RESEARCH ARTICLE OPEN ACCESS

A Compromise Weighted Solution For Multilevel Linear Programming Problems

Osman M. S.[A], Gadalla M. H.[B], Zeanedean R. A.[C], Rabie R. M.[D*]
[a] (El-Asher University, 10th of Ramadan City, Egypt)
[b] (Department of Mechanical Design and Production, Faculty of Engineering, Cairo University, Egypt)
[c] (Department of Operations Research, , Institute of Statistical Studies and Research, Cairo University, Egypt)
[d*] (Department of Operations Research, Institute of Statistical Studies and Research, Cairo University, Egypt)

ABSTRACT
Decision making is the process of selecting a possible course of action from all available alternatives. Many real world physical situations can be categorized as hierarchical optimization problems and be formulated as Multi-level Programming (MLP) models. Instead of solid optimality concept, it is more accuracy adopting the satisfaction concept that play an important role in the analysis of hierarchical structures and no assumptions or information are required regarding the Decision Makers (DMs) utility function. In this article a compromise weighted solution is presented for MLP problems, where a non-dominated solution set is obtained. In weighting approach, the relative weights represent the relative importance of the objective functions for all DMs whose provide their preferences of their decision variables that is the lower and upper-bounds to the decision variables they control. The hierarchical system is converted into Scalar Optimization Problem (SOP) by finding proper weights using the Analytic Hierarchy Process (AHP) so that objective functions can be combined into a single objective. A brief historical overview and a comparative study is presented for some approaches used in solving MLP problem with the solution obtained in the weighting approach with two cases collateral numerical illustrative examples.
Keywords **-** Multi level decision making; hierarchical structures; weighting approach; scalar optimization problem; analytic hierarchy process.

I. INTRODUCTION

Hierarchical data structures are very common in the social and behavioral sciences and Multi-level (ML) decision making models are developed for analyzing hierarchically structured data. So, MLP is an important branch of Operation Research, this problem consists of two or more levels, namely; first level, second level, and so on up to last level. MLP problem is a sequence of many optimization problems in which the constraints region of one is determined by the solution of other DMs. The first (higher, upper) level Decision Maker (DM1) is called the center (leader). The lower-levels Decision Makers (DM2, DM3 …) called followers. They execute their policies after the decision of higher levels DMs and then the leader optimizes his objective independently but may be affected by the reaction of the followers. ML decision making models are used for representing many hierarchical optimization situations in real word strategic, planning, and management such as; financial control, economic analysis, facility location, government regulation, organizational management, conflict resolution, network design, traffic assignment, signal optimization, planning for resource management, defense, transportation, central economic planning at the regional or national level to create model problems concerning organizational design [1, 2, 3]. ML decision making often involves many uncertain factors and it is hard to formulate. Contributions had been delivered by mathematicians, economists, engineers and many other researchers and developers. In first time, Bi-Level Programming (BLP) (as a special case of MLP) is introduced by *Von Stackelberg* in the context of unbalanced economic markets. After that moment this field has obtained a rapid development and intensive investigation in both theories and applications. Much effort has been done on the development of both linear and nonlinear ML decision making modeling and solution methods. The study of MLP problems is not vast and wide as compared with BLP problems in the literature. Over the last three decades, tremendous amount of research effort has been made on MLP for hierarchical decentralized planning problems leading to the publication of many interesting results in the literature and many methodologies have been proposed to solve it potentially [4, 5].

MLP problem can be defined as a *p*-person, non-zero sum game with perfect information in which each player moves sequentially from top to bottom. ML decentralized models is characterized by a center that controls more than one independent divisions on the lower-levels. For instance, by adopting three criteria with respect to; strategic, production and operational planning as objective functions for three different levels, MLP problem; that is Tri-Level Programming (TLP) problem can be set for hierarchical decision situation in firms with three different DMs in three different levels, one DM on each level [6, 7].

Multi objective decision making solutions procedure cannot be directly applied to MLP problem, since in MLP problem, DMs are on different hierarchical levels and each one controls only a subset of the decision variables. There are two main types of uncertainties in modeling MLP problems; one is that the parameter values in the objective functions and constraints of the leader and the followers may be uncertain or inaccurate; another type of uncertainties involves the form of the objective functions and constraint functions. That is, how to determine the relationships among proposed decision variables and formulate these functions for a real decision problem [8].

Unfortunately, MLP problems are difficult to solve and not every problem has a solution even though it has a nonempty compact feasible set and it have been proved to be ***NP***-hard. The features of it, mainly its nonconvexity, make it difficult one, even when all involved functions are linear. Also, there are some difficulties from nonuniqueness of lower-levels optimal solutions and on its optimality conditions [See, 9, 10].

This article will be organized as: a short overview of ML models, its use, history, characteristics and formulation is presented in section 2, AHP concept and non-dominated solution in section 3. Section 4 provides a weighting approach for generating non-dominated solution for MLP problem. Two numerical illustrative examples and a short discussion are presented in section 5. The article will be finalized with its conclusion.

II. MLP problems Characterizing and Formulation

MLP problems are characterized that a DM at a certain level of the hierarchy may have his objective function and decision space determined partially by other levels where each DM controls over some decision variables. So, the followers can take part of the system decision which be concerned by their control variables because they always try to optimize their objective functions but they must take the goal or preference of the leader into consideration. DM1 defines his objective function and decision variables, this information then constrains the DM2's feasible space and so on. So, the preference information is delivered from the upper-levels to the lower-levels sequentially. The geometric properties of the linear MLP problems are obtained in [11] for general max-min problem and presented in [5] for the linear BLP problem. In [7] showed that when all the functions of the MLP problem are linear and its feasible region is a polyhedron, the optimal solution occurs at a vertex of feasible region. MLP is particularly appropriate for problems with the following characteristics [12]:

1) The system has interactive decision making units within a predominantly hierarchical structure.
2) The external effect on a DM's problem can be reflected in both his objective function and his set of feasible decisions.
3) The loss of cost of one level is unequal to the added gain to other level.
4) The order of the play is very important and the choice of the upper-level limits affects the choice or strategy of the lower-levels.
5) The execution of decision is sequential, from upper to lower-levels.
6) Each DM controls only a subset of the decision variables.
7) Each level optimizes its own objective function independently apart from other levels.
8) Each DM is fully informed about all prior choices.

