\varphi'(x) = \frac{r_t L}{\lambda EI_W}\left[-\sinh(\lambda x) - \coth\lambda L\cosh(\lambda x) + \frac{1}{\lambda L}\right] \quad 26$$

$$V_\varphi''(x) = \frac{r_t L}{EI_W}\left[-\cosh(\lambda x) - \coth\lambda L\sinh(\lambda x)\right] \quad 27$$

The constants for the second span (28) through (32)

$$\overline{C_1} = \frac{r_t L}{\lambda^2 EI_W} - \frac{q_{(x)}}{\lambda^4 EI_W} \quad 28$$

$$\overline{C_2} = \frac{r_t}{\lambda^2 EI_W}\left[1 - 2\lambda L\coth(\lambda L)\right] + \frac{q_{(x)}}{\lambda^3 EI_W\sinh(\lambda L)}\left[\cosh(\lambda L - 1\right. \quad 29$$

$$\overline{C_3} = \frac{q_{(x)}}{\lambda^4 EI_W} - \frac{r_t L}{\lambda^2 EI_W} \quad 30$$

$$\overline{C_4} = \frac{q_{(x)}}{\lambda^4 EI_W\sinh(\lambda L)}\left[1 - \cosh(\lambda L)\right] + \frac{r_t L\coth(\lambda L)}{\lambda^2 EI_W} \quad 31$$

$$r_t = \frac{q}{4\lambda}\left[\frac{\lambda L + \frac{1}{\sinh(\lambda L)}(1 - \cosh(\lambda L))}{1 - \lambda L\coth(\lambda L)}\right] \quad 32$$

From the given analysis, both equations are similar but differ in their torsional constants as the present equation allows for modal interaction. The internal torque obtained on the discretization of the continuous span generates significant contributions to internal stresses.

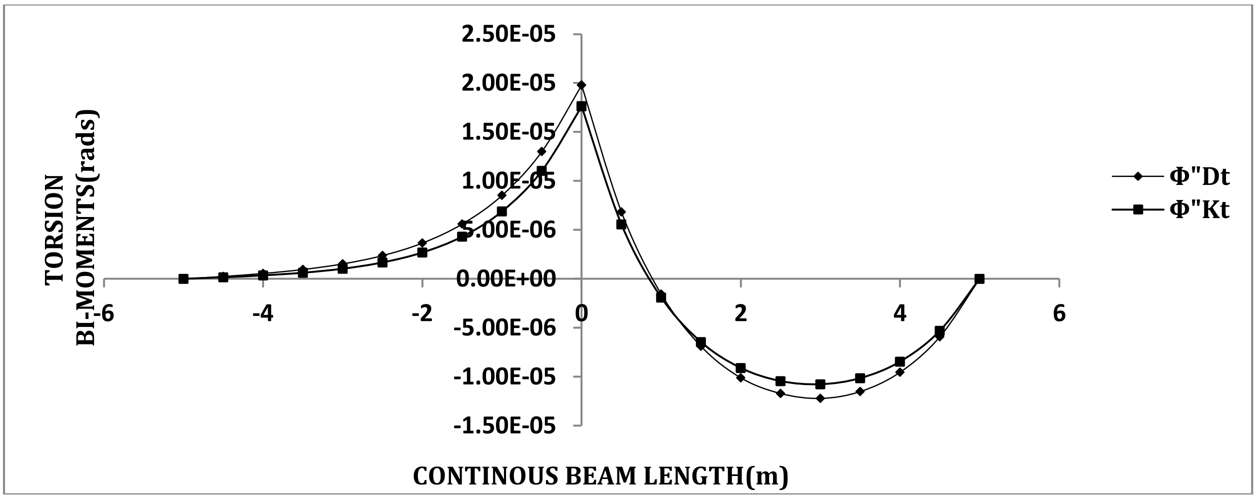

Fig 6. Predicted values of the second order derivative of torsional deflection.

