RESEARCH ARTICLE OPEN ACCESS

Vibration Analysis of CSSF and SSFC Panel

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ABSTRACT
The paper presents a theoretical formulation based on Ibearugbulem's shape function and application of Ritz method. In this study, the free vibration of simply supported panel with one free edge was analyzed. The Ibearugbulem's shape function derived was substituted into the potential energy functional, which was minimized to obtain the fundamental natural frequency. This research focused on aspect ratio from 0.1 to 2.0 with 0.1 increments. For aspect ratio of 1, values of non dimensional natural frequency parameter, λ is 17.35. The values of λ obtained were compared with those of previous research. The comparisons indicates that no significant difference exist between the values of non dimensional natural frequency parameter from this present study and those from previous research works.
Keywords: *fundamental natural frequency, CSSF panel, SSFC panel, free panel, Ritz method.*

I. INTRODUCTION

The exact solution on the vibration of rectangular panels with one free edge has been difficult to attain. Over the years, problems have been treated by the use of trigonometric series as the shape function of the deformed panel or by using method of superposition. Research has been carried out on the problems from equilibrium approach and others solve the problems from energy and numerical approaches. However, no matter the approach used, trigonometric series has been the most dominant shape function used in energy methods. The analysis of the free vibration of panels was well documented by Leissa [1] and includes a variety of geometry and boundary conditions using trigonometric series. Gorman [2] solved problem on free vibration of rectangular panels with different boundary conditions, aspect ratio and Poisson ratios using method of superposition.

II. FORMULATION OF FUNDAMENTAL NATURAL FREQUENCY

Chakraverty [3] gave the maximum kinetic energy functional as

$$K_{max} = \frac{\lambda^2}{2}\iint \rho h W^2(x,y)\partial x \partial y \qquad (1)$$

Making use of the non dimensional parameters, R and Q in x and y coordinates, equation (1) becomes

$$= \frac{ab\lambda^2 \rho h}{2}\iint w^2 \partial R \partial Q \qquad (2)$$

Where ρ is the weight per unit area of the plate, h is the plate thickness and λ is the frequency of panel vibration. The maximum strain energy functional for a thin rectangular isotropic plate under vibration was given by Ibearugbulem [4] as follow:

$$U_{max} = \frac{D}{2}\iint [(W''^{x})^2 + 2(W''^{xy})^2 + (W''^{y})^2]\partial x \partial y \qquad (3)$$

Adding equations (2) and (3) gave the total potential energy functional of rectangular panel under lateral vibration as:

$$\prod max = \frac{aDb}{2}\iint \left[\frac{1}{a^4}(W''^{R})^2 + \frac{2}{a^2b^2}(W''^{RQ})^2 + \frac{1}{b^4}(W''^{Q})^2\right]\partial R \partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2 \partial R \partial Q \qquad (4)$$

Factorizing out b/a^3 gave:

$$\prod max = \frac{Db}{2a^3}\iint \left[(W''^{R})^2 + \frac{2a^2}{b^2}(W''^{RQ})^2 + \frac{a^4}{b^4}(W''^{Q})^2\right]\partial R \partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2 \partial R \partial Q \qquad (5)$$

If the aspect ratio, P = a/b, then:

$$\prod max = \frac{Db}{2a^3}\iint [(W''^{R})^2 + 2P^2(W''^{RQ})^2 + P^4(W''^{Q})^2]\partial R \partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2 \partial R \partial Q \qquad (6)$$

If the aspect ratio, P = b/a, then:

$$\prod max = \frac{Db}{2a^3}\iint \left[(W''^{R})^2 + \frac{2}{P^2}(W''^{RQ})^2 + \frac{1}{P^4}(W''^{Q})^2\right]\partial R \partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2 \partial R \partial Q \qquad (7)$$

From equation (4), if a/b^3 is factorized out then:

$$\prod max = \frac{Da}{2b^3}\iint\left[\frac{b^4}{a^4}(W''^{R})^2 + \frac{2b^2}{a^2}(W''^{RQ})^2 + (W''^{Q})^2\right]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \qquad (8)$$