TLP problem's formulation has different versions that are given in many articles. Linear TLP problem can be formulated as follows [13]:

$$\max_{x_1} \; f_1(X) = c_{11}x_1 + c_{12}x_2 + c_{13}x_3,$$

where, x_2 and x_3 solve:

$$\max_{x_2} \; f_2(X) = c_{21}x_1 + c_{22}x_2 + c_{23}x_3,$$

where, x_3 solves:

$$\max_{x_3} \; f_3(X) = c_{31}x_1 + c_{32}x_2 + c_{33}x_3$$

$$\text{s.t.} \quad A_1x_1 + A_2x_2 + A_3x_3 \le b,$$
$$x_1, x_2, x_3 \ge 0.$$

Where, $X=(x_1, x_2, x_3)$ denote the decision variables under control of DM1, DM2 and DM3 respectively. For i =1, 2, 3, x_i is n_i-dimensional decision variable, and $f_i(X)$ is the related objective function to 1st, 2nd, and 3rd level, respectively. Let $X = x_1 \cup x_2 \cup x_3$ and $n = n_1 + n_2 + n_3$ then, c_{11}, c_{21}, c_{31} are constant row vectors of size $(1\times n_1)$, c_{12}, c_{22}, c_{32} are of size $(1\times n_2)$ and c_{13}, c_{23}, c_{33} are of size $(1\times n_3)$, b is an m-dimensional constant column vector, and A_i is an $m \times n_i$ constant matrix. Each DM has to improve his strategy from a jointly dependent set S ;

$$S = \{X \mid A_1x_1+A_2x_2+A_3x_3 \le b, x_1, x_2, x_3 \ge 0\}.$$

Solution approaches can be classified into four categories; extreme point search, transformation approach, descent and heuristic and evolutionary approach [10]. While, in [14] an additional category is added, interior points approach through the neural network approaches. According to the stages of development, these methods can be classified only into two categories; first one, extreme point search, transformation approach, and descent and heuristic can be referred to as the traditional approaches, and second one, intelligent computation or evolutionary approach and interior point approach are based on more recent developments. Computational methods are diverse from vertex enumeration approaches such as K^{th}-best algorithm [15, 16, 17], *Kuhn-Tucker* approaches, [18, 19] to penalty function approaches [20]. New feasible and efficient algorithms are presented for solving BLP and TLP problems in [21, 22] respectively.

When formulated problems are such difficult classes of optimization problems and consequently it is difficult to obtain exact its optimal solutions, DMs may require approximate optimal solutions. Fuzzy approaches are proposed for obtaining non-dominated solutions using fuzzy membership functions and the tolerance concept which simplifies the representation and the computations for the compromises among levels. The basic concept is the same as implies that each lowest level DM optimizes his objective function, taking a goal or preference of the upper-level DM into consideration. An effective fuzzy method by using the concept of the tolerance membership function of fuzzy set theory to MLP problems is developed in [6] and is extended in [23] for satisfactory solution. A fuzzy approach for MLP problems that is a supervised search procedure with the use of max–min operator presented in [24] to simplify the complex nested structure by utilizing the concept of the degree of satisfaction, in terms of fuzzy membership functions. In [25], further extension for Lai's concept, [6] by introducing the compensatory fuzzy operator. In [26, 27], some developing for alternative MLP techniques based on fuzzy mathematical programming. A fuzzy goal programming procedure for solving quadratic BLP problems presented in [28] and the work in [29] presented for solving BLP and TLP non-linear multiobjective problems as an extension of the fuzzy approach for MLP problems in [22]. In [30], a presentation of a fuzzy goal programming method to overcome such difficulties in MLP problems for proper distribution of decision powers to the DMs to arrive at a satisfaction decision for overall benefit of the organization.

Interactive procedures have met with a great success with such situations include full cooperation among DMs without predetermined preference information, by using interactive and fuzzy interactive methods, MLP problem can be solved, giving the best satisfactory results where the leader satisfies his maximal (updated maximal) satisfaction level and also, each DM in lower-levels accepts his satisfactory level. The basic concept is that the computational complexity with re-evaluation of the problem repeatedly by redefining the elicited membership functions values in the solution search process for searching higher degree of satisfaction and obtaining the satisfactory solutions. In [31], suggestion of an interactive approach for nonlinear bi-level multiobjective decision making problem, while in [32], an interactive fuzzy programming for linear MLP problems is presented. In [33], an interactive fuzzy programming for 0–1 MLP problems through genetic algorithms is proposed. Interactive fuzzy programming approaches for both linear fractional and decentralized BLP problems are presented in [34, 35]. In [36], there is a presentation of weighting method for BLP. A new algorithm for solving BLP problems is presented in [24] and in [37] a global optimization algorithm for solving linear fractional BLP problem.

Recently, an assignment scheme of relative satisfaction for the higher-level DM is proposed in [38] to ensure his leadership and therefore prevent the paradox problem reported in the literature, where lower-level DMs have higher satisfaction degrees than that of the higher-level DM. a fuzzy TOPSIS (technique for order preference by similarity to ideal solution) algorithm is proposed in [39] to solve BL multi-objective decision making problems, the model is a multiple criteria method to identify solutions from a finite set of alternatives based upon simultaneous minimization of distance from an ideal point and maximization of distance from a nadir point for getting the satisfactory solution. an approach based on particle swarm optimization is proposed to solve nonlinear BLP problem in [40] by applying *Kuhn-Tucker* condition to the lower-level problem and transforming the problem into a regular nonlinear programming with complementary constraints, then the approach is applied for getting the approximate optimal solution. Using the concept of chance constraints, an interactive fuzzy programming method for stochastic BLP is proposed in [41]. It has an advantage that candidates for a satisfactory solution can be easily obtained through the combined use of the bisection method and the phase one of the simplex method. An explicit solution to MLP problems is presented in [19], and a new algorithm for solving linear TLP problems in [25] and in [26] a fuzzy mathematical programming applied to MLP problems is developed.