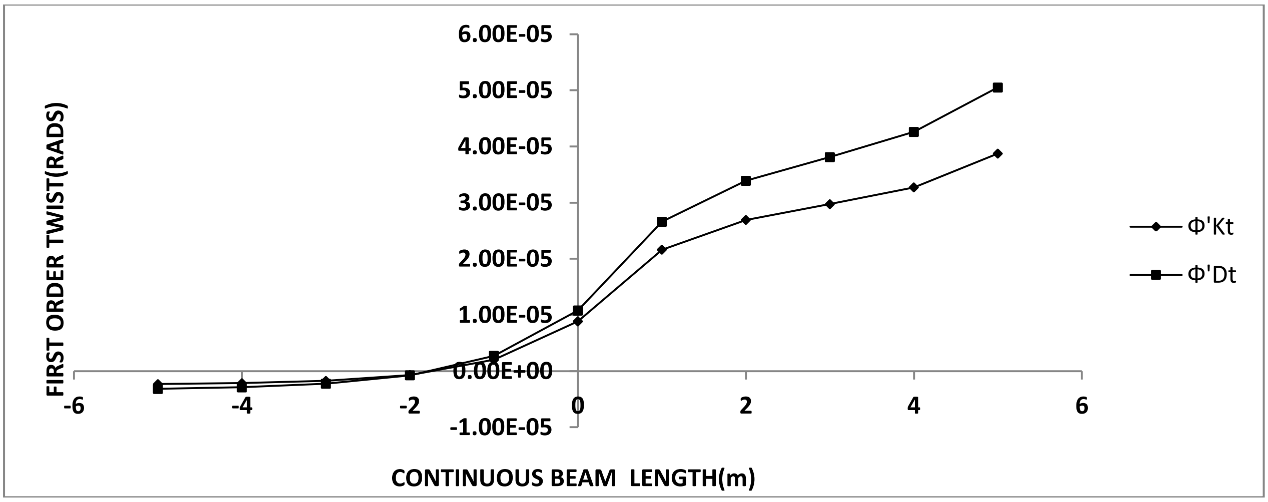

Fig 7. Predicted first order derivative of torsional deflection.

VI. CONCLUSION

The analysis of continuous thin wall box beam presented on the basis of the derivation of torsional equilibrium equation by the variational approach and values obtained by Osadebe was used in the comparative analysis. The results, however goes to show that the present equation which is based on the interactive effect of the combined torsional states generates higher stresses than the conventional equation which was based on the separate summation of the combined torsional states. A closed comparison of the numerical results obtained as shown in the graph clearly shows a difference in results as higher stresses are obtained in the equation involving $\boldsymbol{\lambda_D}$. This interactive effect as shown relative to torsional stresses is paramount in the analysis and design of thin-walled structures in torsion both for economy and safe design with the present formulation in which the internal concentrated torque $\boldsymbol{r_t}$ contributes significantly.

REFERENCES

[1] C.A. Chidolue, Torsional-distortional analysis of thin-walled box girder bridges using Vlasov's theory, Ph.D. thesis, University of Nigeria, Nsukka, 2012.

[2] Osadebe, N.N(1993), "Combined analysis of thin walled box girder structures in pure and warping torsion", Structural Engineering, Analysis and Modeling(SEAM 3), Vol 1. Pg 821-830.

[3] Heins C.P**.** "Bending and torsional design in structural members**"** Lexington books, 1975.

[4] Giginna Emmanuel(1992)."Thin-walled box beam-column stiffness's and their applications in the analysis of frames."Dept. of Civil engineering, University of Nigeria, Nsukka.

[5] Paul A Seaburg (1997)."Torsional analysis of structural steel members" American institute of steel corporation, Inc.

[6] Osadebe, N.N(1993)."Torsional response of self excited axially thin wall box beam "

Journal of the university of science and technology, Kumasi. Structural Engineering, Analysis and Modeling, SEAM Vol. 13,no 1,pg 54-59.
[7] Composite structures, (2009).“Free vibrations of axially loaded thin composite box beams,” volume 90, issue 2, pg 233-244.
[8] Santano Das“Computerized numerical solutions to combined pure and warping torsion in open sections,” Massachusetts institute of technology, 1997.
[9] Nam-Hoi Park et al(2002).Distortional analysis of steel box girders.(steel structures 2).Dept. of Civil and environmental Engineering, Korea university, Seoul.
[10] Osadebe and Mbajiorgu(2006). “Finite element formulation of torsion in thin wall elastic beam with arbitrary open and closed sections,” Nigerian journal of technology volume 25 no 2 pg 36-45.
[11] Osadebe and Chidolue (2012). ‘‘Effect of Flexural-Torsional-Distortional Interactions on the Behaviour of Thin-Walled Mono Symmetric Box Girder Structures” International journal of Engineering research and application