If the aspect ratio, P = a/b, then:

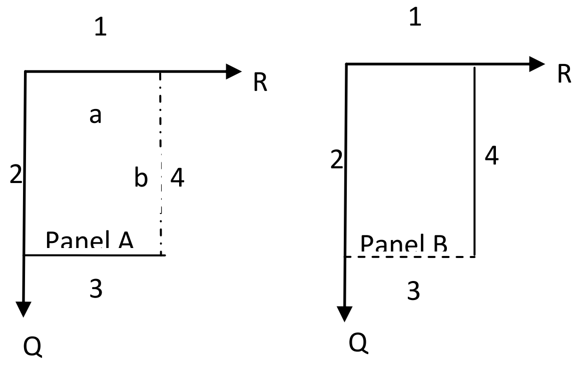
$$\prod max = \frac{Da}{2b^3}\iint\left[\frac{1}{P^4}(W''^{R})^2 + \frac{2}{P^2}(W''^{RQ})^2 + (W''^{Q})^2\right]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \qquad (9)$$

If the aspect ratio, P = b/a, then:

$$\prod max = \frac{Da}{2b^3}\iint [P^4(W''^{R})^2 + 2P^2(W''^{RQ})^2 + (W''^{Q})^2]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \qquad (10)$$

III. IBEARUGBULEM'S SHAPE FUNCTION

Displacement Function for CSSF and SSFC Panel.



Boundary Condition	**Boundary condition**
$W(R=0)=0, W(Q=0)=0$	$W(R=0)=0, W(Q=0)=0$
$W''(R=0)=0, W'(Q=0)=0$	$W'(R=0)=0, W''(Q=0)=0$
$V^R(R=1)=0, W''(Q=1)=0$	$W''(R=1)=0, V^Q(Q=1)=0$

$W^Q = b_4(1.5Q^2 - 2.5Q^3 + Q^4)$ (11)

$W^R = a_4(8R - 4R^3 + R^4)$ (12)

$W = A(8R - 4R^3 + R^4)(1.5Q^2 - 2.5Q^3 + Q^4)$ (13)

W, W^R, W^Q, V^R, V^Q are general deflection shape function, partial shape functions in x and y axes, and partial shear force functions in x and y axes respectively. In line with Ibearugbulem [4], the equation (13) is the displacement function for CSSF panel. In the same vein, the deflection equation for SSFC panel was written as:

$W = A(1.5R^2 - 2.5R^3 + R^4)(8Q - 4Q^3 + Q^4)$

IV. TOTAL POTENTIAL ENERGY FOR CSSF AND SSFC PANEL.

Integrating the squares of the differential equation (13) with respect to R and Q.

$$\int_0^1\int_0^1 (W''^{R})^2\partial R\partial Q = A^2(76.8)(1.587301587\times10^{-3}) = 0.121904761619\,A^2$$

$$\int_0^1\int_0^1 (W'^{R'Q})^2\partial R\partial Q = A^2(31.08571429)(0.019047619051) = 0.5921088436\,A^2$$

$$\int_0^1\int_0^1 (W''^{Q})^2\partial R\partial Q = A^2(12.5968254)(0.8) = 10.07746032\,A^2$$

$$\int_0^1\int_0^1 (W)^2\partial R\partial Q = A^2(12.5968254)(1.587301587\times10^{-3}) = 0.0199946095\,A^2$$