Compromise or coordination is usually needed in order to reach a satisfactory solution, even in noncooperative environments. Most real world decision problems involve multiple criteria that are often conflict in general and it is sometimes necessary to conduct trade-off analysis in multiple criteria decision analysis. Because of the special nature of the problem and the need of adopting some cooperation among DMs, many approaches had been presented to solve MLP problems mostly based on fuzzy and interaction concepts. While BLP is a special case of MLP, and only a special case, is considered in [36], the main motivation in this submission was studying its applicability with the general state, MLP. In literature, any solution approach used for special cases needs more studies while used for the general case. In this article, an extension work of [36] is considered, where the weighting approach allows DMs to provide two issues; their preferences bound for the decision variables that are the lower and upper-bounds for it, and their assigned importance for objective functions in all levels. In weighting approach the hierarchical system will be converted into SOP by finding the proper weights for all objective functions and pairwise comparisons manner using AHP [42, 43, 44] so that objective functions of three levels can be combined into a single objective function, where its relative weights represent the relative importance of DMs's objective functions. After assessing the consistency of the pairwise judgments, a non-dominated solution set is obtained. Perhaps the most creative task in making a decision with the hierarchical situations is to choose the factors that are important for that decision.

III. AHP and Non-dominated solution

AHP is a mathematical technique developed for incorporating multi criteria decision making and designed to solve its complex problems. AHP and similar methods often use pairwise comparison matrices for determining the scores of alternatives with respect to a given criterion, or determining values of a weight vector. There are many papers applied AHP to solve decision problem. For example, in [45], over 100 applications of AHP in the service and government sectors are studied. While, the majority practitioner agreed to use the effective mathematical technique, eigenvector method proposed in [42], some researchers suggested other choices such as mean transformation, or row geometric mean. For example, in [46, 47] a refined method to adjust the maximum entry being one for the weight of alternatives is developed; in [48] a usage of the geometric mean method instead of the eigenvector method. The process requires each DM to provide judgments about the relative importance of his objective and then specify a preference for it for each other's.

AHP can be conducted in three steps; perform pairwise comparisons, assess consistency of pairwise judgments, compute the relative weights and then, it is enables DM to make pairwise comparisons of importance between objectives according to the scale in table (1).

Because human is not always consistent, the theory of AHP does not demand perfect consistency and allows some small inconsistency in judgment and provides a measure of inconsistency. Before computing the weights based on pairwise judgments, the degree of inconsistency is measured by the Consistency Index (CI). Perfect consistency implies a value of zero for CI. Therefore, it is considered acceptable if CI ≤ 10%. For CI values greater than 10%, the pairwise judgments may be revised before the weights are computed.

Option	Numerical value(s)
Equal	1
Marginally strong	3
Strong	5
Very strong	7
Extremely strong	9
Intermediate judgment values for fuzzy inputs	2, 4, 6, 8

Table (1): Gradation scale for quantitative comparison of alternatives

Mathematically, the weighting method can be stated as follows:

$$\max \ (w_1 f_1(X) + w_2 f_2(X) + \ldots + w_p f_p(X)),$$
$$\text{s. t. } X \in S .$$

The weights w_p operating on $f_p(X)$, can be interpreted as ''the relative weight or worth'' of that objective function when compared to other's then, the solution for previous problem is equivalent to the best compromise solution, i.e., the optimal solution relative to a particular preference structure. Moreover, this optimal solution is a non-dominated solution provided all the weights are positive. Allowing negative weights would be equivalent to transforming the maximizing problem to a minimizing one, for which a different set of non-dominated solutions will be exist. The trivial case where all the weights are zero will simply identify $X \in S$ as an optimal solution and will not distinguish between dominated and non-dominated solutions [36].

The concept of non-dominated solution was introduced by *Pareto*, an economist in 1896. A preferred (best) solution is a non-dominated solution which is chosen by the DM his self that is lies in the region of acceptance of all DMs. Non-dominated solution is to design the best alternative by considering the various interactions within the design constraints that best satisfy the DM by way of obtaining some acceptable levels of quantifiable objective functions. This method be distinguished with; a set of quantifiable objective functions, a set of well defined constraints and a process of obtaining some trade-off information, between the stated quantifiable objective functions. The most common strategy for finding non-dominated solutions of MLP problems is to convert it into a SOP. DMs provide their preference and converting MLP problem into a SOP by finding vector of weights for objectives. A feasible solution $X^* \in S$ is a non-dominated solution if there does not exists any other feasible solution $X \in S$ such that:

$$f_p(X^*) \le f_p(X),\ p = 1, 2,\ldots p \text{ and } f_p(X^*) < f_p(X), \text{ for at least one } p.$$

IV. Weighting approach for MLP problems

Weighting approach does not require any assumptions or information regarding DMs utility function. Considering, the procedure of AHP methodology in three steps; inputs (importance of objectives – bounds of variables), model and output (ranked priorities). In the weighting problem $P(w)$ in the absence explicit preference structure, the strategy is to generate all or representative subsets of non-dominated solutions from which a DM can select the suitable solution. A Linear TLP problem is represented as:

$$\begin{array}{l} \max\limits_{x_1} \; f_1(X) \\ \max\limits_{x_2} \; f_2(X) \\ \max\limits_{x_3} \; f_3(X) \\ \text{s.t.} \quad A_1x_1 + A_2x_2 + A_3x_3 \le b, \\ \qquad\qquad\qquad x_1, x_2, x_3 \ge 0. \end{array}$$