Substituting these integral values into functional (6), (7), (9) and (10), respectively gave:

$$\prod max = \frac{DbA^2}{2a^3}[0.12190476 + 1.18421769P^2 + 10.077460P^4] - \frac{ab\lambda^2\rho hA^2}{2}[0.01999496] \qquad (14)$$

$$\prod max = \frac{DbA^2}{2a^3}\left[0.12190476 + \frac{1.18421769}{P^2} + \frac{10.077460}{P^4}\right] - \frac{ab\lambda^2\rho hA^2}{2}[0.01999496] \qquad (15)$$

$$\prod max = \frac{DaA^2}{2b^3}\left[\frac{0.12190476}{P^4} + \frac{1.18421769}{P^2} + 10.077460\right] - \frac{ab\lambda^2\rho hA^2}{2}[0.01999496] \qquad (16)$$

$$\prod max = \frac{DaA^2}{2b^3}[0.12190476P^4 + 1.18421769P^2 + 10.077460] - \frac{ab\lambda^2\rho hA^2}{2}[0.01999496] \qquad (17)$$

Equations (14), (15), (16), (17) were minimized by $\frac{\partial \prod max}{\partial A} = 0$. The values of fundamental natural frequency parameter, k were used in plotting the curves of figures 1 to 8 that show the graphical model of fundamental natural frequency values, for both X-X and Y-Y axes.

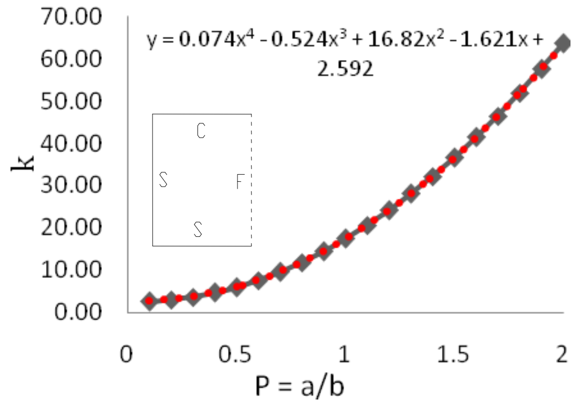


Figure 1: Graph of CSSF Panel P = a/b (note: y = k and x = aspect ratio, P

The free edge of the panel is on X – X axis. With respect to edge length, a, the natural frequency increases as the aspect ratio increases. The line of best fit of the curve of figure 1 is the polynomial equation curve is represented by $y = 0.074x^4 - 0.524x^3 + 16.82x^2 - 1.621x + 2.592$

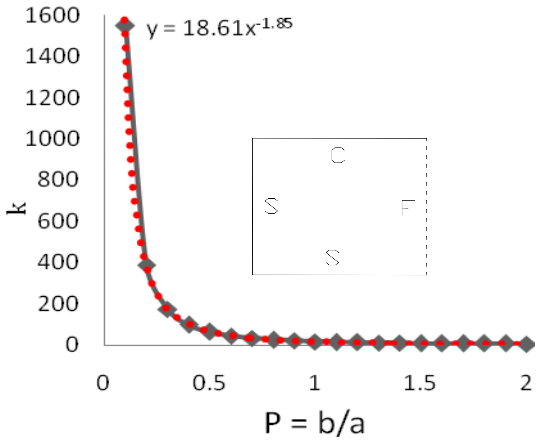


Figure 2: Graph of CSSF Panel P = b/a (note: y = k and x = aspect ratio, p)

The free edge of the panel is on X – X axis. With respect to length (a), the natural frequency decreases as the aspect ratio increases. The line of best fit of the curve of figure 2 is the power equation curve is represented by $y = 18.61x^{-1.85}$

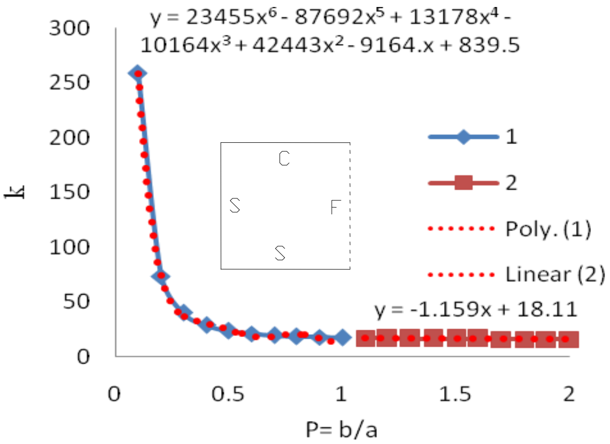


Figure 3: Graph of CSSF Panel P = a/b (note: y = k and x = aspect ratio, p)