Where, $f_1(X)$, $f_2(X)$ and $f_3(X)$ and constraints are linear functions. Solving SOP involves finding $X^* \in S$ such that $f_p(X^*) \ge f_p(X)\ \forall\ X \in S$. The point X^* is said to be global optimum. If strict inequality holds for the objective functions, then X^* is the unique global optimum. If the inequality holds for some neighborhood of X^*, then X^* is a local or relative optimum while it is strict local optimum if strict inequality holds in a neighborhood of X^*. By using the AHP pairwise comparison process, weights or priorities are derived from a set of judgments. While it is difficult to justify weights that are arbitrarily assigned, it is relatively easy to justify judgments and the basis (hard data, knowledge, experience) for the judgments. Suppose already the relative weights of three objective functions are known, for TL hierarchical objective functions a complete pairwise comparison matrix A can be expressed as; $A=[a_{ij}]$ i,j =1, 2, 3 is a matrix of size 3×3 with the following properties; $a_{ij} > 0$, a_{ii} =1, and $a_{ij}=1/a_{ji}$ for i,j=1, 2, 3, where aij is the numerical answer given by the each DM for the question "How many times objective i is more important than objective j?"

$$A= \begin{array}{c c} & \begin{array}{ccc} f_1 & f_2 & f_3 \end{array} \\ \begin{array}{c} f_1 \\ f_2 \\ f_3 \end{array} & \begin{bmatrix} 1 & a_{12} & a_{13} \\ a_{21} & 1 & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \end{array} = \begin{bmatrix} w_1/w_1 & w_1/w_2 & w_1/w_3 \\ w_2/w_1 & w_2/w_2 & w_2/w_3 \\ w_3/w_1 & w_3/w_2 & w_3/w_3 \end{bmatrix}$$

After the normalized matrix, N of pairwise comparison matrix A for a hierarchical TL structure is designed, the normalized principal eigen vector (priority vector) can be obtained by some ways such as averaging across the rows where, it shows the relative weights for objectives. The weighting problem is to find the 3-dimensional weight vector $W=(w_1,w_2,w_3)^T$ such that the appropriate ratios of the components of W reflect or, at least, approximate all the a_{ij} values (i, j=1, 2 ,3), given by DMs. Then, the weighting problem for linear TLP becomes as follows:

$$\begin{array}{l} P(w) = \max \sum\limits_{p=1}^{3} w_p f_p(X) \\ \text{s.t.} \quad A_1x_1 + A_2x_2 + A_3x_3 \le b, \\ \qquad\qquad L_1 \le x_1 \le U_1, \\ \qquad\qquad L_2 \le x_2 \le U_2, \\ \qquad\qquad L_3 \le x_3 \le U_3, \\ \qquad\qquad x_1, x_2, x_3 \ge 0, \end{array}$$

$$\text{where, } w \in W = \left\{ w \mid w \in R^p,\ w_p \ge 0,\ p=1, 2, 3 \text{ and } \sum_{p=1}^{3} w_p = 1 \right\}$$

L_P and U_P are the lower and upper-bounds of decision variables provided by the respective DM. The previous problem, with a single objective function is solved. Here the weighting coefficients convey the importance attached to objective functions. Suppose that the relative importance of all objective functions and the bounds of the variables are known then the preferred solution is obtained by solving $P(w)$ with infinite number of selections through varying of weight vectors as shown in fig. 1.

The weighting approach is adopting in this article because it is related to the interactive techniques and belongs to the cooperative direction for dealing with the hierarchical structure situations in which applying the satisfaction concept is more suitable than optimality concept to achieve overall satisfactory level among all decision makers that satisfies their preferences or importance. The weighting approach generates non-dominated solutions by utilizing various values of W. In such a case the weighting coefficients W do not reflect the relative importance of the objective functions in the proportional sense, but are only parameters varied to locate the non-dominated points [49]. Solution techniques derived in the MLP problems literature often assume uniqueness [5], which is what is done in the exposition of this article as well.

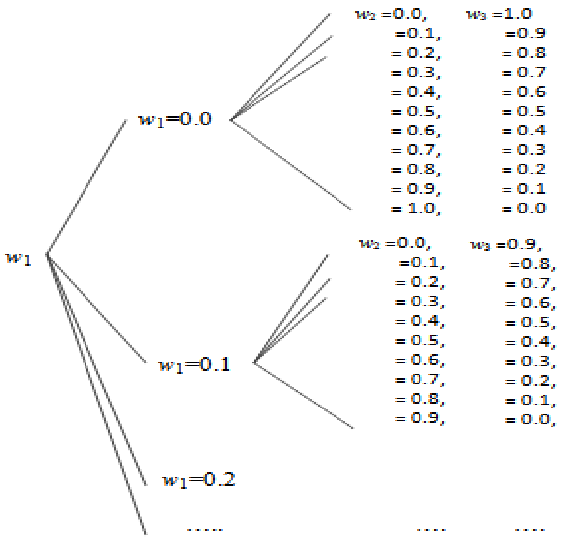

Fig. 1: weighted combinations tree for one decimal value

V. Illustrative Examples

Example 1: Consider the following numerical TLP problem [23];

$$\max_{x_1} f_1(x_1, x_2, x_3) = 7x_1 + 3x_2 - 4x_3,$$

where x_2 and x_3 solve:

$$\max_{x_2} f_2(x_1, x_2, x_3) = x_2,$$

where x_3 solves:

$$\max_{x_3} f_3(x_1, x_2, x_3) = x_3$$

$$\begin{aligned} \text{st:}\quad & x_1 + x_2 + x_3 \le 3, \\ & x_1 + x_2 - x_3 \le 1, \\ & x_1 + x_2 + x_3 \ge 1, \\ & -x_1 + x_2 + x_3 \le 1, \\ & x_3 \le 0.5, \\ & x_1, x_2, x_3 \ge 0. \end{aligned}$$

The pairwise comparison matrix, A of order 3 and its normalized matrix, N for the hierarchical TLP objective functions are given as:

$$A = \begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \end{array} \begin{array}{c} \begin{array}{ccc} f_1 & f_2 & f_3 \end{array} \\ \begin{bmatrix} 1 & 2 & 4 \\ 0.5 & 1 & 1 \\ 0.25 & 1 & 1 \end{bmatrix} \end{array}$$

$$N = \begin{bmatrix} 1/1.75 & 2/4 & 4/6 \\ 0.5/1.75 & 1/4 & 1/6 \\ 0.25/1.75 & 1/4 & 1/6 \end{bmatrix} = \begin{bmatrix} 0.57 & 0.5 & 0.66 \\ 0.29 & 0.25 & 0.17 \\ 0.14 & 0.25 & 0.17 \end{bmatrix}$$

The following priority vector that is normalized relative weights $W=(w_1,w_2,w_3)^T$ can be obtained by;

$$W = \frac{1}{3}\begin{bmatrix} 0.57 + & 0.5 + & 0.66 \\ 0.29 + & 0.25 + & 0.17 \\ 0.14 + & 0.25 + & 0.17 \end{bmatrix} = \begin{bmatrix} 0.58 \\ 0.24 \\ 0.18 \end{bmatrix}$$

The principal eigen value; λ_{max}=1.75(0.58)+ 4(0.24) + 6(0.18) =3.055. Then, the consistency index (CI) = ($\lambda_{max} - n$)/n - 1=(3.055– 3)/2 =0.0275 where, n denote number of comparisons. While Random Consistency Index (RI)=0.58. Then, the Consistency Ratio (CR) = CI/RI = 0.0275/0.58 = 0.474% □0.10% (accepted ratio) and A is a consistent matrix.

Weighting approach for solving TLP problem achieves the non-dominated solution and according to the related increased decimal points for objective functions weights, the number of weight vectors increase more and more. Example 1, shows that while varying the infinite number of different weight vectors; the solution remains more or less the non-dominated one where, for one decimal point, 66 weight vectors are set as in table (2) and it is increasing more and more for two decimal points as shown in table (3) and so on.

w_1	w_2	w_3	x_1, x_2, x_3	$P(w)$	f_1, f_2, f_3	#
0.0	0.0	1.0	1, 1, 1	1.0	6, 1, 1	11
	0.1	0.9	1, 1, 1	1.0	6, 1, 1	
	….	….	….	….	….	
	0.9	0.1	1, 1, 1	1.0	6, 1, 1	
	1.0	0.0	1, 1, 1	1.0	6, 1, 1	
0.1	0.0	0.9	1, 1, 1	1.5	6, 1, 1	10
	0.1	0.8	1, 1, 1	1.5	6, 1, 1	
	….	….	….	….	….	
	0.8	0.1	1, 1, 1	1.5	6, 1, 1	
	0.9	0.0	1, 1, 1	1.5	6, 1, 1	
….	….	….	….	….	….	….
0.5	0.0	0.5	1, 1, 1	3.5	6, 1, 1	6
	0.1	0.4	1, 1, 1	3.5	6, 1, 1	
	0.2	0.3	1, 1, 1	3.5	6, 1, 1	
	0.3	0.2	1, 1, 1	3.5	6, 1, 1	
	0.4	0.1	1, 1, 1	3.5	6, 1, 1	
	0.5	0.0	1, 1, 1	3.5	6, 1, 1	
….	….	….	….		….	
0.9	0.0	0.1	1, 0.5, 0.5	5.9	6.5, 0.5, 0.5	2
	0.1	0.0	1, 0.5, 0.5	5.9	6.5, 0.5, 0.5	
1.0	0.0	0.0	1, 0.5, 0.5	6.5	6.5, 0.5, 0.5	1

Table (2): 66 different weighted combinations and solutions for one decimal value for weights

The individual problems for each DM are calculated in his level subject to the set of constraints to determine his optimal solution as;

f_1^*=8.5 at (1.5, 0, 0.5),
f_2^*=1 at (0, 1, 0) and
f_3^*=0.5 at (0, 0.5, 0.5), (0.5, 1, 0.5) or (1.5, 0, 0.5).

Lower and upper-bounds are arbitrary values or it is assumed by its individual optimal solution in all levels problems. In other words, lower and upper-bounds are represented by obtained minimum & maximum values from the individual problems in all levels. Changes in lower and upper-bounds values reflect the flexibility in the approach that translates the preferences of all decision makers.

Assuming that the lower and upper-bounds provided for the decision variables by DMs are as follows; $0 \le x_1 \le 1.5$, $1 \le x_2 \le 2$ and x_3 =0.5 or all variables in the closed interval [0, 1], with note that these bounds are set from the individual solutions for each level. Hence, the weighting problem is therefore formulated as:

$$P(w) = \max \sum_{p=1}^{3} w_p f_p(X)$$
$$= \max 0.58\,(7x_1 + 3x_2 - 4x_3) + 0.24\,(x_2) + 0.18\,(x_3)$$
$$\begin{aligned} \text{st:} \quad & x_1 + x_2 + x_3 \le 3, \\ & x_1 + x_2 - x_3 \le 1, \\ & x_1 + x_2 + x_3 \ge 1, \\ & -x_1 + x_2 + x_3 \le 1, \\ & 0 \le x_1 \le 1.5, \\ & 1 \le x_2 \le 2, \\ & x_3 = 0.5, \\ & x_1, x_2, x_3 \ge 0. \end{aligned}$$