The free edge of the panel is on X – X axis. With respect to width, b, the natural frequency decreases as the aspect ratio increases,

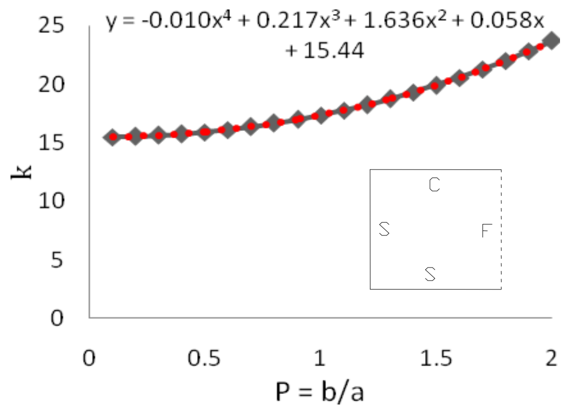


Figure 4: Graph of CSSF Panel P = b/a (note: y = k and x = aspect ratio, p)

The free edge of the panel is on X – X axis. With respect to width (b), the natural frequency increases as the aspect ratio increases, the polynomial equation curve is represented by $y = -0.143x^4 + 1.856x^3 - 0.006x^2 + 0.540x + 22.39$

V. SIMPLE - SIMPLE - FREE - CLAMPED SUPPORTED EDGE PANEL

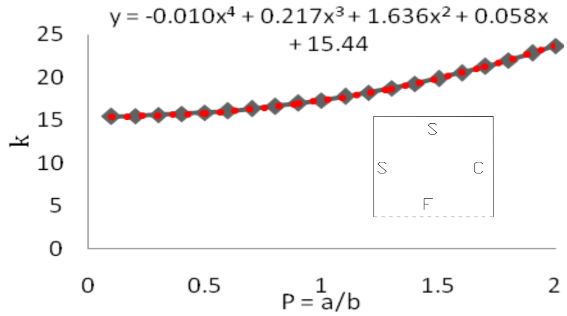


Figure 5: Graph of SSFC Panel P = a/b (note: y = k and x = aspect ratio, p)

The free edge of the panel is on Y – Y axis. With respect to edge length, a, the natural frequency increases as the aspect ratio increases. The line of best fit of the curve of figure 5 is the polynomial equation curve is represented by $y = -0.010x^4 + 0.217x^3 + 1.636x^2 + 0.058x + 15.44$

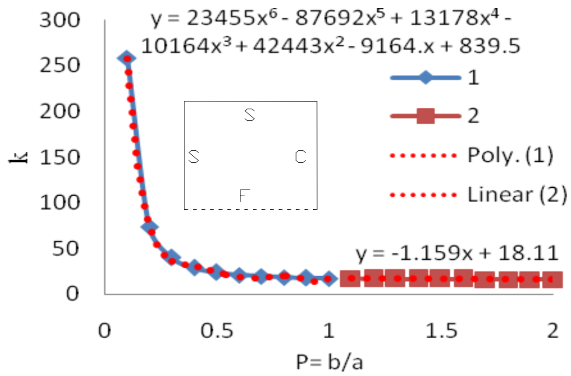


Figure 6: Graph of SSFC Panel P = b/a (note: y = k and x = aspect ratio, p)

The free edge of the panel is on Y-Y axis. With respect to edge length, a, the natural frequency decreases as the aspect ratio increases,