w_1	w_2	w_3	x_1, x_2, x_3	$P(w)$	f_1, f_2, f_3	#
0.56	0.00	0.44	1, 0.5, 0.5	3.86	6.5, 0.5, 0.5	45
	0.01	0.43	1, 0.5, 0.5	3.86	6.5, 0.5, 0.5	
	….	….	….	….	….	
	0.43	0.01	1, 0.5, 0.5	3.86	6.5, 0.5, 0.5	
	0.44	0.00	1, 0.5, 0.5	3.86	6.5, 0.5, 0.5	
0.57	0.00	0.43	1, 0.5, 0.5	3.92	6.5, 0.5, 0.5	44
	0.01	0.42	1, 0.5, 0.5	3.92	6.5, 0.5, 0.5	
	….	….	….	….	….	
	0.42	0.01	1, 0.5, 0.5	3.92	6.5, 0.5, 0.5	
	0.43	0.00	1, 0.5, 0.5	3.92	6.5, 0.5, 0.5	
0.58	0.00	0.42	1, 0.5, 0.5	3.98	6.5, 0.5, 0.5	43
	0.01	0.41	1, 0.5, 0.5	3.98	6.5, 0.5, 0.5	
	….	….	….	….	….	
	0.23	0.19	1, 0.5, 0.5	3.98	6.5, 0.5, 0.5	
	0.24	0.18	1, 0.5, 0.5	3.98	6.5, 0.5, 0.5	
	0.25	0.17	1, 0.5, 0.5	3.98	6.5, 0.5, 0.5	
	….	….	….	….	….	
	0.41	0.01	1, 0.5, 0.5	3.98	6.5, 0.5, 0.5	
	0.42	0.00	1, 0.5, 0.5	3.98	6.5, 0.5, 0.5	
0.59	0.00	0.41	1, 0.5, 0.5	4.04	6.5, 0.5, 0.5	42
	0.01	0.40	1, 0.5, 0.5	4.04	6.5, 0.5, 0.5	
	….	….	….	….	….	
	0.40	0.01	1, 0.5, 0.5	4.04	6.5, 0.5, 0.5	
	0.41	0.00	1, 0.5, 0.5	4.04	6.5, 0.5, 0.5	
0.60	0.00	0.40	1, 0.5, 0.5	4.10	6.5, 0.5, 0.5	41
	0.01	0.39	1, 0.5, 0.5	4.10	6.5, 0.5, 0.5	
	….	….	….	….	….	
	0.39	0.01	1, 0.5, 0.5	4.10	6.5, 0.5, 0.5	
	0.40	0.00	1, 0.5, 0.5	4.10	6.5, 0.5, 0.5	

Table (3): detailed information of set of non-dominated solutions for two decimal values for weights

A non-dominated solution set is generated throughout parametrically varying the weights, then the TLP problem's solution, obtained in two cases;

- Case (1): with unequal lower and upper-bounds

X=(x_1, x_2, x_3) = (1, 0.5, 0.5), f_1, f_2, f_3 = 6.5, 0.5, 0.5 respectively, with $P(w)$=3.98.

- Case (2): with equal lower and upper-bounds

X=(x_1, x_2, x_3) = (0.5, 1, 0.5), f_1, f_2, f_3 =4.5, 1, 0.5 respectively, with $P(w)$=2.94.

Problem's solution is calculated using some approaches such as; the K^{th}-best algorithm in [17], the fuzzy approach in [23], and the interactive approach used in [32]. A brief comparative study for the solutions is presented using the three mentioned approaches beside to the weighting approach for dealing with the TLP problem as one of the famous models for MLP problems. Then, the DMs satisfaction levels in both weighting approach and K^{th}-best algorithm can be calculated as the ratio of the optimal solution for the complete TLP problem over the optimal solution for the individual problem for each DM.

In the fuzzy approach [23], DMs on the upper-levels (only) determine the tolerance values for their decision variables and assuming that x_1, x_2 should be around 0.95, 0.58 respectively, with negative and positive-side tolerances (0.95, 0) and (0.58, 0), respectively. The interactive fuzzy approach in [32] provides a solution concept for TLP problems in full cooperative decision making situations to obtain satisfactory solutions where fuzzy membership functions are built for f_i where i=1, 2, 3 with determined minimal satisfaction levels. Lower and upper-bounds are set for DMs's overall satisfaction levels if needed and it is calculated from the ratio $\mu_{i+1}(f_{i+1})/\mu_i(f_i)$.

As soon as expected, upper-levels DMs start their initial minimal satisfactory levels =1.0, and suppose that lower and upper-bounds of the ratio of overall satisfactory degrees may be set as [0.5, 1.0]. Table (4) shows the results of example 1, using the weighting approach in two different cases for the lower and upper-bounds of decision variables besides other three different approaches. The results include the satisfaction level for all DMs represented by the degrees of the membership functions. Note that the achieved compromised weighted solutions may be the same obtained by other methods or around them.

	Weighting approach		Zhang et al. [17]	Shih et al. [23]	Sakawa et al. [32]
	Case (1)	Case (2)			
x_1	0.5000	1.0000	0.5000	0.9200	0.8450
x_2	1.0000	0.5000	1.0000	0.5800	0.6500
x_3	0.5000	0.5000	0.5000	0.5000	0.5000
f_1	4.5000	6.5000	4.5000	6.1800	5.8650
f_2	1.0000	0.5000	1.0000	0.5800	0.6500
f_3	0.5000	0.5000	0.5000	0.5000	0.5000
$\mu(f_1)$	0.5294	0.7647	0.5294	1.0000	0.6900
$\mu(f_2)$	1.0000	0.5000	1.0000	1.0000	0.6500
$\mu(f_3)$	1.0000	1.0000	1.0000	1.0000	1.0000

Table (4): example 1 results using the weighted approach and other different approaches

Example 2: Consider the following numerical TLP problem [23, 27, 32];

$$\max_{x_1,x_2} f_1(X) = 7x_1 + 3x_2 - 4x_3 + 2x_4$$

,

where x_3 and x_4 solve:

$$\max_{x_3} f_2(X) = x_2 + 3x_3 + 4x_4 ,$$

where x_4 solves:

$$\max_{x_4} f_3(X) = 2x_1 + x_2 + x_3 + x_4$$

st: $x_1+ x_2+ x_3+x_4\le 5,$
$x_1+ x_2 - x_3-x_4\le 2,$
$x_1+ x_2+ x_3 \ge 1,$
$-x_1+ x_2+ x_3\le 1,$
$x_1- x_2+ x_3+2x_4\le 4,$
$x_1+2x_3+3x_4\le 3,$
$x_4\le 2,$
$x_1, x_2, x_3, x_4 \ge 0.$

The optimal solutions of the individual problems will be as;
f_1^*=16.25 at (2.25, 0, 0, 0.25),
f_2^*=5 at (1, 0, 1, 0) and
f_3^*=5 at (1.33, 1.5, 0.83, 0).

Given, the DMs preferences to set the pairwise comparison matrix, *B* as:

$$B = \begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \end{array} \begin{array}{c} \begin{array}{ccc} f_1 & f_2 & f_3 \end{array} \\ \begin{bmatrix} 1 & 3 & 0.5 \\ 0.33 & 1 & 0.2 \\ 2 & 5 & 1 \end{bmatrix} \end{array}$$

The following priority vector $W=(w_1,w_2,w_3)^T$ = (0.31, 0.11, 0.58), the Consistency Ratio (CR) = 0.0072 □0.10, then, *B* is a consistent matrix.

Assuming that the lower and upper-bounds are as follows; $1\le x_1 \le 2.25$, $0 \le x_2 \le 1.5$, $0 \le x_3 \le 1$ and $0\le x_4\le 0.25$ or all variables in the closed interval [0, 2.25]. Hence, the weighting problem objective function is:

$$P(w) = \max \sum_{p=1}^{3} w_p f_p(X) = \max\ 3.33x_1+1.62x_2-0.33x_3+1.64x_4$$

Where, in the two cases the problem solutions are identical as:
$X=(x_1, x_2, x_3, x_4)$ = (2.25, 0, 0, 0.25), f_1, f_2, f_3 = 16.25, 1, 4.75 respectively, with $P(w)$=7.9025. Table (5) shows the results of example 2, using the weighting approach besides other three different approaches.

	Weighting approach Cases (1 & 2)	Sinha [27]	Shih et al. [23]	Surapati et al. [32]
x_1	2.2500	1.5900	1.6400	0.8570
x_2	0.0000	1.0800	0.9800	1.8570
x_3	0.0000	0.6200	0.6800	0.0000
x_4	0.2500	0.0600	0.0000	0.7140
f_1	16.2500	12.0100	11.7000	13.0000
f_2	1.0000	3.18000	3.0200	4.7100
f_3	4.7500	4.9400	4.9400	4.2800
$\mu(f_1)$	1.0000	0.0680	1.0000	0.7110
$\mu(f_2)$	0.2000	0.0080	1.0000	0.9280
$\mu(f_3)$	0.9500	0.0000	1.0000	0.7600

Table (5): example 2 results using the weighted approach and other different approaches

VI. Conclusion

AHP gives the relative weights to form a single objective function while converting the hierarchical system into SOP by finding proper weights so that objective functions of all levels can be combined into a super objective function. In this article, a compromise weighted approach is applied to solve MLP problem with testing the applicability of the weighting approach for the MLP case, checking the outputs accuracy of applying the weighting approach in MLP problems by comparing the results with other applied approaches and the approach was applied on the MLP problem throughout two different cases; equal and unequal (individual values) lower and upper-bounds for the decision variables values for each decision maker in his level. The approach can be applicable as satisfactory or an approximation solution for; ML fractional programming problems and nonlinear MLP problems. The proposed approach is effective tool for finding a satisfactory (near optimal) solution where it can produces results which are very close or improved to the results obtained by most of the other existing methods with observing that even though varying the weight vectors, the solutions remain more or less the same. This approach determines a set of non-dominated solutions and unique characteristic of a MLP problem is with this approach reflected by allowing each DM to determine the importance of his objective regarding to other's and to assign lower and upper-bounds for the decision variables under his control. These bounds are additional constraints. From this set, the DM chooses the most satisfying solution, making implicit trade-offs between all objective functions on the different levels based on some previously un-indicated or non-quantifiable criteria.

REFERENCES

[1] G. Anandalingam, A Mathematical Programming Model of Decentralized Multi-Level Systems, Journal of Operational Research Society, 39 (11) (1988) 1021–1033.