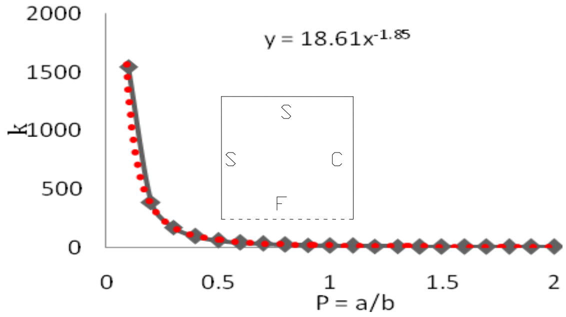


Figure 7: Graph of SSFC Panel P = a/b (note: y = k and x = aspect ratio, p)

The free edge of the panel is on Y-Y axis. With respect to width, b, the natural frequency decreases as the aspect ratio increases. The line of best fit of the curve of figure 7 is the power equation curve is represented by $y = 18.61x^{-1.85}$

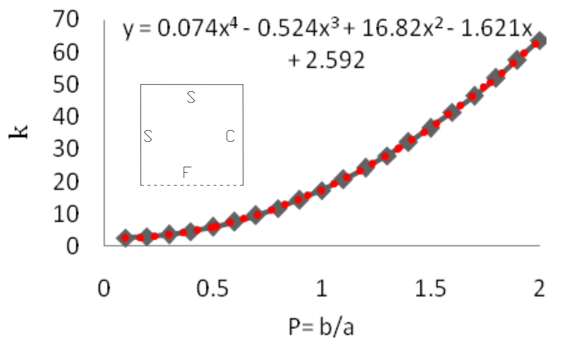


Figure 8: Graph of SSFC Panel P = b/a (note: y = k and x = aspect ratio, p)

The free edge of the panel is on Y-Y axis. With respect to width b, the natural frequency increases as the aspect ratio increases. The line of best fit of the curve of figure 8 is the polynomial equation curve is represented by $y = 0.074x^4 - 0.524x^3 + 16.82x^2 - 1.621x + 2.592$

Comparison of Results for SSFC Panel

The values of λ of SSFC panel from previous researchers were compared with those of the present study. The percentages of this comparison were presented on tables 1 to 3. In table 1, the results of SSFC panel from Lessia [1] and the present result ranges from 0.61% to 7.31% given an average difference of 2.77% which is indicates no significant difference. Tables 2 and 3 displayed the difference between Gorman [2] and present result with aspect ratios P = a/b and P = b/a respectively. The average difference is/are 2.50% and 9.64% respectively. These percentage differences showed that no significant difference exist between the values of λ from the present study and Gorman [2].

Table 1: k values for SSFC Panel

$\lambda = \frac{K}{a^2}\sqrt{\frac{D}{\rho h}}$			
$P = \frac{a}{b}$	Present	Leissa	% diff
0.4	15.7439	15.649	0.61
0.6	16.1160	16.067	0.30
1.0	17.3477	16.685	2.86
1.5	19.8959	18.540	7.31

Table 2: k value for SSFC Panel

$\lambda = \frac{K}{a^2}\sqrt{\frac{D}{\rho h}}$			
$P = \frac{b}{a}$	Present	Gorman	% diff
1.0	17.3477	16.70	3.88
1.5	16.2751	15.94	2.10
2.0	15.9105	15.67	1.53

Table 3: k values for SCFS Panel

$\lambda = \frac{K}{b^2}\sqrt{\frac{D}{\rho h}}$			
$P = \frac{a}{b}$	Present	Gorman	% diff
1.0	17.3477	16.70	3.88
1.5	8.8426	8.137	8.67
2.0	5.9199	5.087	16.37

S. Chakraverty [3] worked on aspect ratio P = 1, with λ = 16.811, giving a difference of 3.19% to the present value.

VI. CONCLUSION

The study obtained new energy functional based on Ritz's total potential energy and Taylor series deflection equation for panels of various support conditions.

The study came up with a new relationship between fundamental natural frequency and aspect ratios.

The study came up with graphical models which can be used in place of the primary equations.
The study has created a new data base of fundamental natural frequencies for different panel and aspect ratio for designers.

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