[2] E. Anderson and N. Joglekar, A Hierarchical Product Development Planning Framework, Production and Operations Management, 14 (3) (2005) 344-361.
[3] O. Ben-Ayed, Bi-Level Linear Programming, Computers and Operations Research, 20 (1993) 485–501.
[4] J. F. Bard, An Algorithm for Solving the Bi-Level Programming Problem, Mathematics of Operation Research, 8 (2) (1983) 260–270.
[5] J. F. Bard, Optimality Conditions for the Bi-Level Programming Problem, Naval Research Logistics Quarterly, 31 (1984) 13–26.
[6] Y.J. Lai, Hierarchical Optimization: a Satisfactory Solution, Fuzzy Sets and Systems, 77 (1996) 321–335.
[7] J. F. Bard, Geometric Algorithmic Developments for a Hierarchical Planning Problem, European Journal of Operational Research, 19 (1985) 372-383.
[8] H.P. Benson, On the Structure and Properties of a Linear Multi-level Programming Problem, Journal of Optimization Theory and Applications, 60 (1989) 353–373.
[9] W. Candler, M.H. Karwan, A Linear Two Level Programming Problem, Computers and Operation Research, 9 (1) (1982) 59–76.
[10] E.S. Lee, H.S. Shih, Fuzzy and Multi-Level Decision Making, 1st edition, London: Springer, 2001, Chapters 1, 2.
[11] J. Falk, A Linear Max-Min Problem, Mathematical Programming, 5 (1973) 169-181.
[12] O. Ben-Ayed, C.E. Blair, Computational Difficulties of Bi-level Linear Programming, Operations Research, 38 (1988) 556–560.
[13] G. Anandalingam and T.L. Friesz, Hierarchical Optimization: an Introduction, Annals of Operations Research, 34 (1) (1992) 1–11.
[14] H.S. Shih, U.P. Wen, E.S. Lee, K.M. Lan and H.C. Hsiao, A Neural Network Approach to Multi-objective and Multi-level Programming Problems, Computers and Mathematics with Applications, 48 (2004) 95-108.
[15] W. F. Bialas and M.H. Karwan, Two Level Linear Programming, Management Science, 30 (1984) 1004–1020.
[16] H. I. Calvete and C. Gale, Solving Linear Fractional Bi-Level Programs, Operations Research Letters, 32 (2004) 143-151.
[17] G. Zhang, J. Lu, J. Montero and Y. Zeng, Model, Solution Concept, and K^{th}-Best Algorithm for Linear Tri-Level Programming, Information Sciences, 130 (2010) 481-492.
[18] J. Fortuny-Amat and B. McCarl, A Representation and Economic Interpretation of a Two-Level Programming Problem, the Journal of the Operational Research Society, 32 (9) (1981) 783–792.
[19] J. F. Bard and J. E. Falk, An Explicit Solution to the Multi-Level Programming Problems, Computers and Operations Research, 9 (1982) 77–100.
[20] L. Vicente, G. Savard and J. J. J´udice, Discrete Linear Bi-Level Programming Problem, Journal of Optimization Theory and Applications, 89 (1996) 597–614.
[21] P. Lasunon, J. Wetweerapong and T. Remsungnen, A New Algorithm for Solving Bi-Level Linear Programming Problems, Proceeding of the 16th Annual Meeting in Mathematics, Thailand (2011) 59-66.
[22] P. Lasunon and T. Remsungnen, A New Algorithm for Solving Tri-Level Linear Programming Problems, Int. J. Pure Appl. Sci. Technol., 7 (2) (2011) 149-157.
[23] H.S. Shih, Y.J. Lai and E.S. Lee, Fuzzy Approach for Multi-Level Programming Problems, Computers and Operations Research, 23 (1) (1996) 73–91.
[24] R. E. Bellman and L.A. Zadeh, Decision-Making in a Fuzzy Environment, Manage. Sci., 17 (1970) 141–164.
[25] H.S. Shih and E.S. Lee, Compensatory Fuzzy Multiple Level Decision Making , Fuzzy Sets and Systems, 14 (2000) 71–87.
[26] S. Sinha, Fuzzy Mathematical Programming Applied to Multi-Level Programming Problems, Computers and Operations Research, 30 (2003)a 1259–1268.
[27] S. Sinha, Fuzzy Programming Approach to Multi-Level Programming Problems, Fuzzy Sets and Systems, 136 (2003)b 189–202.
[28] B. B. Pal and B.N. Moitra, A Fuzzy Goal Programming Procedure for Solving Quadratic Bi-Level Programming Problems, Int. J. Intell. Syst., 18 (5) (2003) 529–540.
[29] M.S. Osman, M.A. Abo-Sinna, A.H. Amer and O.E. Emam, A multi-level Nonlinear Multi-Objective Decision Making under Fuzziness, J. Appl. Math. Comp., 153 (2004) 239–252.
[30] P. Surapati and K. R. Tapan, Fuzzy Goal Programming Approach to Multi-level Programming Problems, European Journal of Operational Research, 176 (2007) 1151–1166.
[31] X. Shi and H. Xia, Interactive Bi-Level Multi-Objective Decision Making, Journal of the Operational Research Society, 48 (9) (1997) 943-949.
[32] M. Sakawa, I. Nishizaki and Y. Uemura, Interactive Fuzzy Programming for Multi-Level Linear Programming Problems, Computers & Mathematics with Applications, 36 (2) (1998) 71–86.
[33] M. Sakawa, I. Nishizaki and M. Hitaka, Interactive Fuzzy Programming for Multi-Level 0–1 Programming Through Genetic Algorithms, European Journal of Operational Research, 114 (3) (1999) 580–588.
[34] M. Sakawa and I. Nishizaki, Interactive Fuzzy Programming for Two-Level Linear Fractional

Programming Problems, Fuzzy Sets and Systems, 119 (2001) 31-40.

[35] M. Sakawa and I. Nishizaki, Interactive Fuzzy Programming for Decentralized Two-Level Linear Programming Problems, Fuzzy Sets and Systems, 125 (2002) 301-315.

[36] S. Mishra, Weighting Method for Bi-level Linear Fractional Programming Problems, European Journal of Operational Research, 183 (2007) 296–302.

[37] W. Guangmin, Z. Gao and Z. Wan, A Global Optimization Algorithm for Solving the Bi-Level Linear Fractional Programming Problem, Computers & Industrial Engineering, 63 (2012) 428–432.

[38] L. Chen and H. Chen, Considering Decision Decentralizations to Solve Bi-level Multi-Objective Decision-Making Problems: A fuzzy approach, Applied Mathematical Modeling, In Press, Corrected Proof, Available online March 2013.

[39] I. A. Baky and M. A. Abo-Sinna, TOPSIS for Bi-level MODM Problems, Applied Mathematical Modeling, 37 (3) (2013) 1004-1015.

[40] Y. Jiang, X. Li, C. Huang and X. Wu, Application of Particle Swarm Optimization Based on CHKS Smoothing Function for Solving Nonlinear Bi-level Programming Problem, Applied Mathematics and Computation, 219 (9) (2013) 4332-4339.

[41] M. Sakawa and T. Matsui, Interactive Fuzzy Programming for Stochastic Two-Level Linear Programming Problems through Probability Maximization, Artificial Intelligence Research, 2 (2) (2013) 109-124.

[42] T. L. Saaty, The Analytic Hierarchy Process, McGraw-Hill, New York, 1980.

[43] T. L. Saaty, Multi-criteria Decision Making: The Analytic Hierarchy Process, RWS Publications, Pittsburgh, PA, 1990.

[44] T. L. Saaty, Fundamentals of Decision Making and Priority Theory with the Analytic Hierarchy Process, RWS Publications, Pittsburgh, PA, 1994.

[45] S. Zanakis, T. Mandakovic, S. Gupta, S. Sahay and S. Hong, A Review of Program Evaluation and Fund Allocation Methods within the Service and Government Sectors, Socio-Economic Planning Science, 29 (1) (1995) 59-79.

[46] V. Belton and T. Gear, On a Shortcoming of Saaty's Method of Analytic Hierarchies, Omega, 11 (3) (1983) 228-230.

[47] V. Belton and T. Gear, Feedback: the Legitimacy Of Rank Reversal- a Comment, Omega, 13 (3) (1985)143-144.

[48] J. Barzilai, Deriving Weights from Pairwise Comparison Matrices, Journal of the Operational Research Society, 48 (1997)1226-1232.

[49] V. Chankong and Y. Y. Haimes, Multi-Objective Decision Making: Theory and Methodology, Elsevier, North-Holland, 1983.